Dark Energy and the Cosmological Constant

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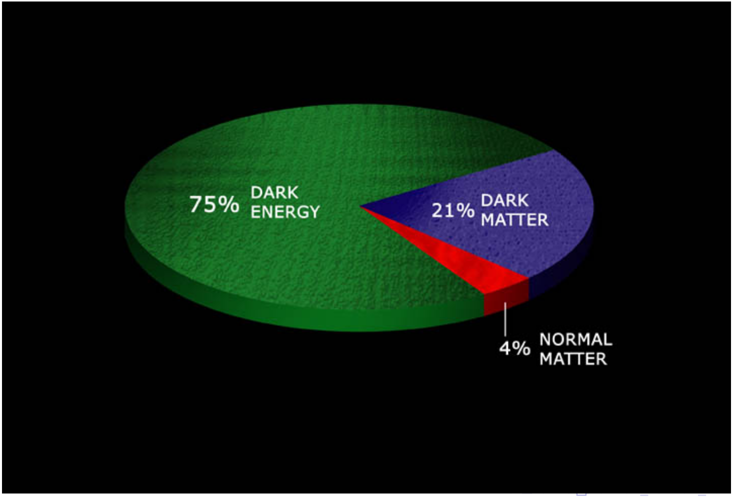
Energy budget



An accelerating universe

Einstein's equations accomodate a Cosmological Constant:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

An accelerating universe requires a Cosmological Constant or some form of *Dark Energy* ($P/\rho < -1/3$):

$$\begin{aligned} H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \end{aligned}$$

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First Friedmann Equation rewritten:

$$1 = \Omega_i + \Omega_\Lambda + \Omega_k$$

$$\Omega_i = \frac{8\pi G}{3H^2}\rho_i, \;\; \Omega_\Lambda = \frac{\Lambda}{3H^2}, \;\; \Omega_k = \frac{-k}{a^2 H^2}$$

Cosmological constant history

- Einstein postulates Cosmological Constant to obtain a static universe (positive energy density + positive curvature)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

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- Dual interpretation of the cosmological constant:

$$\begin{array}{ccc} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} & + & \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \\ & \Updownarrow & \\ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} & = & 8\pi G \left(T_{\mu\nu} - \frac{\Lambda}{8\pi G} g_{\mu\nu} \right) \end{array}$$

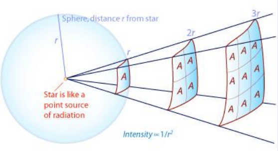
The Plan...

1. Experimental Evidence
   a. Type Ia Supernovae
   b. CMBR Anisotropies
2. The Cosmological Constant = Vacuum Energy ?
   a. Vacuum Energy
   b. The Smallness Problem
   c. The Casimir Effect
   d. The Coincidence Problem
3. Further Solutions to Dark Energy
   a. Quintessence

Distances

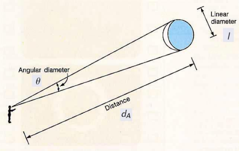
Luminosity Distance

$$F = \frac{S}{4\pi d_L^2} \Rightarrow d_L = \frac{r}{a(z)} = r(1+z)$$



Angular-Diameter Distance

$$\theta = \frac{l}{d_A} \Rightarrow d_A = ra(z) = \frac{r}{1+z}$$



Type Ia Supernovae

Type Ia Supernovae are used as *Standard Candles*

1. Integrate along a null geodesic to obtain the coordinate r in terms of the scale factor $a(t) = 1/(1+z(t))$

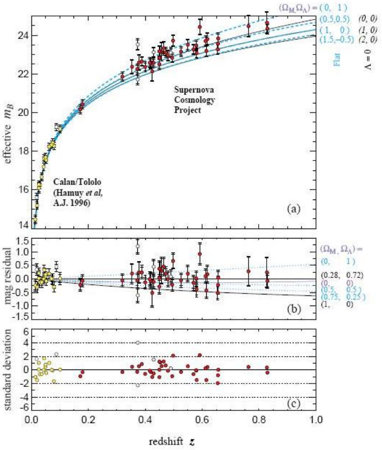
$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^r \frac{dr}{(1-kr^2)^{1/2}}$$

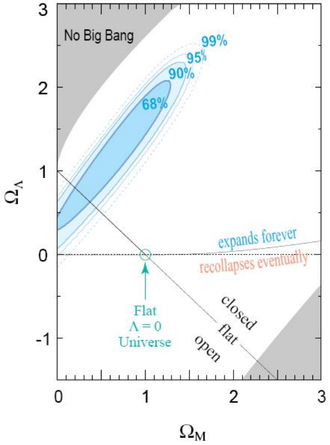
2. The Friedmann equations are used to derive the evolution of the scale factor $a(t)$ for a given energy composition Ω_i.

3. Assuming matter and dark energy only, the luminosity distance is:

$$d_L = r(1+z) = H_0^{-1}\left[z + \frac{1}{2}\left(1 + \Omega_{DE} - \frac{1}{2}\Omega_M\right)z^2\right] + \mathcal{O}(z^3)$$

Supernova Cosmology Project results (Hubble diagram and constraints on $\Omega_M - \Omega_\Lambda$ plane):



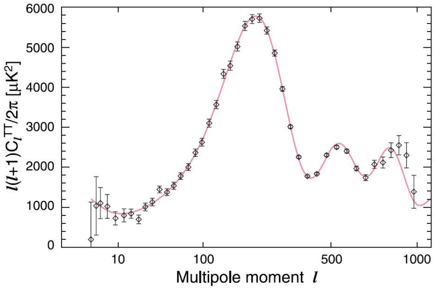


Type Ia Supernovae - remarks and drawbacks:

- A 40% difference appears between peak brightness in nearby supernova - this can be reduced to 15%
- Evolution effects - intrinsic differences between Type Ia Supernovae at high and low redshifts
- Obscuration by dust - dimming and reddening of the incoming signals

CMBR Anisotropies

CMBR Anisotropies were observed first in 1992 by COBE satellite, here is more recent WMAP 5-yr data:

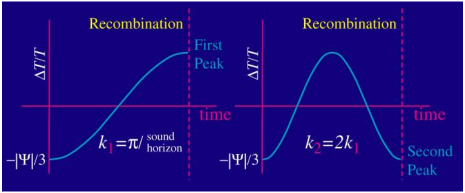


$$\begin{aligned} C(\theta \approx \pi/l) &= \left\langle \frac{\Delta T(\vec{n})}{T} \frac{\Delta T(\vec{n'})}{T} \right\rangle \\ &= \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l P_l(cos\theta) \end{aligned}$$

FRW geometry ($\{\Omega_i\}$) dictates how we these anisotropies translate to perturbations in the energy density before recombination

Before recombination the universe was a fluid of photons and charged particles

Inhomogeneities in the energy density cause sound waves to propagate through space



Modes caught at extrema of their oscillations become peaks in the CMB Power Spectrum

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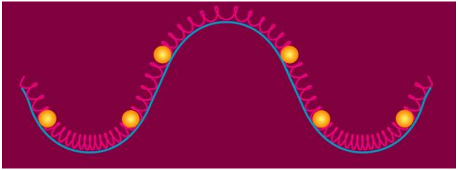

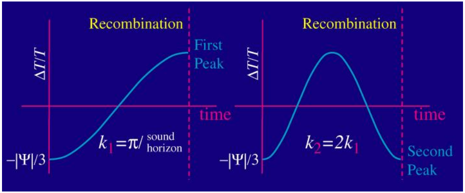

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The angular size of the observed acoustic modes:

$$\theta_s \approx \frac{r_s}{d_{sls}}$$

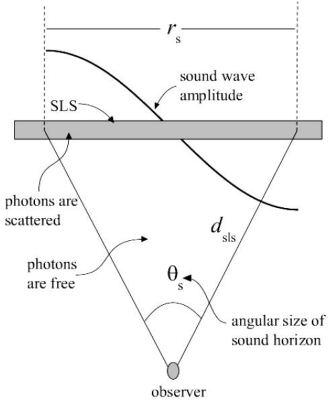
where the **sound horizon**:

$$r_s(z_{dec}, \Omega_i) \approx \int_0^{t_{dec}} c_s dt$$

$$c_s \approx c[3(1 + 3\Omega_b/4\Omega_R)]^{-1/2}$$

and recall the **angular-diameter distance**:

$$d_{sls} = \frac{r_{sls}}{1 + z_{dec}}$$



Assuming a mattered dominated universe, the sound horizon is:

$$r_s(z_{dec}, \Omega_i) \approx \int_0^{t_{dec}} c_s dt \approx \frac{c/\sqrt{3}}{H_0\sqrt{\Omega_M}} \int_{z_{dec}}^{\infty} (1+z)^{-5/2} dz$$

The angular-diameter distance will depend on the universe's energy composition:

$$d_{sls} = \frac{r_{sls}}{1+z_{dec}} \approx \frac{c\Omega_M^{-m}}{H_0} \; where \qquad m = 1 \Leftrightarrow \Omega_{DE} = 0$$

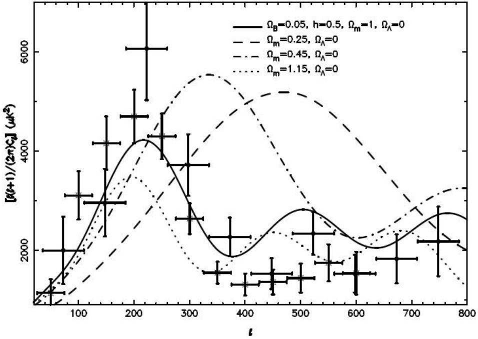
$$m \approx 0.4 \Leftrightarrow \Omega_k = 1 - \Omega_{tot} = 0$$

The location of the first peak is nearly constant for a flat universe

$$\begin{aligned} l_{peak} \approx \frac{d_{sls}}{r_s} \quad & \propto \quad \Omega_M^{-1/2} if\, \Omega_{DE} = 0 \\ & \propto \quad \Omega_M^{0.1} if\, \Omega_k = 0 \end{aligned}$$

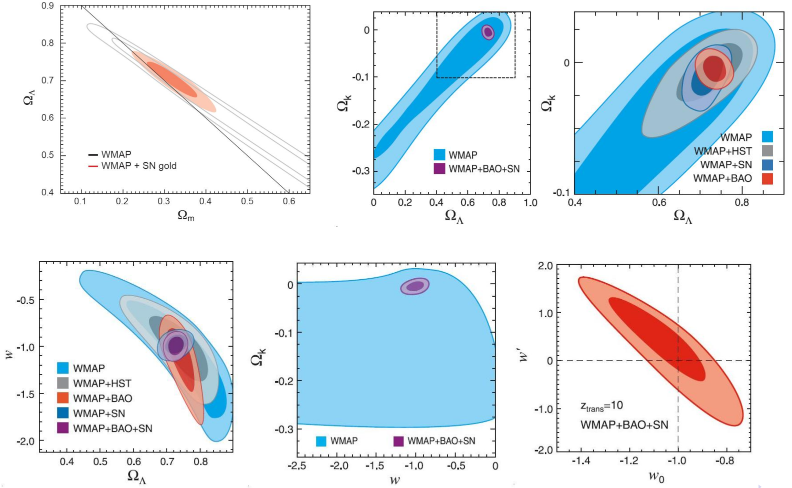
A more rigorous computation shows that:

$$\begin{aligned} l_{peak} &\approx 220\Omega_M^{-1/2} \textit{ for } \Omega_{DE} = 0 \\ &\approx 220 \textit{ for } \Omega_k = 0. \end{aligned}$$



Data points are from first results of MAXIMA and BOOMERANG experiment, curves correspond to various models

5 year (top left is 3 year) WMAP results constraining dark energy density and equation of state, and spatial curvature:



Vacuum Energy

- The Cosmological Constant as a perfect fluid:

$$p_\Lambda = w_\Lambda \rho_\Lambda = -\rho_\Lambda$$

With repulsive gravitational charge. Consider the relative geodesic acceleration:

$$\nabla \cdot \mathbf{g} = -4\pi G(\rho + 3p)$$

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- In **Quantum Field Theory**, each mode of a field has a zero-point energy:

$$1/2\hbar\omega_p$$

  Summing over all modes gives the energy density of the Vacuum:

$$\rho_{vac} = \frac{1}{2}\hbar \int_{IRcutoff}^{UVcutoff} \frac{d^3\mathbf{k}}{(2\pi)^3}\omega_k \Rightarrow \rho_{vac} = \hbar\frac{k_{cutoff}^4}{16\pi^2}$$

The Smallness Problem

- For a **Planck Mass** cutoff energy , $M_{Pl} = (8\pi G)^{-1/2} \approx 10^{18} GeV$, the Vacuum Energy density is:

$$\rho_{vac}^{(Pl)} \approx 10^{109} J/m^3$$

- The observed value of the Cosmological Constant energy density,

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- A discrepancy of **120 orders of magnitude!**
- To sufficiently lower our cutoff, we would have to assume that our theories describe physics up to an energy of $10^{-12} GeV$, an unreasonably low value.

Casimir Effect

Sean Carroll, in a review of the Cosmological Constant, remarks:

"...And the vacuum fluctuations themselves are very real, as evidenced by the Casimir effect."

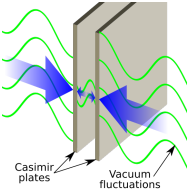
Weinberg, in a different review of the cosmological constant, writes:

"Perhaps surprisingly, it was a long time before particle physicists began seriously to worry about (quantum zero-point fluctuation contributions in Λ) despite the demonstration in the Casimir effect of the reality of zero-point energies"

Casimir Effect and Vacuum Energy:

- Casimir predicted that there is an attracting force between to parallel conducting plates in vacuum

$$\frac{F_c}{A} = -\frac{\hbar c \pi^2}{240 a^4}$$



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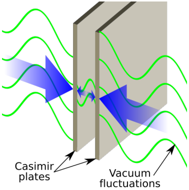


Reservations:

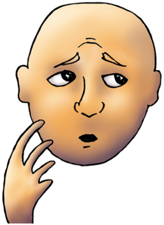
- Casimir's derivation assumes that the waves vanish on the boundaries
- The force *does* depend on interactions and not only on "pure" vacuum energy
- R.L.Jaffe shows that the Casimir Effect can be derived without using the zero-point energy term

The Coincidence Problem

- Dark Energy density is constant as the universe evolves, matter scales as $1/a^3$

in "cosmological" math $\Rightarrow \rho_M = \rho_{DE}$

- Why do we live in such a period when $\rho_M/\rho_{DE} \approx 1$? Is it a pure coincidence?



Quintessence

Time dependent Dark Energy density - in hope to predict the present energy density without fine tuning

From Quintessence:

$$L_{quin} = \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)$$

Follows a time dependent $w(t)$:

$$\rho_q(t) = \frac{1}{2}\dot{\phi}^2 + V;\ p_q(t) = \frac{1}{2}\dot{\phi}^2 - V;\ w_q = \frac{1-(2V/\dot{\phi}^2)}{1+(2V/\dot{\phi}^2)}$$

$\star$ If $\dot{\phi} \ll V$ the Cosmological Constant equation of state is reproduced.

For a given evolution $a(t) \Rightarrow V(t)$ and $\phi(t)$ can be constructed

For the case of only Scalar Field energy density and $k = 0$:

$$V(t) = \frac{3H^2}{8\pi G}\left[1 + \frac{\dot{H}}{3H^2}\right];\ \phi(t) = \int dt \left[-\frac{\dot{H}}{3\pi G}\right]^{1/2}$$

Examples:

- Power law expanding universe $a(t) = a_0 t^n$:

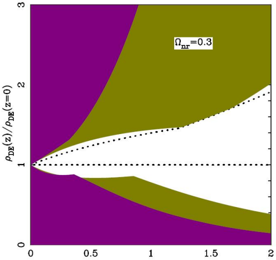
$$V(\phi) = V_0 exp\left(-\sqrt{\frac{2}{n}}\sqrt{8\pi G}\phi\right)$$

- Exponentially expanding universe
$a(t) \propto exp(\alpha t^f),\ f = \beta/4 + \beta,\ 0 < f < 1,\ \alpha > 0$

$$V(\phi) \propto (\sqrt{8\pi G}\phi)^{-\beta}\left(1 - \frac{\beta^2}{6}\frac{1}{8\pi G\phi^2}\right)$$

Drawbacks:

- No predictive power - every $a(t)$ can be modeled by a suitable $V(\phi)$
- Degeneracy in Lagrangians - an observed $w(a)$ can be derived by more than one Lagrangian
- Cosmological Constant must be set to zero
- No field theoretical justification to the potentials used
- Observations today do not prefer a time varying $w(t)$



Conclusion

- Observations favor non-zero dark energy density and (nearly) flat universe
- The Cosmological Constant provides a good (and simplest!) description for the observation
- There are conceptual problems with a Cosmological Constant
- Alternative models attempt to offer a deeper understanding