Fixing the Standard Model: Neutrino oscillations

Mischa Spelt

17 December 2008

Universiteit Utrecht

The history of the neutrino

The neutrino: electrically neutral, spin $\frac{1}{2}$ fermion, only weakly interacting; 3 leptonic flavours, very small mass (Pauli: $\sim m_e$, definitely $< 0.01 m_p$; current estimate: $\sum m_i < 0.7$ eV).

Eleven Nobel Prizes (in)directly connected to the neutrino:

Year / era		N.P.
1915–30	James Chadwick	1935
1930	Wolfgang Pauli	1945
1933	Enrico Fermi	1938
1953	**Frederick Reines**	1995
1962	**Leon Lederman, Melvin Schwartz, Jack Steinberger**	1988
1964	**Ray Davis Jr.**	2002
1987	**Masatoshi Koshiba**	2002

The Homestake experiment

Don Harmer, John Bahcall and Raymond Davis
(appeared in *Mercury*, March/April 1990, source: John Bahcall's website)

The Homestake experiment

- Radiochemical
- Solar neutrinos

  $$4p^+ + 2e^- \longrightarrow {}^4\mathrm{He} + 2\nu_e$$

  detected through

  $$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{35}\mathrm{Ar} + e^- .$$

- 615×10^3 kg C_2Cl_4.
  Depth underground: 1478 m.
  Running time: $\sim$ 30 years.

The solar neutrino problem

Result of the Homestake experiment:

$$\frac{\Phi_{\mathrm{Cl}}^{\mathrm{HS}}}{\Phi_{\mathrm{Cl}}^{\mathrm{th}}} = 0.34 \pm 0.03.$$

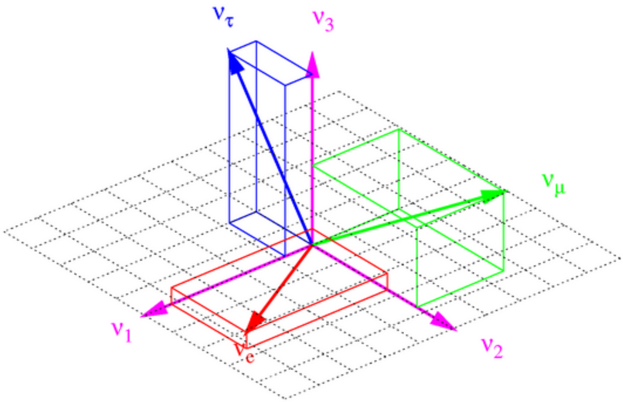
Confirmed by

- Kamiokande (1980's)
- Superkamiokande ($>$ 1995)
- GALLEX (1990's)
- GNO ($>$ 1997)
- SAGE

"If the Standard Solar Model is (more or less) correct something strange is happening"
The solution: neutrino oscillations!

What is happening?

"A neutrino with flavour *A* changes to flavour *B* and back in-flight."



Source: `http://nu.phys.laurentian.ca/~fleurot/oscillations/NeutrinoMixing.png`

Detailed example: two flavour mixing

- Flavour neutrinos ν_α, mass states ν_i. Unitary 2×2 mixing matrix U, $\nu_\alpha = \sum_i U^*_{\alpha i} \nu_i$ parametrised by one angle:

$$\begin{pmatrix} \nu_\mathrm{e} \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}.$$

- Propagation: $\nu_i(t) = \nu_i(0) e^{-\mathrm{i} E_i t/\hbar}$
- Probability amplitude to measure an electron neutrino:

$$P(\nu_\mathrm{e} \to \nu_\mathrm{e}) = \left| \frac{\nu_\mathrm{e}(t)}{\nu_\mathrm{e}(0)} \right|^2 = 1 - \sin^2(2\theta) \sin^2\left(\frac{(E_2 - E_1)t}{2} \right);$$

$$P(\nu_\mathrm{e} \to \nu_\mu) = 1 - P(\nu_\mathrm{e} \to \nu_\mathrm{e})$$

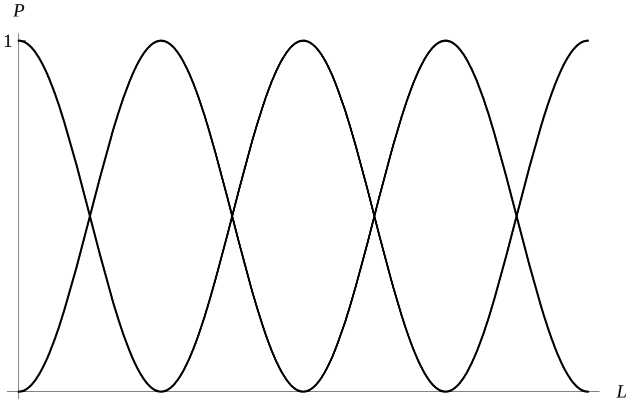
- Approximate:

$$E_i(p) = \sqrt{(pc)^2 + (m_i c^2)^2} \simeq pc + \frac{(m_i c^2)^2}{2pc} \to E + \frac{m_i^2}{2E}.$$

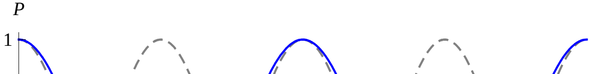
Universiteit Utrecht

- Writing $\Delta m^2 = m_1^2 - m_2^2$, $t = Lc$:

$$P(\nu_{\rm e} \to \nu_{\rm e}) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$



The probability $P(\nu_{\rm e} \to \nu_{\rm e})$ (top curve) and $P(\nu_{\rm e} \to \nu_\mu)$ (bottom curve) for a neutrino that starts out in the electron neutrino eigenstate, for mixing angle $\theta = 45^\circ$.



Pontecorvo–Maki–Nakagawa–Sakata matrix

- For three flavours ν_α $(\alpha = e, \mu, \tau)$: $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$.
- U is called the **PMNS matrix**, it is the neutrino analog of the CKM matrix.
- After lepton rephasing, it can be parametrised by three mixing angles θ_{12}, θ_{23}, θ_{13} and three phases δ, ϕ_1, ϕ_2; *e.g.*

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-\mathrm{i}\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{\mathrm{i}\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi$$

with $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$; $\Phi \equiv \begin{pmatrix} e^{\mathrm{i}\phi_1} & 0 & 0 \\ 0 & e^{\mathrm{i}\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

For three flavours

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_{i=1}^{3} U_{\alpha i}^* e^{-\mathrm{i}E_i T/\hbar + \mathrm{i}p_i L/\hbar} U_{\beta i} \right|^2 .$$

With analagous procedure as before,

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\,\mathrm{Re} \sum_{i>j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-\mathrm{i}\Delta m_{ij}^2 L/(2E)} .$$

- Only depends on $\Delta m_{ij}^2 = m_i^2 - m_j^2$, not m_i separately.
- Depends oscillatorily on experimental parameters L, E. If $\Delta m^2 L/4E \gtrsim 0.1$ this limits sensitivity.
- Of the three phases δ, ϕ_1, ϕ_2, only δ enters.
- No matter effects have been taken into account, this only works in vacuum!

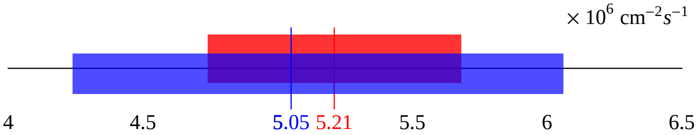
Universiteit Utrecht

The solution to the solar neutrino problem

Sudbury Neutrino Observatory (SNO). Real-time heavy-water Cherenkov detector, (1 kton of $\mathrm{D_2O}$, 2073 m underground),

$$\nu_\mathrm{e} + \mathrm{D}^+ \to 2\mathrm{p} + \mathrm{e}^-, \qquad \nu + \mathrm{D}^+ \to \mathrm{p} + \mathrm{n} + \nu.$$

Result: $\sum_{\text{flavours}} \Phi_\nu$ is conserved.



Theoretical value (red) vs. experimental SNO value (blue)

No right handed neutrino $\implies$ no neutrino mass

Recall: All fields have a left-handed and right-handed component, obtained by acting with the projection operators $\frac{1}{2}(1 \mp \gamma^5)$.

The neutrino fields only have left-handed components in agreement with the experiment of M. Goldhaber, L. Grodzins, and A. W. Sunyar, *Phys. Rev.* **109**, 1015 (1958), Available from: `http://prola.aps.org/abstract/PR/v109/i3/p1015_1`.

$$\begin{aligned}\mathcal{L} &= -\overline{\psi}(\partial\!\!\!/ + m)\psi \\ &= -\overline{\psi_{\rm L}}\partial\!\!\!/\psi_{\rm L} - \overline{\psi_{\rm R}}\partial\!\!\!/\psi_{\rm R} - m\overline{\psi_{\rm L}}\psi_{\rm R} - m\overline{\psi_{\rm R}}\psi_{\rm L},\end{aligned}$$

How do we add neutrino masses to the Standard model?

Assume the existence of a right-handed neutrino field ν_R:

$$\mathcal{L}_\mathrm{Dirac} = -m_\mathrm{D}\left(\overline{\nu_\mathrm{R}}\nu_\mathrm{L} + \overline{\nu_\mathrm{L}}\nu_\mathrm{R}\right), \qquad m_\mathrm{D} = \frac{1}{\sqrt{2}}yv,$$

cf. the Standard Model course.

Problem: If $m \lesssim 1 - 2$ eV, $y \sim 10^{-13} - 10^{-12}$!

Majorana fermions

- **Charge conjugation**: $\psi^{\mathrm{c}} = \mathcal{C}\gamma^0\psi^*$.
- **Majorana fermion:** $\psi^{\mathrm{c}} = \psi$, $\psi_{\mathrm{R}} = \psi_{\mathrm{L}}^{\mathrm{c}}$.
- **Majorana mass term:** $\mathcal{L}_{\mathrm{Maj.}} = -\frac{1}{2} m_{\mathrm{M}} \left(\overline{\psi_{\mathrm{L}}^{\mathrm{c}}} \psi_{\mathrm{L}} + \overline{\psi_{\mathrm{L}}} \psi_{\mathrm{L}}^{\mathrm{c}} \right)$.

Total mass term

$$\begin{aligned}\mathcal{L}_{\nu-\mathrm{mass}} = & -m_{\mathrm{D}}\left(\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{L}} + \overline{\nu_{\mathrm{L}}}\nu_{\mathrm{R}}\right) \\ & -\frac{1}{2}m_{\mathrm{L}}\left(\overline{\psi^{\mathrm{c}}_{\mathrm{L}}}\psi_{\mathrm{L}} + \overline{\psi_{\mathrm{L}}}\psi^{\mathrm{c}}_{\mathrm{L}}\right) \\ & -\frac{1}{2}m_{\mathrm{R}}\left(\overline{\psi^{\mathrm{c}}_{\mathrm{R}}}\psi_{\mathrm{R}} + \overline{\psi_{\mathrm{R}}}\psi^{\mathrm{c}}_{\mathrm{R}}\right).\end{aligned}$$

In matrix form:

$$\begin{aligned}\mathcal{L}_{\nu-\mathrm{mass}} = & -\frac{1}{2}\begin{pmatrix}\overline{\nu^{\mathrm{c}}_{\mathrm{L}}} & \overline{\nu_{\mathrm{R}}}\end{pmatrix}\begin{pmatrix}m_{\mathrm{L}} & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\nu_{\mathrm{L}} \\ \nu^{\mathrm{c}}_{\mathrm{R}}\end{pmatrix} + \text{h.c.} \\ \equiv & -\frac{1}{2}\overline{N^{\mathrm{c}}_{\mathrm{L}}}\cdot M\cdot N_{\mathrm{L}} + \text{h.c.}\end{aligned}$$

Diagonalising the mass matrix

- Insert unitary matrix $U = (U^\dagger)^{-1}$:

  $$\mathcal{L}_{\nu-\mathrm{mass}} = \frac{1}{2}(\overline{N_{\mathrm{L}}^{\mathrm{c}}}U)(U^\dagger M U)(U^\dagger N_{\mathrm{L}}) + \text{ h.c..}$$

  and use U to diagonalise M. $U^\dagger N_{\mathrm{L}}$ are mass eigenstates.

- $$\mathcal{L}_{\nu-mass} = -\frac{1}{2}\left(m_1\overline{\nu_1^{\mathrm{c}}}\nu_1 + m_2\overline{\nu_2^{\mathrm{c}}}\nu_2\right) + \text{h.c.}$$

  with

  $$m_{2,1} = \frac{1}{2}\left[m_{\mathrm{L}} + m_{\mathrm{R}} \pm \sqrt{(m_{\mathrm{L}} - m_{\mathrm{R}})^2 + 4m_{\mathrm{D}}^2}\right].$$

The seesaw mechanism

Consider $m_{\mathrm{L}} = 0$, $m_{\mathrm{D}} \ll m_{\mathrm{R}}$. From

$$m_{2,1} = \frac{1}{2}\left[m_{\mathrm{L}} + m_{\mathrm{R}} \pm \sqrt{(m_{\mathrm{L}} - m_{\mathrm{R}})^2 + 4m_{\mathrm{D}}^2}\right],$$

in approximation,

$$m_1 \simeq m_{\mathrm{D}}\frac{m_{\mathrm{D}}}{m_{\mathrm{R}}}, \qquad m_2 \simeq m_{\mathrm{R}}.$$

Solves two major issues if m_{R} is large... but **how to justify these assumptions?**

- $m_{\mathrm{L}} = 0$ follows from Standard Model gauge group.
- $m_{\mathrm{D}} \ll m_{\mathrm{R}}$ — GUT?

The seesaw mechanism (3 flavours)

With three flavours,

$$\frac{1}{2}\overline{N_{\mathrm{L}}^{\mathrm{c}}} \cdot M \cdot N_{\mathrm{L}} + \text{ h.c.}$$

with $N_{\mathrm{L}} = \begin{pmatrix} \nu_{\mathrm{e}} & \nu_{\mu} & \nu_{\tau} & \nu_{s_1}^{\mathrm{c}} & \nu_{s_2}^{\mathrm{c}} & \nu_{s_3}^{\mathrm{c}} \end{pmatrix}^{\mathsf{T}}$, $M = \begin{pmatrix} M^{\mathrm{L}} & (M^{\mathrm{D}})^{\mathsf{T}} \\ M^{\mathrm{D}} & M^{\mathrm{R}} \end{pmatrix}$.

If $M^{\mathrm{L}} = 0$, and $\lambda(M^{\mathrm{R}}) \gg \lambda(M^{\mathrm{D}})$, right-handed neutrinos decouple and the left-handed neutrino mass matrix can be diagonalised:

$$U^{\dagger} M_{\mathrm{light}} U = \mathrm{diag}(m_1, m_2, m_3),$$

(U is the PMNS-matrix), and

$$\nu_{\alpha} = \sum_{k=1}^{3} U_{\alpha k} \nu_k .$$

Parameters of the PMNS matrix

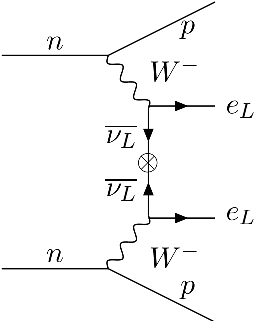
- **The parameters of the PMNS matrix.**

$$
\begin{aligned}
\Delta m_{12}^2 &= 7.92^{+0.09}_{-0.09} \times 10^{-5} \ \ \mathrm{eV}^2 \\
\sin^2\theta_{12} &= 0.314^{+0.18}_{-0.15} \\
\Delta m_{23}^2 &= 2.4^{+0.21}_{-0.26} \times 10^{-3} \ \ \mathrm{eV}^2 \\
\sin^2\theta_{23} &= 0.44^{+0.41}_{-0.22} \\
\sin^2\theta_{13} &= 0.9^{+2.3}_{-0.9} \times 10^{-2} \overset{?}{=} 0
\end{aligned}
$$

- What is θ_{13}? (CHOOZ)
- Are the flavour states Majorana or not? (i.e. are ϕ_1 and ϕ_2 physically relevant and if so, how do we measure them).
- What is the mass hierarchy?
- What is δ?

Are neutrinos Majorana or not?

- Massive states are Majorana. How about flavour states?
- Most sought-for: **neutrinoless double-beta decay**.



Feynman diagram of neutrinoless double-beta decay.
Source: F.-X. Josse-Michaux, Recent developments in thermal leptogenesis: the role of flavours in various seesaw realisations, Master's thesis, 2008, `arXiv:0809.4960`

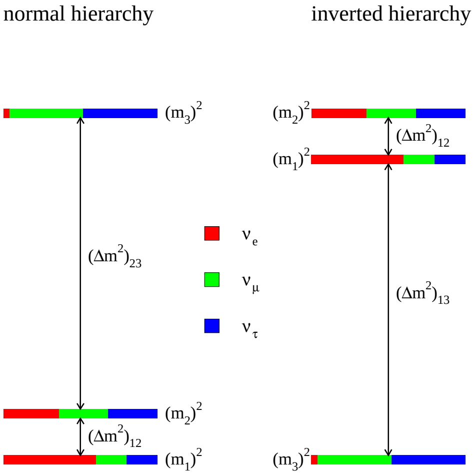
- Heidelberg–Moscow experiment: “observed!”
- Amplitude / Half-life of neutrinoless double beta decay:

$$P(0\nu\beta\beta) \propto \frac{1}{T_{1/2}} \simeq \left| \sum_i U_{ei}^2 m_i \right|^2 \equiv |m_{\rm ee}|^2$$

- Current estimates:

$$\begin{aligned} T_{1/2}(\text{Heidelberg-Moscow}) &\geq 1.9 \times 10^{25} \text{ years} \implies |m_{\rm ee}| \leq 0.55 \text{ eV} \\ T_{1/2}(\text{CUORICINO}) &\geq 1.8 \times 10^{24} \text{ years} \implies |m_{\rm ee}| \leq 1.1 \text{ eV}. \end{aligned}$$

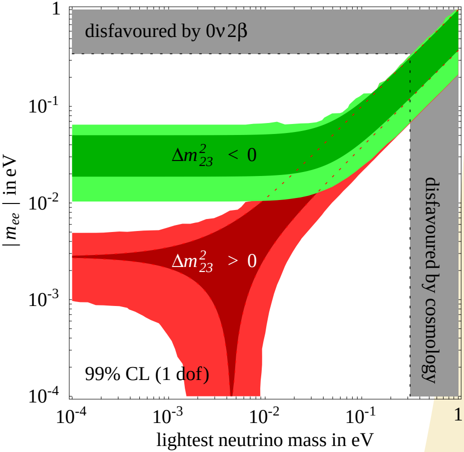
The mass hierarchy problem



Source: A. de Gouvêa, TASI lectures on neutrino physics, 2004, `arXiv:hep-ph/0411274`

The mass hierarchy problem

- Hierarchy has effect on mass bounds, cosmological observables, . . .
- Normal hierarchy seems to be simpler and preferred in GUT models.



From A. Strumia and F. Vissani, *Nucl. Phys. B* **726**, 294 (2005)

How much (if any) *CP*-violation is there?

- Only the CP-violating phase δ in oscillations, e.g. $\mathcal{P} - \overline{\mathcal{P}}$:

$$16 \cos\theta_{12} \sin\theta_{12} \cos\theta_{23} \sin\theta_{23} \cos^2\theta_{13} \sin(\delta) \prod_{i,j\ (\circlearrowleft)} \sin\left(\frac{\Delta m_{ij}^2 L}{4E}\right).$$

- Sakharov conditions for leptogenesis:
  - Lepton number violation: $\nu_\mathrm{R} \to H^+ + e^-$
  - C and CP violation: $\Gamma(\nu_\mathrm{R} \to H^+ + e^-) \neq \Gamma(\overline{\nu_\mathrm{R}} \to H^- + e^+)$
  - Out of thermal equilibrium: $\Gamma(\nu_\mathrm{R} \to H^+ + e^-) \neq \Gamma(H^+ + e^- \to \nu_\mathrm{R})$
- Fukgita and Yanagida (1986): This is possible *if* right-handed Majorana neutrinos exist.
- After leptogenesis, sphaelerons will provide baryogenesis.
- **Problem:** Hard to detect: high ν_R mass, small asymmetry.

Summary and outlook

- Solar and atmospheric neutrino problems are definitely solved by oscillations $\Longrightarrow$ neutrinos are massive.
- Likely candidate: seesaw theories. Many variants (mass-matrix limits, *ad-hoc* or GUT-down, extra Higgs scalars / Higgs triplets / exotic particles, . . .)
- Current mixing model can explain all current experiments, but not all parameters are known: Dirac / Majorana, absolute mass scale, mass hierarchy, amount of CP violation, number of (light) neutrinos, . . .
- Several new / improved experiments are being planned.
- *Not discussed:* Structure formation, cosmological implications (Yasha), sterile neutrinos as Dark Matter candidates (Rob?)

The end

Thank you for staying awake 😉

Some general review articles

- Very introductory review by C. Giunti, Coherence and Wave Packets in Neutrino Oscillations, 2003, `arXiv:hep-ph/0302026`
- Somewhat more specialised review by R. Mohapatra and A. Smirnov, Neutrino mass and new physics, 2006, `arXiv:hep-ph/0603118`
- Extensive review with recent experimental results by U. Dore and D. Orestano, Experimental results on neutrino oscillations, 2008, `arXiv:0811.1194`

Universiteit Utrecht