# Topological Defects: Cosmic Strings

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# Introduction

Cosmic string creation: two toy models

Deriving a scaling solution

Cosmic strings as a source of geometric perturbations

Cosmic strings genericity in SUSY GUT

Gravitational effects of cosmic strings

Possible observations of cosmic strings

Conclusion

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# Global strings

- Start with a complex scalar field $\phi(x)$ and

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi), \qquad V = \frac{1}{2}\lambda(|\phi|^2 - \frac{1}{2}\eta^2)^2$$

- a global U(1) symmetry with $\phi \to \phi e^{i\alpha}$
- the Euler-Lagrange equations become

$$[\partial^2 + \lambda(|\phi|^2 - \frac{1}{2}\eta^2)]\phi = 0$$

# Global strings 2

- Ground state $\phi = (\eta/\sqrt{2})e^{i\alpha_0}$ breaks U(1) symmetry
- Mass of scalar particle becomes $m_s^2 = \lambda\eta^2$
- Static solution with non-zero energy density
- Ansatz

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s\rho)\mathrm{e}^{in\psi}$$

# Global strings 3

- Introduce $\xi \equiv m_s \rho$, then

$$f'' + \frac{1}{\xi} f' - \frac{n^2}{\xi^2} f - \frac{1}{2}(f^2 - 1) f = 0$$

$$\xi \to 0 \quad f \to 0 \qquad \text{and} \qquad \xi \to \infty \quad f \to 1$$

- Writing $f = 1 - \delta f$, $\delta f \sim n^2/\xi^2$, we find

$$\mathcal{E} = |\dot{\phi}|^2 + |\nabla \phi|^2 + V(\phi)$$

# Local strings

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

- where $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
- Now the field equations become

$$[D^2 + \lambda(|\phi|^2 - \frac{1}{2}\eta^2)]\phi = 0,$$

$$\partial_\nu F^{\mu\nu} + ie(\phi^* D^\mu \phi - D^\mu \phi^* \phi) = 0$$

# Local strings 2

- Still scalar particle with mass $m_s = \sqrt{\lambda}\eta$
- Nambu-Goldstone boson incorporated into vector field, gains mass $m_v = e\eta$
- Choose radial gauge $A_\rho = 0$

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s\rho)\mathrm{e}^{in\psi}, \qquad A^i = \frac{n}{e\rho}\hat{\psi}^i a(m_v\rho)$$

# Local strings 3

- For large $\xi = m_s \rho$ we now get

$$f \sim 1 - f_1 \xi^{-1/2} \exp(-\beta\xi) \qquad a \sim 1 - a_1 \xi^{1/2} \exp(-\xi)$$

- with $\beta = m_s / m_v$.
- Note that now the energy is finite and we find for the energy per unit length

$$\mu = \int \rho \mathrm{d}\rho \mathrm{d}\phi \mathcal{E}(\rho) = \pi \eta^2 \epsilon(\beta)$$

# Local strings 4

- Similar calculation for more complicated Lie group $G$
- Denote vacuum manifold by $\mathcal{M}$ and little group of a $\phi_0 \in \mathcal{M}$ by H then

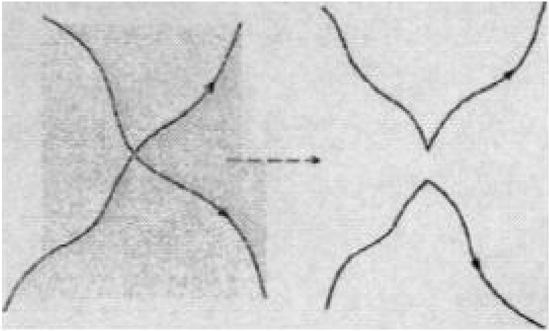
$$\mathcal{M} = G/H$$

- Vortices are formed if $\pi_1(\mathcal{M})$ non-trivial
- This is equivalent to $\pi_0(H)$ is non-trivial

# Strings from a topological viewpoint

- For simplicity assume $\mathcal{M} = S^1$ and look in 2 dimensions
- Assume after phase-transition you have a closed path on which $\phi$ assumes all values of $\mathcal{M}$ once
- Then there is a point where $\phi$ has to leave $\mathcal{M}$
- In 3 dimensions this point becomes a string and represents trapped energy.
- Strings can not end, either form loops or go on for ever

# Intercommuting strings



Ruth Durrer, "Topological defects in Cosmology",
New Astronomy Reviews 43 (1999) 111-156

# Energy loss in loops

- take $v(t)$ the number of infinite strings inside a horizon of size $t$, then

$$\rho_\infty \sim \frac{v(t)\mu t}{t^3}$$

- Number of intersections $\sim v(v-1)/t^4$

$$\frac{dn}{dt} \sim pv(v-1)/t^4$$

$$\frac{d}{dt}(\rho_\infty a^3) = -\mu t \frac{dn}{dt} a^3 \sim a^3 \mu p v(v-1)/t^3$$

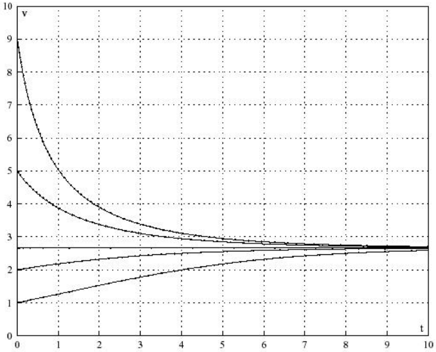
# Finding the scaling solution

- Assuming we are in radiation era, we derive

$$\dot{v} = \frac{v}{2}(1/2 - p(v - 1))$$

- Has two equilibria $v = 0$ and $v = 1 + 1/(2p)$
- From bifurcation theory it follows that $v = 1 + 1/(2p)$ is a stable static solution for $p > 0$.

# Finding the scaling solution




- Energy density in strings becomes

$$\Omega_\infty = \frac{\rho_\infty}{\rho} = \frac{8\pi G}{3H^2}\rho_\infty \sim G\mu$$

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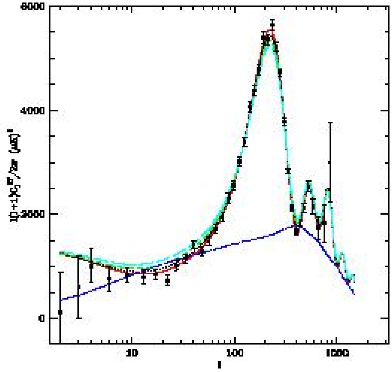
# Source of geometric perturbations

- Cosmic strings create perturbations of the order

$$\Omega_\infty = \frac{\rho_\infty}{\rho} = \frac{8\pi G}{3H^2}\rho_\infty \sim G\mu$$

- For GUT scale strings this of the order $10^{-6}$
- However unable to fit to both CMB and large scale structure formation
- Strings do not contribute for more than 10% to large scale structure formation

# WMAP CMB Data




l. Pogosian, S. Tye, I. Wasserman and M. Wyman, Observationl constraints on cosmic string production during brane inflation, Phys. Rev. D68 (2003)

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# Research by Rocher and Jeannerot

Assuming

- No monopoles formed after inflation (monopole problem)
- Inflaton field is included in model as pair of Higgs fields
- Baryogenesis occurs via leptogenesis so $U(1)_{B-L}$ symmetry broken at end inflation
- R-parity either contained in $U(1)_{B-L}$ or group breaks down to $G_{SM} \times Z_2$
- Rank between 4 and 8 including $SU(5)$, $SO(10)$ and $E_6$

All symmetry breaking patterns that satisfy this, create cosmic strings at the end of inflation

# Deriving string metric

- Work in Minkowski space
- String tension is equal to mass density from Lorentz invariance along the string
- The string stress tensor becomes

$$T^{\mu\nu} = \mu \mathrm{diag}(1, 0, 0, -1)\delta(x)\delta(y)$$

# Linearized Einstein equations

- Introduce $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$
- Linearized Einstein equations become

$$\partial^2 h_{\mu\nu} = -16\pi G(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T)$$

- This has the solution

$$h_{\mu\nu} = 8G\mu \ln(\rho/\rho_0)\mathrm{diag}(0, 1, 1, 0)$$

- Can be matched to exact solution by coordinate transformation

$$[1 - 8\pi G\mu \ln(\rho/\rho_0)]\rho^2 = (1 - 4G\mu)^2 R^2$$



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# Deriving the metric

- To order $G^2\mu^2$

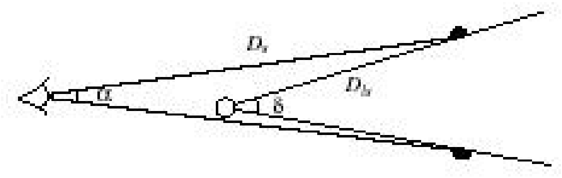
$$ds^2 = dt^2 - dz^2 - dR^2 - (1 - 4G\mu)^2 R^2 d\psi^2$$

- Introduce new angular coordinate $\bar{\psi} = (1 - 4G\mu)\psi$

$$ds^2 = dt^2 - dz^2 - dR^2 - R^2 d\bar{\psi}^2$$

- However $\bar{\psi}$ runs from 0 to $2\pi - \delta$ with $\delta = 8\pi G\mu$

# Cone-shaped metric




$$\alpha = \frac{D_{ls}}{D_s}\delta \sin\theta$$

# CMB Signature

- A string moving with transverse velocity **v** creates the discontinuity

$$\delta T/T = 8\pi G\mu\gamma v_\perp$$

- From this two bounds on $G\mu$ can be derived

$$G\mu \leq 1.3 \times 10^{-6}\sqrt{\frac{B\lambda}{0.1}}$$

$$G\mu \leq 3.3 \times 10^{-7}$$



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# Gravitational radiation

- String loops form cusps which emit strong pulses of gravitational radiation
- Pulses should be observable by LIGO or LISA
- Indirect observations through pulsar timing, put a bound on the density of gravitational radiation.
- From this a bound on $G\mu$ can be derived

$$G\mu \leq 10^{-7}$$

# Observation of cosmic string lensing

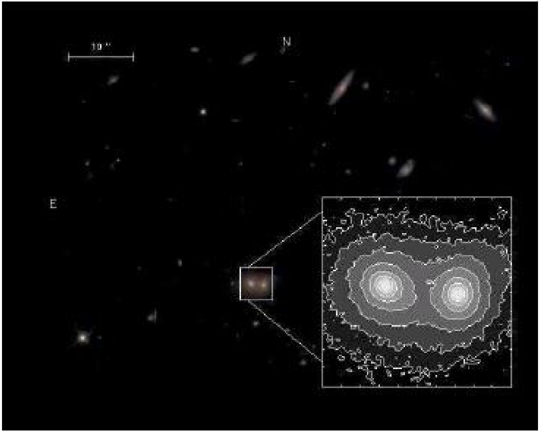
- In 2003 discovery of CSL-1 by Sazhin et al.
- Two systems equal in mass, red-shift and radiation

| | $B$ | $V$ | $R$ |
|---|---|---|---|
| $A$ | $22.73 \pm 0.15$ | $20.95 \pm 0.13$ | $19.67 \pm 0.20$ |
| $B$ | $22.57 \pm 0.15$ | $21.05 \pm 0.13$ | $19.66 \pm 0.20$ |

- Would require a $G\mu$ in the order

$$G\mu \geq 4 \times 10^{-7}$$

# 2006 HUBBLE data rules out lensing




Eric Agol, Craig J. Hogan and Richard M. Plotkin, "Hubble imaging excludes cosmic string lens", e-Print: arXiv:astro-ph/0603838v3

# Possible observation of cosmic string loop

- Anomalous fluctuations observed in quasar brightness in two images in the system Q0957+561 in 1995
- Usually a fluctuation intrinsic to the quasar first appears in image A and 417.1 days later in image B.
- Further fluctuations are caused for both images differently by individual stars in lensing galaxy.
- These can not explain the simultaneous fluctuations that were observed



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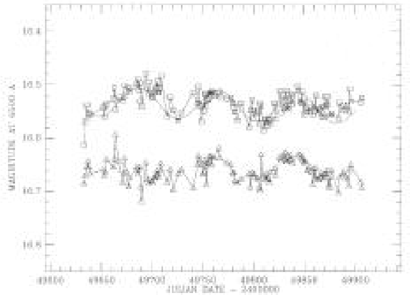
# Possible observation of cosmic string loop 2



R. Schild, I.S. Masnyak, B.I. Hnatyk and V.I. Zhdanov, "Anomalous fluctuations in observations of Q0957+561 A,B: smoking gun of a cosmic string?", e-Print: arXiv:astro-ph/0406434v1 (2004)



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# Explanation by string loop

- Oscillating cosmic string loops create brightness fluctuations

$$\Delta m \approx 5.6 \left(\frac{\theta_I}{3''}\right)^{-6} \left(\frac{\theta_R}{1.5''}\right)^4 \left(\frac{\mu}{10^{22}\mathrm{g/cm}}\right)^2 (1+v_3)^{-2}$$

- Observed $\Delta m$ is approximately 4%
- Depending on values of parameters $3 \times 10^{-8} < G\mu < 6 \times 10^{-7}$
- Numerical simulations can reproduce experimental data



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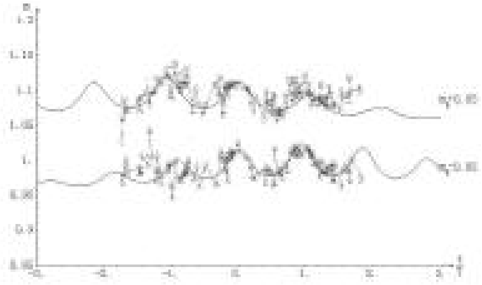
# Explanation by string loop



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# Explanation by binary system

- A binary system creates brightness fluctuations

$$\Delta m \approx 0.04 \left(\frac{T}{100\mathrm{days}}\right)^{-4} \left(\frac{\theta_r}{1.5''}\right)^{8} \left(\frac{\theta_l}{3''}\right)^{-6} \left(\frac{D_l}{1.2\mathrm{pc}}\right)^{4}$$

- To explain observed data the mass of both elements of the binary system has to be 78 solar masses at minimal distance of 1.2 pc
- It is very unlikely that such a large system so close is not yet observed
- System could be further away, but would then also have to be heavier

# Conclusion

- Cosmic strings are formed as defects by phase transitions in the early universe
- Strings do not die out or overpopulate the universe
- CMB data ruled them out as source of prime-ordial density perturbations
- SUSY GUT seems to demand cosmic strings
- One possible observation of a string loop still open