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Deriving a scaling solution

Cosmic strings as a source of geometric perturbations

Cosmic strings genericity in SUSY GUT

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Topological Defects: Cosmic Strings

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Global strings

- Start with a complex scalar field \( \phi(x) \) and

\[
\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi), \quad V = \frac{1}{2} \left( |\phi|^2 - \frac{1}{2} \eta^2 \right)^2
\]

- a global U(1) symmetry with \( \phi \rightarrow e^{i\alpha} \)

- the Euler-Lagrange equations become

\[
[\partial^2 + \lambda(|\phi|^2 - \frac{1}{2} \eta^2)]\phi = 0
\]
- Ground state $\phi = (\eta/\sqrt{2})e^{i\alpha_0}$ breaks U(1) symmetry
- Mass of scalar particle becomes $m_s^2 = \lambda \eta^2$
- Static solution with non-zero energy density
- Ansatz

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s \rho)e^{in\psi}$$
Global strings 3

- Introduce $\xi \equiv m_s \rho$, then

\[ f'' + \frac{1}{\xi} f' - \frac{n^2}{\xi^2} f - \frac{1}{2} (f^2 - 1) f = 0 \]

$\xi \to 0$ \quad $f \to 0$ \quad and \quad $\xi \to \infty$ \quad $f \to 1$

- Writing $f = 1 - \delta f$, $\delta f \sim n^2 / \xi^2$, we find

$$\mathcal{E} = |\dot{\phi}|^2 + |\nabla \phi|^2 + V(\phi)$$
Local strings

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi) \]

- where \( D_\mu = \partial_\mu + ieA_\mu \) and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \).
- Now the field equations become

\[
[D^2 + \lambda(|\phi|^2 - \frac{1}{2} \eta^2)]\phi = 0, \]

\[
\partial_\nu F^{\mu\nu} + ie(\phi^* D^\mu \phi - D^\mu \phi^* \phi) = 0
\]
Local strings 2

- Still scalar particle with mass $m_s = \sqrt{\lambda} \eta$
- Nambu-Goldstone boson incorporated into vector field, gains mass $m_v = e \eta$
- Choose radial gauge $A_\rho = 0$

$$\phi = \frac{\eta}{\sqrt{2}} f(m_s \rho) e^{im\psi}, \quad A^i = \frac{n}{e \rho} \hat{\psi}^i a(m_v \rho)$$
Local strings 3

- For large $\xi = m_s \rho$ we now get
  \[ f \sim 1 - f_1 \xi^{-1/2} \exp(-\beta \xi) \quad a \sim 1 - a_1 \xi^{1/2} \exp(-\xi) \]

- with $\beta = m_s / m_v$.

- Note that now the energy is finite and we find for the energy per unit length
  \[ \mu = \int \rho d\rho d\phi E(\rho) = \pi \eta^2 \epsilon(\beta) \]
Local strings 4

- Similar calculation for more complicated Lie group $G$
- Denote vacuum manifold by $\mathcal{M}$ and little group of a $\phi_0 \in \mathcal{M}$ by $H$ then

$$\mathcal{M} = G/H$$

- Vortices are formed if $\pi_1(\mathcal{M})$ non-trivial
- This is equivalent to $\pi_0(H)$ is non-trivial
Strings from a topological viewpoint

- For simplicity assume $\mathcal{M} = S^1$ and look in 2 dimensions
- Assume after phase-transition you have a closed path on which $\phi$ assumes all values of $\mathcal{M}$ once
- Then there is a point where $\phi$ has to leave $\mathcal{M}$
- In 3 dimensions this point becomes a string and represents trapped energy.
- Strings can not end, either form loops or go on for ever
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Intercommuting strings

Energy loss in loops

- take \( v(t) \) the number of infinite strings inside a horizon of size \( t \), then

\[
\rho_\infty \sim \frac{v(t) \mu t}{t^3}
\]

- Number of intersections \( \sim v(v - 1)/t^4 \)

\[
\frac{dn}{dt} \sim p v(v - 1)/t^4
\]

\[
\frac{d}{dt}(\rho_\infty a^3) = -\mu t \frac{dn}{dt} a^3 \sim a^3 \mu p v(v - 1)/t^3
\]
Finding the scaling solution

- Assuming we are in radiation era, we derive
  \[
  \dot{v} = \frac{v}{2} \left(1/2 - p(v - 1)\right)
  \]

- Has two equilibria \( v = 0 \) and \( v = 1 + 1/(2p) \)
- From bifurcation theory it follows that \( v = 1 + 1/(2p) \) is a stable static solution for \( p > 0 \).
Finding the scaling solution

- Energy density in strings becomes

\[ \Omega_\infty = \frac{\rho_\infty}{\rho} = \frac{8\pi G}{3H^2} \rho_\infty \sim G\mu \]
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Cosmic strings create perturbations of the order

$$\Omega_\infty = \frac{\rho_\infty}{\rho} = \frac{8\pi G}{3H^2} \rho_\infty \sim G\mu$$

- For GUT scale strings this of the order $10^{-6}$
- However unable to fit to both CMB and large scale structure formation
- Strings do not contribute for more than 10% to large scale structure formation
WMAP CMB Data

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Research by Rocher and Jeannerot

Assuming

- No monopoles formed after inflation (monopole problem)
- Inflaton field is included in model as pair of Higgs fields
- Baryogenesis occurs via leptogenesis so $U(1)_{B-L}$ symmetry broken at end inflation
- R-parity either contained in $U(1)_{B-L}$ or group breaks down to $G_{SM} \times Z_2$
- Rank between 4 and 8 including $SU(5)$, $SO(10)$ and $E_6$

All symmetry breaking patterns that satisfy this, create cosmic strings at the end of inflation
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Deriving string metric

- Work in Minkowski space
- String tension is equal to mass density from Lorentz invariance along the string
- The string stress tensor becomes

\[ T^{\mu \nu} = \mu \text{diag}(1, 0, 0, -1) \delta(x)\delta(y) \]
Linearized Einstein equations

- Introduce $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$
- Linearized Einstein equations become

$$\nabla^2 h_{\mu\nu} = -16\pi G (T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T)$$

- This has the solution

$$h_{\mu\nu} = 8 G \mu \ln(\frac{\rho}{\rho_0}) \text{diag}(0, 1, 1, 0)$$

- Can be matched to exact solution by coordinate transformation

$$[1 - 8\pi G \mu \ln(\frac{\rho}{\rho_0})] \rho^2 = (1 - 4 G \mu)^2 R^2$$
Deriving the metric

- To order $G^2 \mu^2$

$$ds^2 = dt^2 - dz^2 - dR^2 - (1 - 4G\mu)^2 R^2 d\psi^2$$

- Introduce new angular coordinate $\bar{\psi} = (1 - 4G\mu)\psi$

$$ds^2 = dt^2 - dz^2 - dR^2 - R^2 d\bar{\psi}^2$$

- However $\bar{\psi}$ runs from 0 to $2\pi - \delta$ with $\delta = 8\pi G\mu$
Cone-shaped metric

\[ \alpha = \frac{D_{ls}}{D_s} \delta \sin \theta \]
CMB Signature

- A string moving with transverse velocity $v$ creates the discontinuity

$$\delta T / T = 8\pi G_{\mu} \gamma v_{\perp}$$

- From this two bounds on $G_{\mu}$ can be derived

$$G_{\mu} \leq 1.3 \times 10^{-6} \sqrt{\frac{B\lambda}{0.1}}$$

$$G_{\mu} \leq 3.3 \times 10^{-7}$$
Gravitational radiation

- String loops form cusps which emit strong pulses of gravitational radiation
- Pulses should be observable by LIGO or LISA
- Indirect observations through pulsar timing, put a bound on the density of gravitational radiation.
- From this a bound on $G\mu$ can be derived
  \[ G\mu \leq 10^{-7} \]
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Observation of cosmic string lensing

- In 2003 discovery of CSL-1 by Sazhin et al.
- Two systems equal in mass, red-shift and radiation

<table>
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<th></th>
<th>B</th>
<th>V</th>
<th>R</th>
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<td>A</td>
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<td>20.95 ± 0.13</td>
<td>19.67 ± 0.20</td>
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<td>B</td>
<td>22.57 ± 0.15</td>
<td>21.05 ± 0.13</td>
<td>19.66 ± 0.20</td>
</tr>
</tbody>
</table>

- Would require a $G\mu$ in the order

\[ G\mu \geq 4 \times 10^{-7} \]
2006 HUBBLE data rules out lensing

Possible observation of cosmic string loop

- Anomalous fluctuations observed in quasar brightness in two images in the system Q0957+561 in 1995
- Usually a fluctuation intrinsic to the quasar first appears in image A and 417.1 days later in image B.
- Further fluctuations are caused for both images differently by individual stars in lensing galaxy.
- These can not explain the simultaneous fluctuations that were observed.
Possible observation of cosmic string loop 2

Explanation by string loop

- Oscillating cosmic string loops create brightness fluctuations

\[ \Delta m \approx 5.6 \left( \frac{\theta_I}{3''} \right)^{-6} \left( \frac{\theta_R}{1.5''} \right)^4 \left( \frac{\mu}{10^{22} \text{g/cm}} \right)^2 (1 + v_3)^{-2} \]

- Observed \( \Delta m \) is approximately 4%

- Depending on values of parameters
  \[ 3 \times 10^{-8} < G\mu < 6 \times 10^{-7} \]

- Numerical simulations can reproduce experimental data
Explanation by binary system

- A binary system creates brightness fluctuations

$$\Delta m \approx 0.04 \left( \frac{T}{100 \text{days}} \right)^{-4} \left( \frac{\theta_r}{1.5''} \right)^8 \left( \frac{\theta_I}{3''} \right)^{-6} \left( \frac{D_I}{1.2 \text{pc}} \right)^4$$

- To explain observed data the mass of both elements of the binary system has to be 78 solar masses at minimal distance of 1.2 pc

- It is very unlikely that such a large system so close is not yet observed

- System could be further away, but would then also have to be heavier
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Conclusion

- Cosmic strings are formed as defects by phase transitions in the early universe
- Strings do not die out or overpopulate the universe
- CMB data ruled them out as source of prime-ordial density perturbations
- SUSY GUT seems to demand cosmic strings
- One possible observation of a string loop still open