Gravitational Waves in Cosmology

"The weakest link"

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Student Seminar '08

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Outline

- Introduction
- Basic Theory of GW’s
- Cosmological GW’s
- Candidate Processes for Production
- Experimental Bounds
- GW Detection and Upcoming Experiments

Introduction

- Radiative Solutions are predicted by General Relativity (tiny ripples of spacetime)
- Because of extremely weak coupling, observation of such waves requires highly sophisticated technology
- The only available laboratory is the Universe itself. We obviously have no control over the conditions of the experiments.
- Sources can be of Astrophysical (massive bodies) or Cosmological (early Universe) nature

Motivation

- **Why are we so interested in Gravitational Radiation?**
  - **Detection of Gravitational waves will not only be undisputed support for GR but will also open a new window to the cosmos**
  - **Study of Astrophysical systems that comprise sources of GW's**
  - **Probe for a snapshot of the early Universe and validate or exclude cosmological models**
  - **Challenge to push current technology to its extreme limits**

Motivation

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General Relativistic Context

- GR provides us with a set of dynamical equations for the geometry of spacetime.

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

- These equations are highly nonlinear wrt the metric and cannot be solved analytically in the generic case.
- One has to resort to a linearized version of the above equations in order to come up with radiative solutions for small field perturbations.
- Interactions?

Weak Field Approximation

- **Take metric perturbations around Minkowski spacetime**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| << 1$$

and evaluate all quantities up to 1st order in the perturbation

- **The Christoffel connection reads :**

$$\Gamma^{\lambda}_{\mu\nu} = \tfrac{1}{2}(\eta^{\lambda\rho} + h^{\lambda\rho})[\partial_\nu h_{\mu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho h_{\mu\nu}]$$
$$= \tfrac{1}{2}\eta^{\lambda\rho}[\partial_\nu h_{\mu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho h_{\mu\nu}] + \cancel{O(h^2)}$$

- **And the Ricci tensor :**

$$R_{\mu\nu} = \partial_\mu \Gamma^{\lambda}_{\lambda\nu} - \partial_\nu \Gamma^{\lambda}_{\lambda\mu} + \Gamma\Gamma - \Gamma\Gamma$$
$$= \tfrac{1}{2}\eta^{\lambda\rho}[\partial_\mu\partial_\nu h_{\lambda\rho} + \partial_\mu\partial_\lambda h_{\nu\rho} - \partial_\mu\partial_\rho h_{\nu\lambda} - \partial_\lambda\partial_\nu h_{\mu\rho} - \partial_\lambda\partial_\mu h_{\nu\rho} + \partial_\lambda\partial_\rho h_{\mu\nu}]$$
$$= \tfrac{1}{2}[\partial_\mu\partial_\nu h^{\lambda}_{\lambda} - \partial_\mu\partial^{\lambda} h_{\nu\lambda} - \partial_\nu\partial^{\lambda} h_{\mu\lambda} + \Box h_{\mu\nu}] \equiv R^{(1)}_{\mu\nu}$$

Gauge freedom

- **The 10 Einstein equations are not functionally independent due to the 4 Bianchi identities**

$$\nabla^{\mu} G_{\mu\nu} = 0$$

the metric is underdetermined

- **General covariance imposes diffeomorphism invariance on the theory (via coordinate xfms). This is the gauge freedom of GR :** $x^{\mu} \rightarrow x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$
- **So from the 6 remaining d.o.f. one can choose to "fix the gauge" by imposing certain properties on the metric and get rid of 4 more non-physical d.o.f.**
- **Remember EM gauge invariance**

Gauge freedom

- Now the perturbation will transform as :

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

and

$$h' = h - 2\partial_\mu \xi^\mu$$

trace reverse : $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

- We can always transform to a gauge that is convenient, here the Lorenz gauge :

$$\nabla^\mu \bar{h}_{\mu\nu} = 0$$

- So the Einstein equation can be recast in the simple form

$$\Box \bar{h}_{\mu\nu} = 16\pi G_N T_{\mu\nu}$$

The TT gauge

- **Vanishing source term**

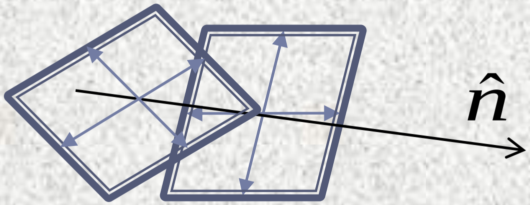
$$\Box \bar{h}_{\mu\nu} = 0 \qquad\qquad \Box \xi_\mu = 0$$

- **This allows us to bring the metric to the transverse traceless form :**

$$h^{0i} = 0 \qquad \partial_0 h^{00} = 0 = h^{00} \qquad h = 0$$

and for a single plane wave

$$h_{ij}^{\ TT} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



- **Generic way of finding TT :** $P_{ij} = \delta_{ij} - n_i n_j$

$$h_{ij}^{TT} = \left[P_{ik} P_{jl} - \tfrac{1}{2} P_{ij} P_{kl} \right] h_{kl}$$

Perturbing around a non-flat metric

- **Minkowski is boring for cosmology.**

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- **The Christoffel connection reads :**

$$\Gamma^{\lambda}_{\mu\nu} = \tfrac{1}{2}(\bar{g}^{\lambda\rho} + h^{\lambda\rho})[\partial_\nu \bar{g}_{\mu\rho} + \partial_\nu h_{\mu\rho} + \partial_\mu \bar{g}_{\nu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho \bar{g}_{\mu\nu} - \partial_\rho h_{\mu\nu}]$$

$$= \tfrac{1}{2}\bar{g}^{\lambda\rho}[\partial_\nu \bar{g}_{\mu\rho} + \partial_\mu \bar{g}_{\nu\rho} - \partial_\rho \bar{g}_{\mu\nu}] + \tfrac{1}{2}\underline{h^{\lambda\rho}}[\partial_\nu \bar{g}_{\mu\rho} + \partial_\mu \bar{g}_{\nu\rho} - \partial_\rho \bar{g}_{\mu\nu}]$$

$$+ \tfrac{1}{2}\bar{g}^{\lambda\rho}[\partial_\nu h_{\mu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho h_{\mu\nu}] + \cancel{O(h^2)}$$

$$h^{\lambda\kappa}\bar{g}_{\kappa\sigma}\bar{g}^{\sigma\rho}$$

$$= \bar{\Gamma}^{\lambda}_{\mu\nu} \underbrace{- h^{\lambda\kappa}\bar{g}_{\kappa\sigma}\bar{\Gamma}^{\sigma}_{\mu\nu} + \frac{1}{2}\bar{g}^{\lambda\rho}[\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho} - \partial_\rho h_{\mu\nu}]}$$

and we can define :

$$\Gamma^{\lambda}_{\mu\nu} = \bar{\Gamma}^{\lambda}_{\mu\nu} + \delta\Gamma^{\lambda}_{\mu\nu}$$

$$\delta\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\bar{g}^{\lambda\rho}[\bar{\nabla}_\mu h_{\nu\rho} + \bar{\nabla}_\nu h_{\mu\rho} - \bar{\nabla}_\rho h_{\mu\nu}]$$

- And the **Riemann** tensor :

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\bar{\Gamma}^{\rho}_{\sigma\nu} - \partial_{\nu}\bar{\Gamma}^{\rho}_{\sigma\mu} + \partial_{\mu}\delta\Gamma^{\rho}_{\sigma\nu} - \partial_{\nu}\delta\Gamma^{\rho}_{\sigma\mu} = \bar{R}^{\rho}_{\sigma\mu\nu} + \delta R^{\rho}{}_{\sigma\mu\nu}$$

where we used the Riemann normal coord.

General covariance implies: $\delta R^{\rho}{}_{\sigma\mu\nu} = \bar{\nabla}_{\mu}\delta\Gamma^{\rho}_{\sigma\nu} - \bar{\nabla}_{\nu}\delta\Gamma^{\rho}_{\sigma\mu}$

- The **Ricci** tensor : $\delta R^{\rho}{}_{\mu\rho\nu} = \bar{\nabla}_{\rho}\delta\Gamma^{\rho}_{\mu\nu} - \bar{\nabla}_{\nu}\delta\Gamma^{\rho}_{\mu\rho} = \ldots$

$$= -\tfrac{1}{2}\bar{\Box}h_{\mu\nu} - \tfrac{1}{2}\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}h + \tfrac{1}{2}\bar{\nabla}_{\rho}\bar{\nabla}_{(\mu}h^{\rho}{}_{\nu)} = \delta R_{\mu\nu}$$

- And **Einstein** eqn : $\delta G_{\mu\nu} = \delta R_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\delta R$

in the gauge $\bar{\nabla}^{\mu}\bar{h}_{\mu\nu} = 0$ and $h = 0$

transversality tracelessness

$$\delta G_{\mu\nu} = -\tfrac{1}{2}\bar{\Box}h_{\mu\nu} + \bar{R}_{\rho\mu\nu\sigma}h^{\rho\sigma} = 0$$

TT part in nonvacuum spacetime

- **Residual symmetry** $\Box\xi_\mu = 0$ → **TT gauge**

- **In a spacetime with a nonvanishing source term one also gets 'spurious' l-l and l-t contributions** Φ, Θ, Ξ_i

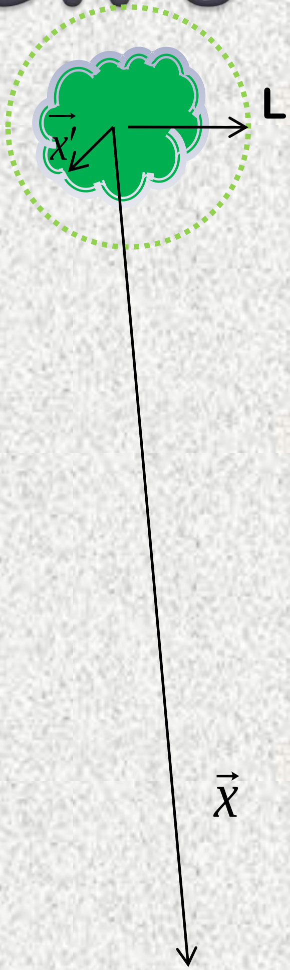
"They are not objective and are not detectable by any conceivable experiment. They are merely sinuosities in the co-ordinate system, and the only speed of propagation relevant to them is the speed of thought."

A. S. Eddington

- **TT part is gauge invariant observable and obeys a wave-like equation** $\boxed{\Box h_{ij}^{TT} = 16\pi\sigma_{ij}}$

$$\sigma_{ij} = \Pi_{ij,lm} T_{lm}$$

Generation of GW's



- Consider a spacetime with a source :

$$\Box \bar{h}_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

- Introducing the Green's function :

flat

$$\Box G(x-x') = \delta^{(4)}(x-x')$$

$$G(x-x') = \frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{x}'|}\delta\left(t-|\vec{x}-\vec{x}'|-t'\right)$$

$$\bar{h}_{\mu\nu} = 8\pi G_N \int d^3\vec{x}' G(x-x') T_{\mu\nu}(x')$$

- For r = |x| >> L remember quadrupole formula ($|\vec{x}-\vec{x}'| \simeq r + \ldots$)

$$\boxed{\bar{h}_{ij}(x) = \frac{2}{r}\frac{\partial^2}{\partial t^2}\int d^3\vec{x} T_{00}(t-r,\vec{x}') x^i{}' x^j{}'}$$

Wave zone approximation

- **Linearized Einstein eqns → no self-interactions**
- **Metric perturbations can be idealized as plane waves :**

$$h_{ij}(\eta) = \sqrt{8\pi G_N} \sum_{P=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \epsilon_{ij}^P(\vec{k}) h_{\vec{k}}^{\,P}(\eta) e^{-i\vec{k}\vec{x}}$$

- **As a free field theory of a relativistic spin-2 particle**

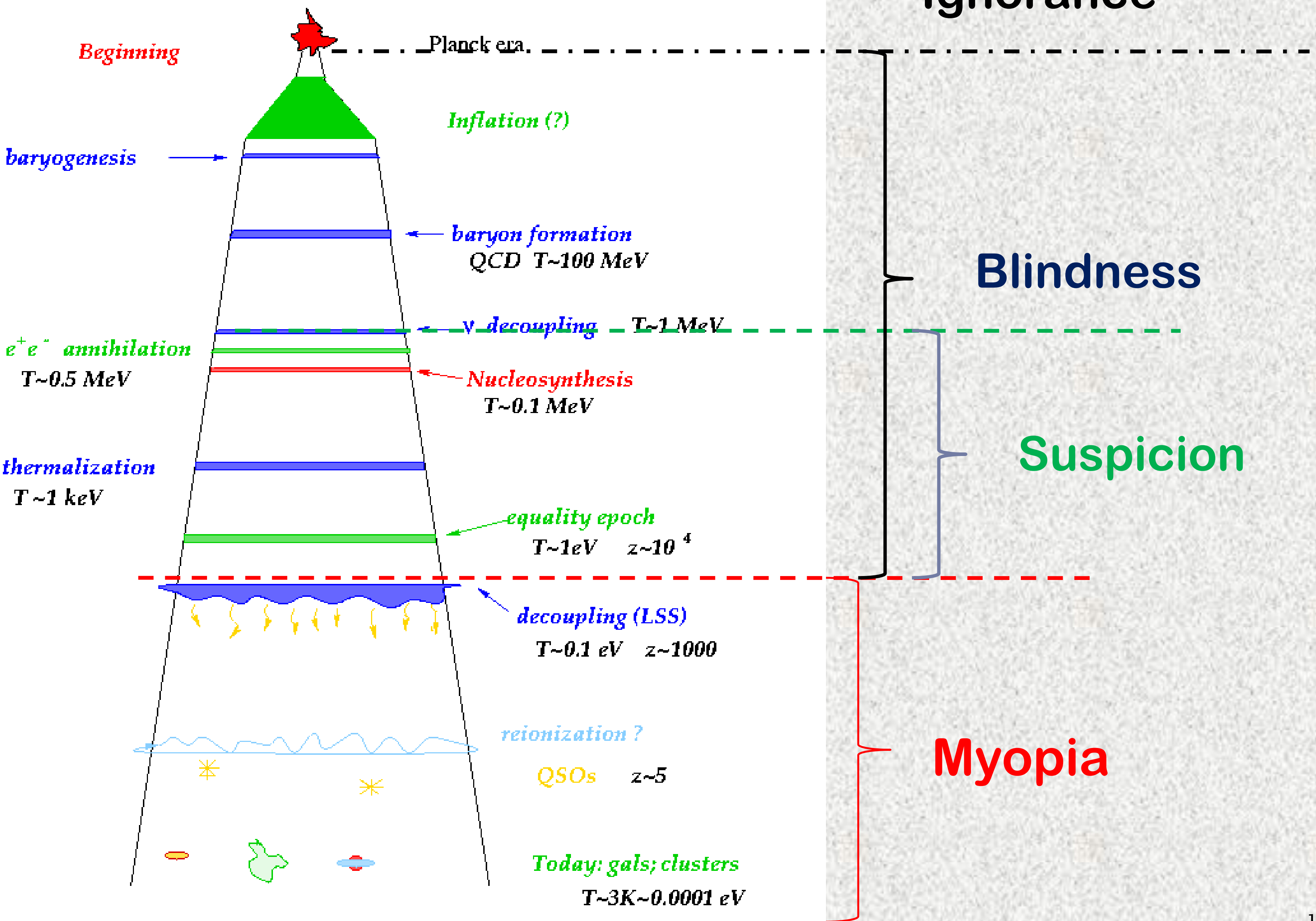
$$h_{ij}(\eta) = \sqrt{8\pi G_N} \sum_{P=+,\times} \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}} [\hat{b}^P(\vec{k}) h_{\vec{k}}^{\,P}(\eta) e^{-i\vec{k}\vec{x}} + \hat{b}^{\dagger P}(\vec{k}) h_{\vec{k}}^{\,*P}(\eta) e^{i\vec{k}\vec{x}}] \epsilon_{ij}^P(\hat{\Omega})$$

$$\left[\hat{b}_{\vec{k}}, \hat{b}_{\vec{k}'}^{\dagger}\right] = \delta(\vec{k} - \vec{k}') \qquad \left[\hat{b}_{\vec{k}}, \hat{b}_{\vec{k}'}\right] = \left[\hat{b}_{\vec{k}}^{\dagger}, \hat{b}_{\vec{k}'}^{\dagger}\right] = 0$$

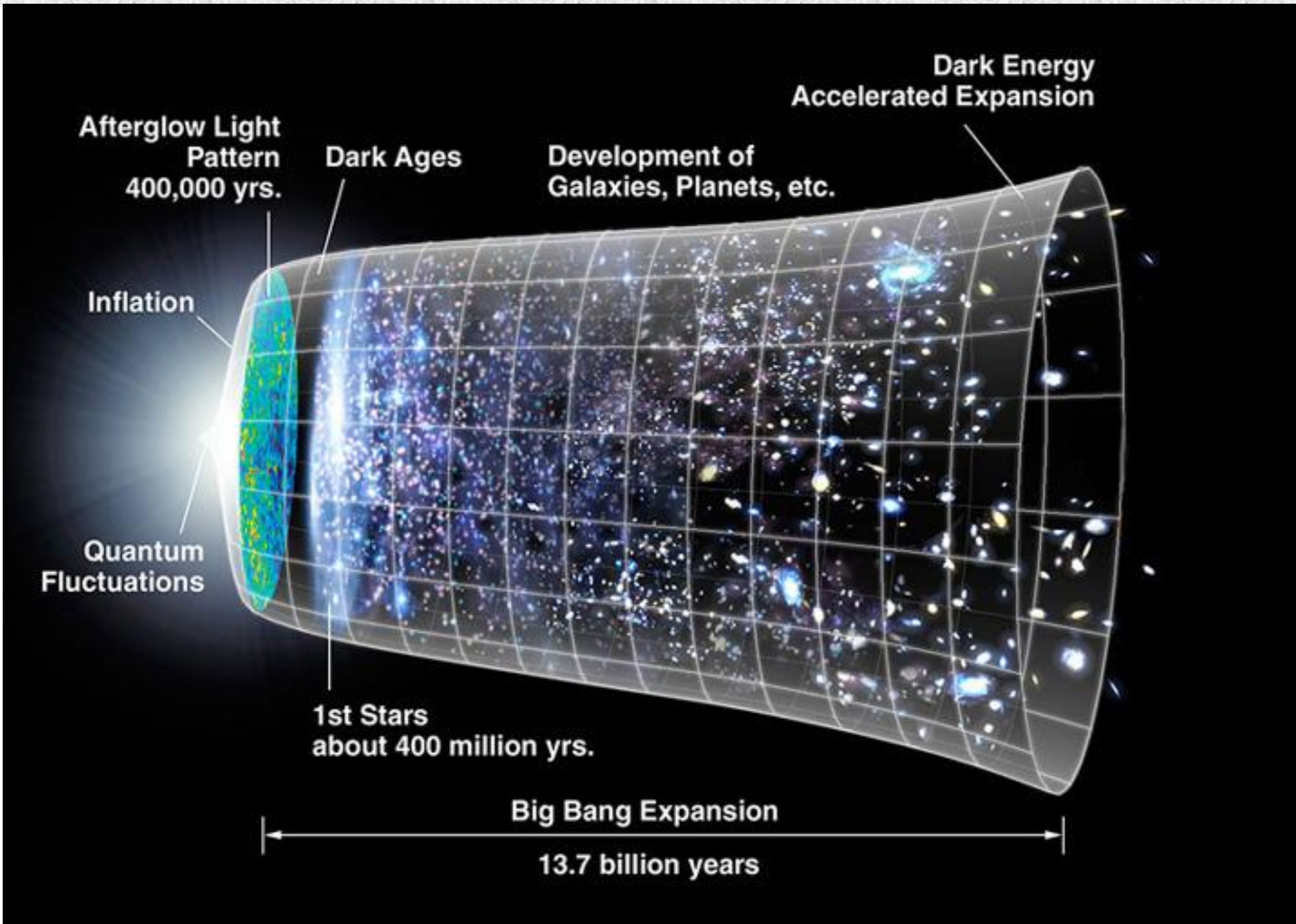
Gravitational waves in Cosmology

- In Cosmology we are looking for the Stochastic background and Ω_{gw}
  - Astro vs. Cosmo
  - Thermal history of the Universe
  - Graviton decoupling
  - Redshift and cooling of background radiation
  - CMBR correspondence

Ignorance



Beginning
Planck era
Inflation (?)
baryogenesis
baryon formation
QCD T~100 MeV
ν decoupling T~1 MeV
e^+e^- annihilation
T~0.5 MeV
Nucleosynthesis
T~0.1 MeV
thermalization
T ~1 keV
equality epoch
T~1eV z~10 ^{4}
decoupling (LSS)
T~0.1 eV z~1000
reionization ?
QSOs z~5
Today: gals; clusters
T~3K~0.0001 eV
Blindness
Suspicion
Myopia

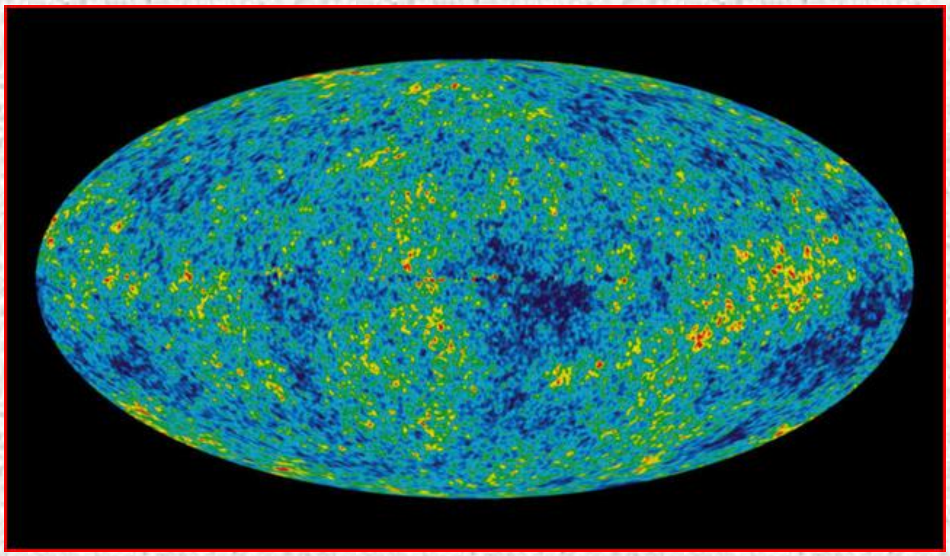

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

EM Analogue

- Consider the case of CMB
  - Photons decouple at z=1089 and T=3000K
  - Equilibrium gives a black body spectrum
  - Photons carry energy
  - Photons cool down with expansion

$$dE = 2hf\left(\frac{1}{e^{hf/kT}-1}\right)\left(\frac{4\pi V f^2 df}{c^3}\right)$$

$$d\rho_{em} = \frac{dE}{V} = \frac{8\pi h}{c^3}\frac{f^3 df}{e^{hf/kT}-1}$$



Graviton bath temperature

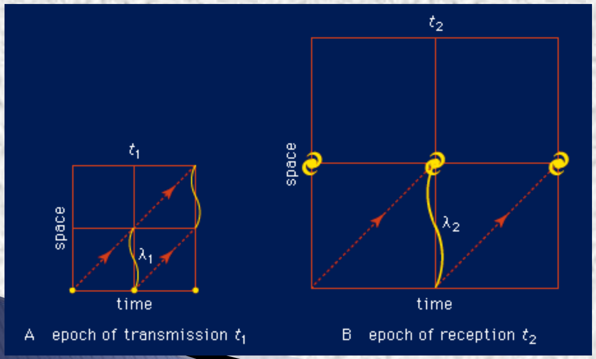
- Recall neutrino decoupling and temperature relation with SM relativistic d.o.f.
- Gravitons decouple below Planck temperature $T_{Pl} = \frac{M_{Pl}c^2}{k} \sim 10^{32} K$

  when (at least!) : $g_{*S}(T_{dec}) = 106.75$
- And of course the physical wavelength is redshifted

$$f = \frac{k}{2\pi a} \Rightarrow f_0 = f_* \frac{a_*}{a_0}$$

- Assuming adiabatic evolution since emission :

$$g_{*S} T_*^3 a_*^3 = g_S T_0^3 a_0^3$$



- So $f_0 \simeq 10^{-13} f_* \left(\frac{100}{g_{*S}}\right)^{1/3} \left(\frac{1GeV}{T_*}\right)$

  and $T_{g_0} \leq T_0 \left(\frac{3.91}{106.75}\right)^{1/3} \simeq 0.9K$

Energy Density

- Gravitational radiation carries energy

$$t_{00} = \frac{1}{16\pi G_N}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle = \frac{1}{16\pi G_N}\left\langle \dot{h}_{ij}(x)\dot{h}_{ij}(x) \right\rangle = \rho_{gw}$$

- Express graviton density spectrum by a useful quantity

$$\Omega_{gw}(f) = \frac{1}{\rho_{cr}} \frac{d\rho_{gw}}{d\ln f} \qquad , \rho_{cr} = \frac{3H_0^2}{8\pi G}$$

frequency dependence
(spectrum, peak, amplitude?)

- This includes a measuring uncertainty of H_0 . Better use

$$h_0{}^2 \Omega_{gw}$$

Energy Density

- **Stochastic isotropic background :**

$$h_{ij}(t) = \sum_{P=+,\times} \int_{-\infty}^{+\infty} df \int d\hat{\Omega} h_P(f,\hat{\Omega}) e^{-2\pi i f t} \epsilon_{ij}^P(\hat{\Omega})$$

- **Spectral density :**

$$< h_P^*(f,\hat{\Omega}) h_Q^*(f',\hat{\Omega}') >= \delta(f-f')\delta^2(\hat{\Omega},\hat{\Omega}') \frac{1}{8\pi} \delta_{PQ} S_h(f)$$

$$\Omega_{gw}(f) = \frac{4\pi^2}{3H_0^{\;2}} f^3 S_h(f)$$

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Production Mechanisms

- In order to have a background of gravitational radiation from the early Universe, which is strong enough for us to detect in the present time, we have to look for extremely violent phenomena that occur throughout the Universe.

- Modern cosmological theories provide us with such scenarios. Some of them are:
  - Fluctuation amplification during inflation
  - Phase transitions
  - Topological defects (cosmic strings)
  - Brane world scenarios
  - etc

Fluctuation amplification during Inflation

- Quantum fluctuations are amplified in an inflating Universe [Grishchuck-Starobinsky]

- Linearized eqns of TT perturbations give:

$$\Box h_{ij}(x) = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g})g^{\mu\nu}\partial_\nu)h_{ij}(x) = 0$$

- In FLRW : $g_{\mu\nu} = diag[-1, a^2(t), a^2(t), a^2(t)]$ $\sqrt{-g} = a^3(t)$

$$\frac{1}{a^3}\left[-\partial_t(a^3\partial_t) + \frac{a^3}{a^2}\nabla^2\right]h_{ij}(x) = \left[-\partial_t{}^2 - \frac{3\dot{a}(t)}{a(t)} + \frac{1}{a^2(t)}\nabla^2\right]h_{ij}(x) = 0$$

$$\left[-\partial_t{}^2 - \frac{3\dot{a}(t)}{a(t)} + \frac{1}{a^2(t)}\nabla^2\right]h_k(\eta)e^{i\vec{k}\vec{x}} = 0$$

Fluctuation amplification during Inflation

- **In conformal time :** $d\eta = \frac{dt}{a(t)}$

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -a^2(\eta)d\eta^2 + a^2(\eta)\delta_{ij}dx^i dx^j \qquad \sqrt{-g} = a^4(t)$$

- **And the box equation becomes**

$$h_k'' + 2\frac{a'}{a}h_k' + k^2 h_k = 0$$

$$\psi_k''(\eta) + \left[k^2 - U(\eta)\right]\psi_k = 0$$

$$\psi_k(\eta) = a h_k(\eta)$$

$$\psi_k' = a' h_k + a h_k'$$

$$\psi_k'' = a'' h_k + 2a' h_k' + a h_k''$$

where we used the wave zone expansions

Fluctuation amplification during Inflation

- End up with an effective "potential" which gives a time varying harmonic oscillator $U(\eta) = \frac{a''}{a}$
- Simple case : de Sitter inflation

$$a = -\frac{1}{H_I \eta}$$

- Here we have to distinguish between sub-Hubble and super-Hubble modes depending on value of k.

$$\begin{cases} h_k \sim \frac{1}{2k}\frac{1}{a}e^{-ik\eta} & , \quad \frac{a}{k} << H_I^{-1} \\ h_k \sim A_k + B_k \int \frac{d\eta}{a^2} & , \quad \frac{a}{k} >> H_I^{-1} \end{cases}$$

Energy Density

- **Recall :** $$\rho_{gw} = \frac{1}{32\pi G_N}\left\langle \dot{h}_{ij}(t,\vec{x})\dot{h}_{ij}(t,\vec{x})\right\rangle_V$$

$$\dot{h}_{ij}\dot{h}_{ij} = \left(\frac{1}{a^2}\psi'_{ij} - \frac{a'}{a^2}\psi_{ij}\right)\left(\frac{1}{a^2}\psi'_{ij} - \frac{a'}{a^2}\psi_{ij}\right) = \frac{1}{a^4}\left(\psi'\psi' - 2aH\psi'\psi + a^2H^2\psi\psi\right)$$

$$\simeq \frac{1}{a^4}\psi'\psi'$$

$$\rho_{gw} = \frac{1}{32\pi G_N a^4}\int_V \frac{d^3x}{V}\frac{d^3\vec{k}}{(2\pi)^{2/3}}\frac{d^3\vec{k}'}{(2\pi)^{2/3}}\psi'_{ij}(\eta,\vec{k})e^{-i\vec{k}\vec{x}}\psi'^{*}_{ij}(\eta,\vec{k}')e^{-i\vec{k}'\vec{x}}$$

$$= \frac{1}{32\pi G_N a^4 V}\int_V d^3\vec{k}\,\psi'_{ij}(\eta,\vec{k})\psi'^{*}_{ij}(\eta,\vec{k}')$$

Sources and Preheating

$$T_{\mu\nu} = \partial_\mu \phi_a \partial_\nu \phi_a - g_{\mu\nu}\left(\tfrac{1}{2} g^{\rho\sigma} \partial_\rho \phi_a \partial_\sigma \phi_a + V\right)$$

$$T_{ij}^{TT}(\vec{k}) = \Pi_{ij,lm}(\hat{k}) \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\vec{p}) \phi_a(\vec{k}-\vec{p})$$

$$\psi_{ij}''(\eta,\vec{k}) + \left[k^2 - \cancel{U(\eta)}\right]\psi_{ij}(\eta,\vec{k}) = \underline{16\pi G_N T_{ij}^{TT}(\vec{k})}$$

- **After approximating for sub-Hubble modes with the help of a few assumptions we work out the Green's function solution :**

$$\psi_{ij}(\eta,\vec{k}) = \frac{16\pi G}{k} \int_{\eta_i}^{\eta} d\eta' \sin\left[k(\eta-\eta')\right] a(\eta') T_{ij}^{TT}(\eta',\vec{k})$$

- **Matching this solution with the one without a source**

$$\psi_{ij}(\eta,\vec{k}) = A_{ij}(\vec{k}) \sin\left[k(\eta-\eta_f)\right] + B_{ij}(\vec{k}) \cos\left[k(\eta-\eta_f)\right]$$

will give us the amplitudes that will propagate to today

Sources and Preheating

$$T_{\mu\nu} = \partial_\mu \phi_a \partial_\nu \phi_a - g_{\mu\nu}\left(\tfrac{1}{2} g^{\rho\sigma} \partial_\rho \phi_a \partial_\sigma \phi_a + V\right)$$

$$T_{ij}^{TT}(\vec{k}) = \Pi_{ij,lm}(\hat{k}) \int \frac{d^3\vec{p}}{(2\pi)^{3/2}} p_l p_m \phi_a(\vec{p}) \phi_a(\vec{k}-\vec{p})$$

$$S_k(\tau_f) = \frac{4\pi G_N k^3}{V} \int d\Omega \sum_{i,j} \left\{ \left| \int_{\tau_i}^{\tau_f} d\tau' \cos(k\tau') a(\tau') T_{ij}^{TT}(\tau', \vec{k}) \right|^2 + \left| \int_{\tau_i}^{\tau_f} d\tau' \sin(k\tau') a(\tau') T_{ij}^{TT}(\tau', \vec{k}) \right|^2 \right\}$$

$$\Omega_{gw}(k) \propto \left(\frac{d\rho_{gw}}{d\ln k} \right)_{\tau > \tau_f} = \frac{S_k(\tau_f)}{a^4}$$

- **Matching this solution with the one without a source**

$$\psi_{ij}(\eta, \vec{k}) = A_{ij}(\vec{k}) \sin\left[k(\eta - \eta_f)\right] + B_{ij}(\vec{k}) \cos\left[k(\eta - \eta_f)\right]$$

will give us the amplitudes that will propagate to today

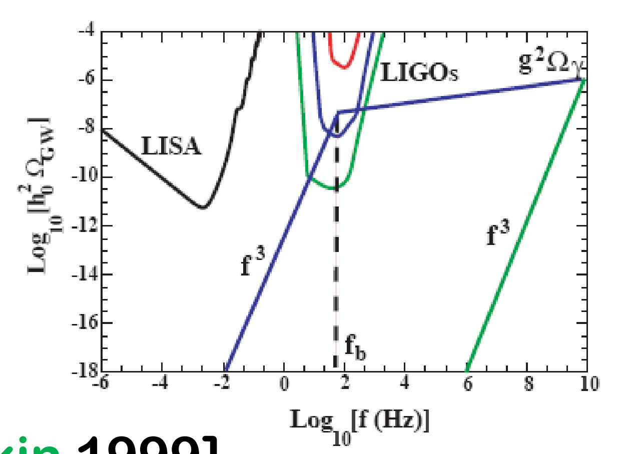
Inflationary models

- In Standard Inflation spectrum is flat or descending and COBE bound sets it too weak to be detected $\sim 10^{-13}$ even for LISA
- Bound is at Ultra low frequencies, so a theory with ascending spectrum could be detectable
- There are such theories :
  - Pre-big-bang scenario (super-inflation) [Veneziano-1991]

    $a \propto (-\eta)^{(1-\sqrt{3})/2}$ spectral slope $n_T=3$
  - Bouncing Universe includes
    - a slow contraction phase $a_E \propto (-\eta)^{\epsilon}$
    - a superinflation phase $a_E \propto (-\eta)^{1/2}$
    - Radiation dominating era

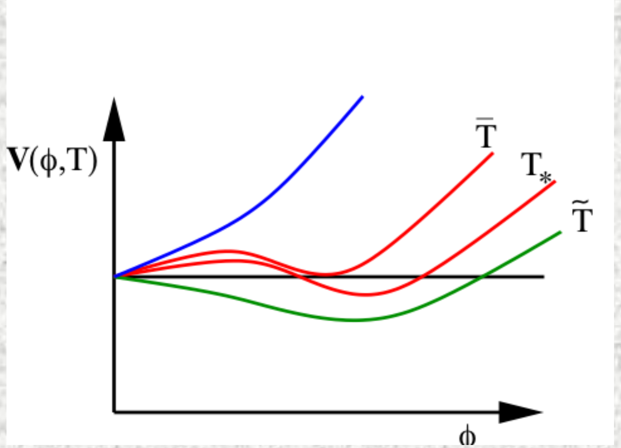
    and $n_T \sim 2+2\varepsilon$ or $n_T \sim 3$
  - Quintessential Inflation [Peebles-Vilenkin 1999]
    - non standard equation of state

      $a \propto \sqrt{\eta} \rightarrow a \propto -1/\eta$ and $n_T \sim 1$



EW Phase Transition

- Phase transitions in the early Universe can produce GW's. More specifically EW
- Primordial Universe is initially in a false vacuum state in high temperatures
- As T drops below the Higgs mass scale, the broken true vacuum shows but is still hidden by an energy barrier
- Tunneling takes action

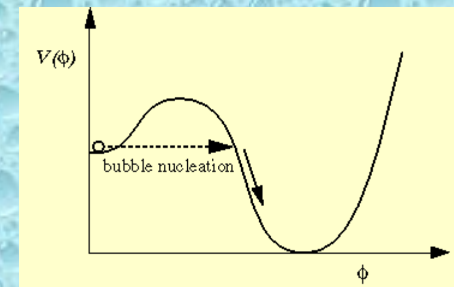


Bubble Nucleation

- Bubbles of true vacuum are nucleated and expanding

$$\Gamma = \Gamma_0 e^{\beta t} \sim H$$

$$f_{peak} \approx \beta / 2\pi \approx 10^{-4} - 5\times 10^{-3}\,Hz$$

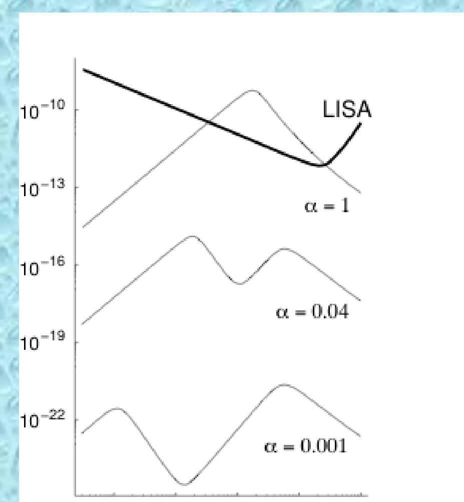


- The latent heat left over by the transition becomes kinetic energy transferred to the bubble walls (and also reheats the plasma) [Kosowsky, Kamionkowski, Turner]

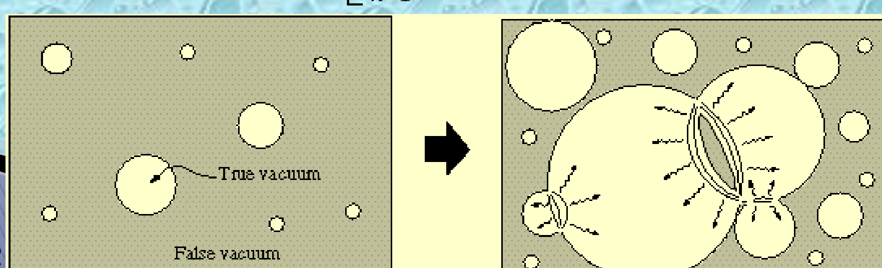
$$\alpha = \frac{E_{f.v.}}{aT_*^4}$$

- Isotropy is spontaneously lost

- GW's are produced on collision surface or even by turbulent areas in plasma



$$T_{ij}(\hat{k},\omega) = \frac{1}{2\pi}\int_0^\infty dt e^{i\omega t}\left[\sum_{n=1}^{N} e^{i\omega \hat{k}\vec{x}_n}\int_{S_n} d\Omega \int_0^R dr r^2 e^{-i\omega \hat{k}\vec{x}} T_{ij}(r,t)\right]$$

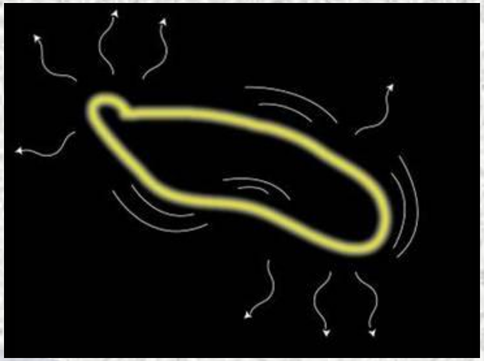


Type of transition

- Strongly 1st order transition is needed
- SM predicts smooth crossover (m_H>100GeV)
- MSSM needs light stop for 1st order
- NMSSM can provide strong 1st order (also gives baryon asymmetry

Cosmic Strings

- Topological defects are produced after a phase transition via the Kibble mechanism
- Cosmic strings are stringy formations of enormous mass densities that extend throughout the universe
- Quantity of interest $\mu \sim \frac{dm}{dl}$
- Loops can be created
- Pulsate relativistically and decay emitting GW's

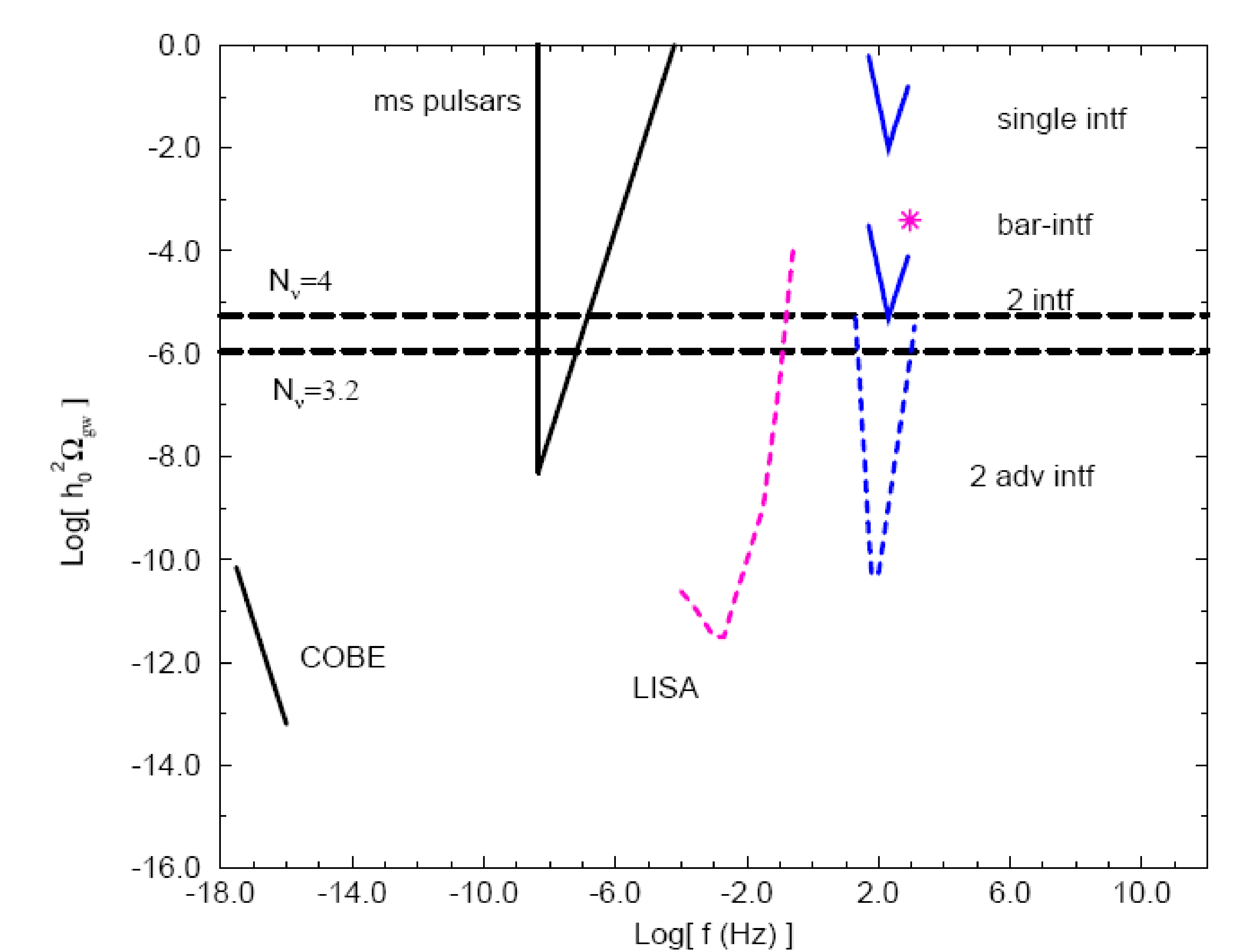


Bounds

- Direct and indirect experimental evidence restrict amplitude of GW's

- Currently operating GW experiments :
  - LIGO (2x) (Luisiana-Washington, USA)
  - VIRGO (Pisa, Italy)
  - GEO 600 (Hannover, Germany)
- Important indirect bounds also given by
  - BBN
  - COBE and the Sachs-Wolfe effect
  - msec Pulsars

Bounds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-2.0
-4.0
-6.0
-8.0
-10.0
-12.0
-14.0
-16.0
-18.0
-14.0
-10.0
-6.0
-2.0
2.0
6.0
10.0
Log[f (Hz)]
Log[$h_0^2 \Omega_{gw}$]
ms pulsars
single intf
bar-intf
2 intf
N_ν=4
N_ν=3.2
2 adv intf
COBE
LISA

Detection

- **Principle of detection:**

No geodesic deviation in 1st order, only "stretching" of coordinates (physical distance travelled by a photon)

$$\frac{dL}{L} \sim 10^{-21}$$

- **2 different types of detection (none of which work yet)**
  - **Resonant bar detectors [J. Weber '60s]**
  - **MiniGRAIL spherical cryogenic antenna (Leiden)**
  - **Laser interferometers** 10 to 10^4 Hz
    - **LIGO**
    - **VIRGO**
    - **GEO 600**
    - **TAMA 300**

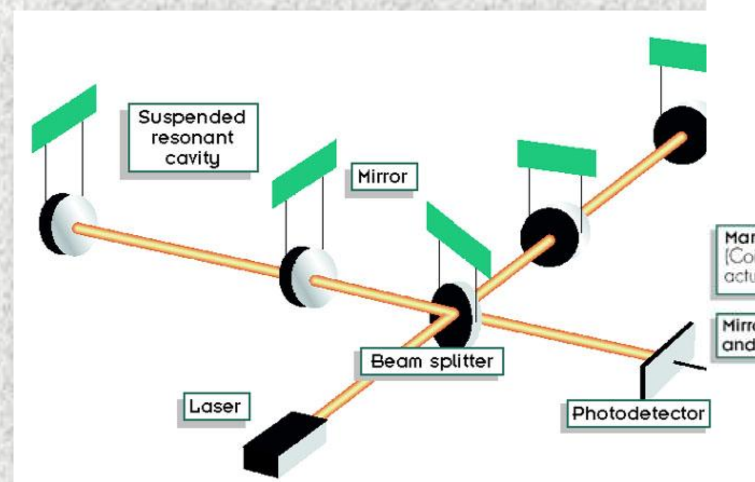
Detection

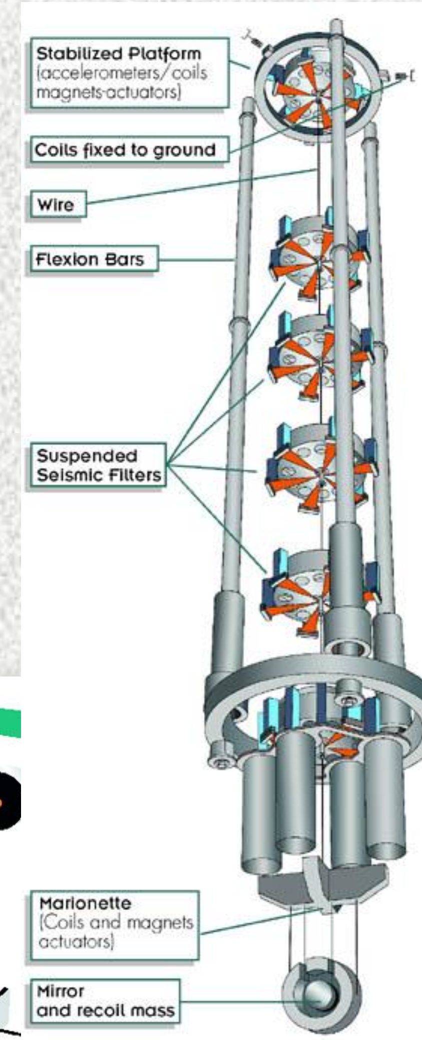
LIGO

3,000km

LIGO

VIRGO





Correlating Detectors

- Consider detector as an input/output system

$$S_i(t) = s_i(t) + n_i(t)$$

- The signal could be smaller than the noise.
- Correlate $S_{12} = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' S_1(t) S_2(t') Q(t-t')$

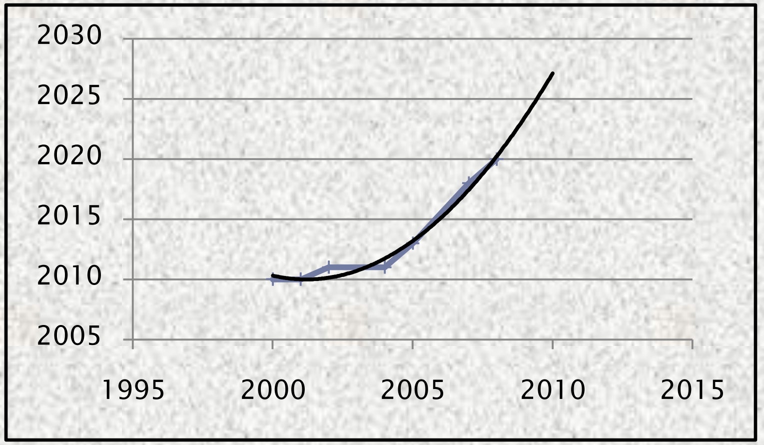
- Optimal filtering [Michelson-1987]

- Observing for a period of time T :

$$S = < S_1, S_2 > \approx < s_1, s_2 > + < n_1, n_2 >$$

$$\left.\begin{array}{l} < s_1, s_2 > \propto \left|\tilde{h}(f)\right|^2 \Delta f T \\ < n_1, n_2 > \propto \left|\tilde{n}(f)\right|^2 \sqrt{\Delta f T} \end{array}\right\} \quad \Omega_{min} \propto \frac{\left|\tilde{n}(f)\right|^2}{\sqrt{\Delta f T}}$$

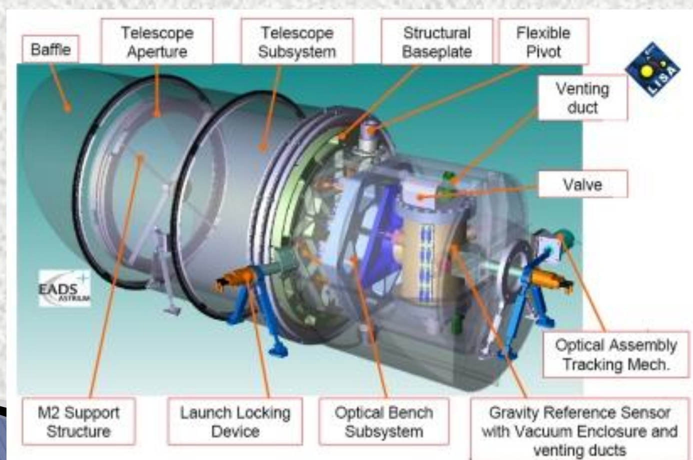
LISA

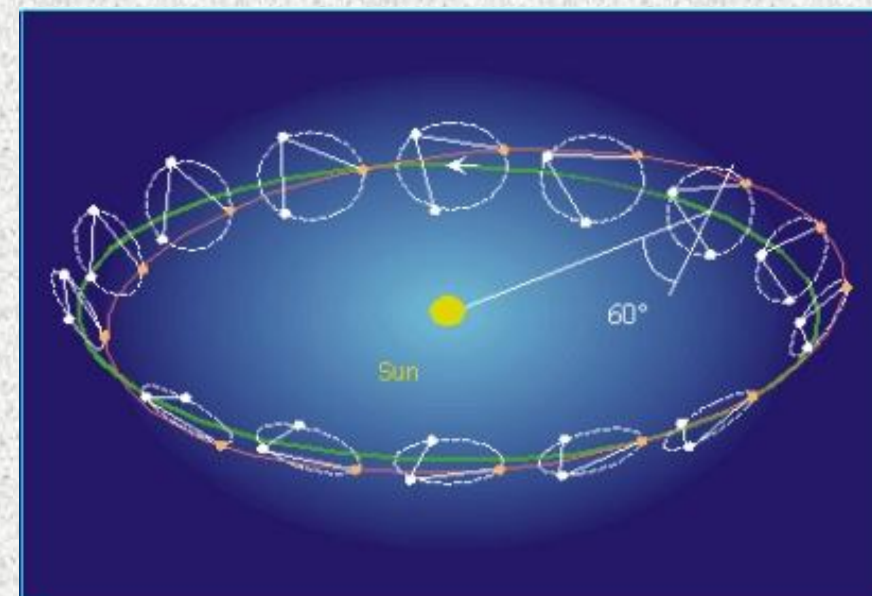


- **Laser Interferometer Space Antenna**
- **Joint project planned by NASA and ESA**

LISA

- 3 spacecrafts equipped with same instrumentation
- armlength $L \simeq 5 \times 10^6 km$
- Low frequency band $10^{-5} Hz \leq f \leq 1 Hz$ with peak sensitivity at mHz (10^{-12})
- Possible because of no ground noise
- 20º wrt earth's position, with a 60º tilt
- Proof mass! (cool)





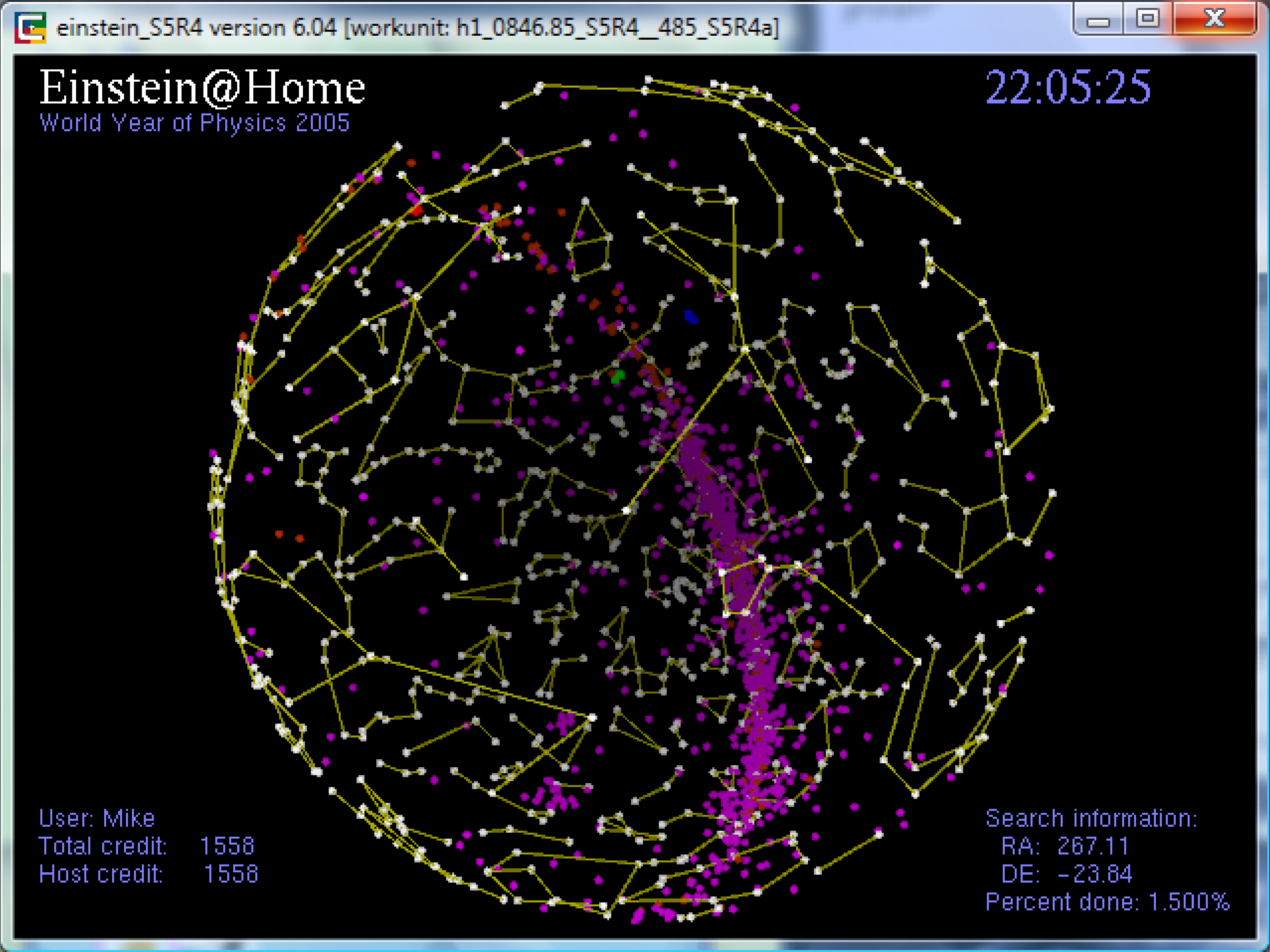
Conclusions

- We overviewed the basic mechanisms of the early Universe that are expected (if they ever happened) to produce a stochastic gravitational wave spectrum.

- Considering the predicted numerical restrictions some mechanisms are not strong enough to be detected even by close future experiments.

- If some signal shows up it will provide a snapshot of the early Universe and will feed more arguing around cosmological models

- But detecting a tiny signal and extracting a stochastic background from it is still a great challenge
- Hopefully LISA will give some useful data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Einstein@Home
World Year of Physics 2005
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Host credit: 1558
Search information:
RA: 267.11
DE: -23.84
Percent done: 1.500%