What light can supersymmetry shed on dark matter?

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Abstract

This article reviews the Lightest Supersymmetric Particle (LSP) as a dark matter candidate. The existence of WIMP dark matter is motivated and its properties are detailed. An introduction to supersymmetry and the MSSM are given, including a derivation of the super-Poincaré algebra and the superfield formalism. Arguments for the LSP as a WIMP and the Neutralino as the LSP are presented. The Neutralino is derived from the electroweak breaking of the MSSM. An overview of direct, indirect and collider detection methods are given, as well as current experimental progress.

1 Introduction

Our current best phenomenological models of cosmology, together with our latest experimental results, tell us, with good precision, that 95 percent of the energy in the universe is in need of new physics. Of this unexplained energy, twenty-two percent is believed to be invisible matter similar in composition to that found in the Standard Model of particle physics, but necessarily exclusive to it. This invisible matter is more commonly known as dark matter. In this article we introduce an extension to the Standard Model that attempts to include dark matter. This extension is namely the introduction of a new space-time symmetry, supersymmetry, that transforms bosons into fermions and vice versa. Supersymmetry has strong theoretical motivations but has to date never been detected. By enforcing this new symmetry along with a number of physical assumptions, a candidate particle for dark-matter enters into the theory.

Including this introduction, this article is split into five main sections. In the next, or second, section we present the evidence for dark matter and calculate what properties a dark matter particle must have if it were in thermal equilibrium at the beginning of the radiation era. This leads directly to the definition of a Weakly Interacting Massive Particle (WIMP). In the third section we introduce supersymmetry, beginning from a derivation of the super-Poincaré algebra, developing the superfield formalism in some detail and giving an overview of how a supersymmetric model is built. In the fourth section the Minimal Supersymmetric Standard Model is introduced, and the Lightest Supersymmetric Particle (LSP) is argued to be stable and hence a valid WIMP candidate. The Neutralino is then put forward as the LSP and shown to be derived from the electroweak breaking of the MSSM. In the fifth, or last, section we give an overview of direct, indirect and collider detection methods and also present the current experimental progress in this field.

2 Dark Matter

2.1 Evidence

The original motive for postulating the existence of dark matter came from observations of spiral galaxy rotation curves [1]. From our understanding of Newtonian physics, we expect the rotational velocity of an object in a galaxy to be dependent on its radius from the centre

$$v_c(r) = \sqrt{G\frac{M(r)}{r}}. \tag{1}$$

If the object is outside the main mass distribution of the galaxy $r > r_M$, then this distribution is essentially constant, and the rotational velocity goes as

$$v_c(r)\Big|_{r>r_M} \propto \frac{1}{\sqrt{r}}. \tag{2}$$

However, when we observe the rotational velocities of objects on the outskirts of spiral galaxies we find these to be roughly constant

$$v_c(r)\Big|_{r>r_M} \approx const. \tag{3}$$

See figure 1 for an example of the observed versus expected Newtonian rotational velocities in the NGC 6503 spiral galaxy.

To explain this anomaly, a spherical dark matter halo is postulated with a density

$$\rho_{\text{halo}}(r) \propto \frac{1}{r^2}. \tag{4}$$

This then gives a mass distribution $M'(r)$ outside the original bulk mass distribution of the galaxy $M(r)$ with the property that

$$M'(r)\Big|_{r>r_M} = \int \rho_{\text{DM}} dV\Big|_{r>r_M} \propto r. \tag{5}$$

Hence the rotational velocity outside the original mass distribution of the galaxy is constant as observed. From figure 1 it is clear that if there is a dark matter halo, more than 90 percent of the galaxies mass must be dark. From sky observations we know the luminous energy density is $\Omega_{\text{lum}} \sim 0.01$[1]. Therefore we conclude that $\Omega_{\text{DM}} \gtrsim 0.1$.

More precise values for the dark matter energy density come from our models of structure formation, the era during which galaxies and stars began to form. Currently the leading model postulates that once the universe has cooled enough, non-baryonic dark matter clumps under the force of gravity while baryonic matter remains too strongly coupled to electrons via the Coulomb interaction to feel gravity. Later, when the baryonic matter has decoupled, it falls into the gravitational potential wells already formed by the dark matter. For the dark matter to successfully form these wells it must in general be cold (non-relativistic) during the era of structure formation. The most popular model of cold dark matter is ΛCDM, also known as the concordance model[21]. Using the latest CMB data from the WMAP experiment[22], the ΛCDM model gives ($h = 0.72$)

$$\begin{aligned}\Omega_{\text{b}} &= 4.6 \pm 0.1\%, \\ \Omega_{\text{DM}} &= 22 \pm 2\%.\end{aligned}$$

[1]The energy density of a particular quantity X is defined as $\Omega_{\text{X}} = \rho_{\text{X}}/\rho_{\text{crit}}$ where $\rho_{\text{crit}} = 3H^2/8\pi G$ is the critical density. The critical density corresponds to the universe having no spatial curvature.

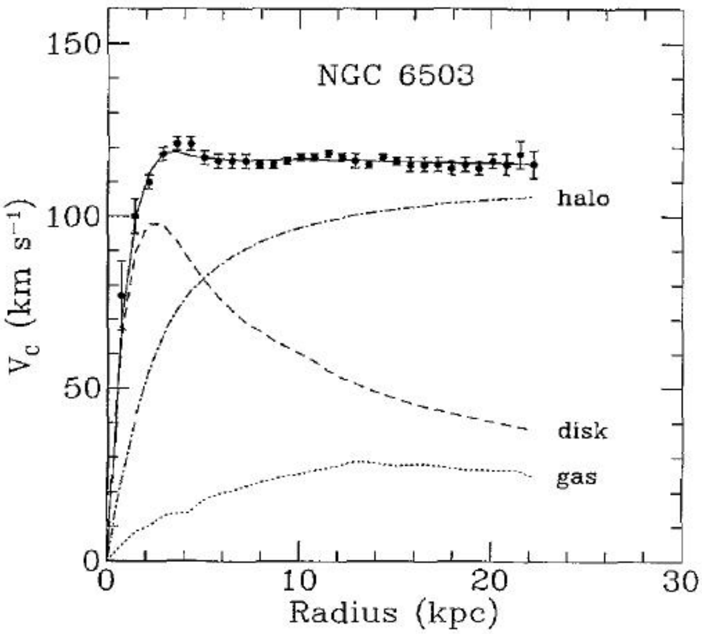

Figure 1: Rotation curve for the spiral galaxy NGC 6503 (From K.G Begeman *et al.*[11])

2.2 Properties

2.2.1 Thermal Freeze Out

We now consider how a postulated cold dark matter particle χ could give rise to the dark matter energy densities that we perceive to exist today. The assumption is made that the particle was in thermal equilibrium at the beginning of the radiation era. There also exist non-thermal explanations of how these relic densities can arise, such as from phase transitions, but these are not discussed here. This section closely follows Kolb and Turner[12], but digresses to make a rough estimate of the relic density freeze out.

In the beginning the particle χ is assumed to be in thermal equilibrium, with equal rates of creation and annihilation into lighter particles l: $\chi + \bar{\chi} \leftrightarrow l + \bar{l}$. As the universe expands and cools, the particle density n_χ can be solved for by using the Boltzmann equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_A v \rangle [n_\chi^2 - (n_\chi^{\mathrm{EQ}})^2], \tag{6}$$

where $\langle \sigma_A v \rangle$ is the velocity averaged annihilation cross section of χ (refer to appendix B for a derivation and explanation of this form of the Boltzmann equation). Clearly in the absence of interactions the density is inversely proportional to the comoving volume

$$n_\chi \big|_{\cancel{\langle \sigma_A v \rangle}} \propto a^{-3}, \tag{7}$$

as we would expect. This suggests it may be more convenient to express our density as a comoving quantity. Using the property that the entropy per comoving volume is conserved, $S = sa^3 = \mathrm{const}$, we define the comoving density as

$$N_\chi := \frac{n_\chi}{s} \tag{8}$$

$$\Rightarrow \; N_\chi\big|_{\cancel{\langle\sigma_A v\rangle}} = const. \tag{9}$$

We also use the fact that temperature scales as $t \propto 1/T^2$ during the radiation era, to define a new time coordinate

$$\tau := \frac{m}{T} \propto m\sqrt{t} \tag{10}$$

In terms of these new coordinates the Boltzmann equation (120) becomes

$$\frac{\tau}{N_\chi^{\mathrm{EQ}}}\frac{dN_\chi}{d\tau} = -\frac{\Gamma_A}{H}\left[\left(\frac{N_\chi}{N_\chi^{\mathrm{EQ}}}\right)^2 - 1\right], \tag{11}$$

where $\Gamma_A := n_\chi^{\mathrm{EQ}}\langle\sigma_A v\rangle$ is the annihilation rate. Qualitatively we see that when Γ_A/H drops below $< O(1)$ it implies $\Delta N/N < O(1)$ i.e. annihilation freezes out meaning the comoving number density freezes in. We will therefore denote the time when freeze out occurs by τ_f, and define it as $\Gamma(\tau_f) \simeq H(\tau_f)$.

More precisely, we consider a simple analytic solution to the Boltzmann equation by making some key assumptions. Most crucially, we assume that $\langle\sigma_A v\rangle$ has no temperature dependence i.e. we set $\langle\sigma_A v\rangle$ constant. In the radiation era $H(\tau) = H(m)\tau^{-2}$, so we can rewrite (11) as

$$\frac{dN_\chi}{d\tau} = -\lambda\tau^{-2}(N_\chi^2 - N_\chi^{\mathrm{EQ}^2}), \tag{12}$$

where

$$\lambda = \frac{\langle\sigma_A v\rangle s}{H(m_\chi)} = 0.264\; g_{*S}\; g_*^{-1/2}\; m_{Pl} m_\chi \langle\sigma_A v\rangle, \tag{13}$$

$$N_\chi^{\mathrm{EQ}} = 0.145\; g_{*S}^{-1}\; g\; \tau^{\frac{3}{2}} e^{-\tau}. \tag{14}$$

By denoting the departure from equilibrium by $\Delta := N_\chi - N_\chi^{\mathrm{EQ}}$, we may again rewrite the Boltzmann equation[12] as

$$\Delta' := \frac{d\Delta}{d\tau} = -\frac{dN_\chi^{\mathrm{EQ}}}{d\tau} - \lambda\tau^{-2}\Delta(2N_\chi^{\mathrm{EQ}} + \Delta). \tag{15}$$

At early times $\tau \ll \tau_f$, the comoving density stays close to equilibrium and we may take Δ and Δ' to be small. At late times $\tau \gg \tau_f$, after freeze out, the comoving density has far departed from equilibrium such that $\Delta \simeq N_\chi \gg N_\chi^{\mathrm{EQ}}$. To a good approximation we set $N_\chi^{\mathrm{EQ}\prime}$ and N_χ^{EQ} to zero. Equation (15) then becomes

$$\Delta' \simeq -\lambda\tau^{-2}\Delta^2, \tag{16}$$

which may be integrated from τ_f to $\tau = \infty$ to give the comoving density today as

$$N_{\chi 0} \approx \Delta_\infty \simeq \frac{\tau_f}{\lambda}. \tag{17}$$

It remains to solve for τ_f. An approximate solution can be found by solving the freeze out criterion $\Gamma(\tau_f) \simeq H(\tau_f)$ for τ_f. This yields $\tau_f \approx 20/m_\chi$.

Substituting into (17) the freeze out time τ_f, as well as values for the Planck mass m_{Pl}, the entropy density today s_0 and the relativistic degrees of freedom g_*, we find

$$N_{\chi 0} \approx \frac{10^{-9}[pb \cdot c][\mathrm{GeV}]}{m_\chi \langle \sigma_A v \rangle}. \tag{18}$$

Therefore the comoving number density is inversely proportional to the annihilation cross section. Figure 2 illustrates how a greater cross section gives a lower freeze out relic density.

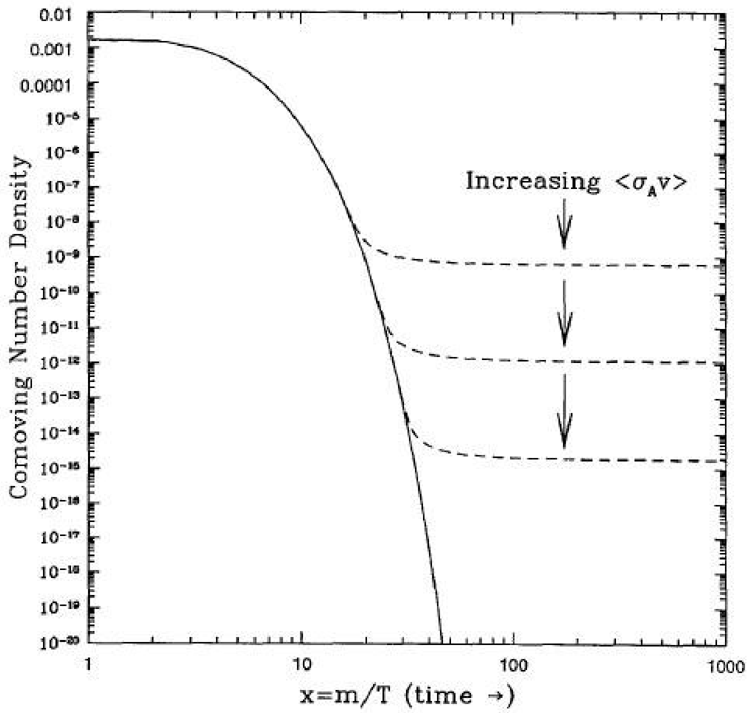

Figure 2: Freeze out of the comoving number density (From Kolb and Turner[12])

It is now possible to compute the energy density of the particle χ today as

$$\Omega_\chi h^2 = \frac{m_\chi}{\rho_c}(n_\chi) \tag{19}$$

$$= \frac{m_\chi}{\rho_c}(s_0 N_{\chi 0}) \tag{20}$$

$$\approx \frac{0.1 pb \cdot c}{\langle \sigma_A v \rangle} \tag{21}$$

where ρ_c is the critical density. We see that the energy density is independent of the mass of the potential cold dark matter candidate, depending only on the velocity averaged annihilation cross section. Setting $\Omega_\chi = \Omega_{\mathrm{DM}} \approx 0.2$ and solving for $\langle \sigma_A v \rangle$ gives

$$\langle \sigma_A v \rangle \sim 1 pb \cdot c \simeq \frac{\alpha^2}{(M_W)^2}, \tag{22}$$

where α is a typical coupling constant ($\sim 10^{-2}$) and M_W is the mass of the weak gauge boson. That is, the cross section is of the order of weak scale interactions. We conclude that for a cold dark matter particle to give the correct thermal relic energy density, it must be weakly interacting.

2.2.2 WIMPs

In the previous section we found that a cold dark matter particle should be weakly interacting to give the correct thermal relic density. We will call such particles Weakly Interacting Massive Particles (WIMPs). WIMPs are colour neutral due to their non-baryonic nature. They must be electrically neutral (or interact very weakly) to classify as being dark. Lastly, they must be stable or have a very long lifetime (close to the age of the universe) to survive as a thermal relic.

There are many possible WIMP candidates in the literature. Once popular were the light neutrino and the heavy fourth generation neutrino. The former, however, is too hot to explain structure formation and the latter has been ruled out by direct detection experiments, as well as having no inherent reason for being stable. Still popular WIMP candidates include the lightest supersymmetric particle (LSP), the lightest Kaluza-Klein modes from extra dimensional theories and stable fermions from little Higgs models. Here we only discuss the LSP as a WIMP candidate, beginning with a detailed introduction to supersymmetry.

3 Supersymmetry

3.1 Motivation

Supersymmetry (SUSY) is a postulated new symmetry of spacetime that has yet to be verified, or falsified, by experiment. The characteristic property of supersymmetry is that it transforms bosonic states into fermionic states and vice versa. That is, given a supersymmetric generator Q, we have

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle \quad \text{and} \quad Q|\text{Fermion}\rangle = |\text{Boson}\rangle.$$

Adding this new symmetry to our existing spacetime symmetries involves extending the Poincaré Algebra. This is discussed in detail in the following section. As we shall see, extending the Standard Model of particle physics with supersymmetry has many useful consequences.

The Standard Model suffers from what is known as the naturalness (or fine-tuning) problem. Specifically, that the scalar masses of the theory (namely the Higgs boson mass) pick up their largest one-loop corrections to the tree level mass from the top quark loop, which is quadratically divergent. This means that the two biggest contributions to the Higgs boson mass

$$m_H^2 = m_{\text{tree}}^2 - \frac{\lambda_t^2}{8\pi^2}{\Lambda_{\text{UV}}}^2 + \ldots \sim (200\text{GeV})^2 \tag{23}$$

are the positive tree level mass and a negative term dependent on the ultraviolet cut-off Λ_{UV} squared, where the minus sign is due to the closed fermion loop. The ultraviolet cut-off is effectively the point at which the Standard Model breaks and we expect new physics to occur, therefore this value is ideally very large (close to the Planck scale for example). This is thus the naturalness problem: two very large contributions must be precisely tuned such that their difference gives the much smaller expected Higgs mass. Supersymmetry solves this problem by providing a bosonic superpartner to the top quark, the scalar top, that will have an identical but positive one-loop contribution to the Higgs mass, thereby exactly cancelling the quadratically divergent terms. The two cancelling loops are shown in figure 3. Not given in equation (23) are the logarithmically divergent corrections to the Higgs boson mass. These corrections are generally not cancelled by supersymmetry, but can be omitted from the fine tuning discussion as they are proportional to

$$m_H^2 \ln \frac{\Lambda_{\rm UV}}{m_H}, \tag{24}$$

and are thus of the same order as the tree level mass.

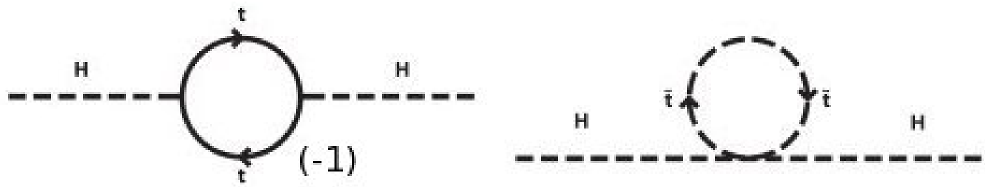

Figure 3: One loop contributions to the Higgs mass from the top quark (left) and its superpartner the scalar top quark (right)

Certain supersymmetric extensions to the Standard Model, particularly the Minimal Supersymmetric Standard Model (MSSM) to be introduced in section 4, have the property that their gauge coupling constants unify at some energy scale. For the MSSM the energy scale where the the evolutions of the gauge coupling constants meet up is 10^{16} GeV, as shown in figure 4. This scale is often referred to as the Grand Unified Theory (GUT) scale, believed by some to be the energy at which a larger GUT internal symmetry group breaks into the Standard Model symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. That supersymmetry gives a progression from the Standard Model to GUTs is another motivation for its existence.

However, the most interesting motivation for supersymmetry in the context of this report is that the MSSM, together with a symmetry that insures baryon and lepton numbers are conserved, has the property that its lightest superpartner is stable and only weakly interacting i.e. it has all the properties of a WIMP (see section 2.2.2). Supersymmetry thus provides a compelling dark matter candidate, whose derivation is the main focus of the rest of this section.

3.2 Super-Poincaré Algebra

3.2.1 The Poincaré Lie Algebra

The Poincaré group consists of all spacetime transformations, namely; translations, rotations and boosts. Translations are generated by the energy-momentum operators P_μ, rotations by the angular momentum operators J_i and boosts by the operators K_i. The generators J_i and K_i together give all proper orthochronous Lorentz tansformations, thereby forming a subgroup of the Poincaré group (the Lorentz group) and obeying the sub-algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [K_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} J_k. \tag{25}$$

Often the rotation and boost generators are combined into an antisymmetric second rank tensor $M_{\mu\nu}$, where $M_{ij} = \epsilon_{ijk} J_k$ and $M_{0i} = -M_{i0} = -K_i$. The commutation relations of the Poincaré

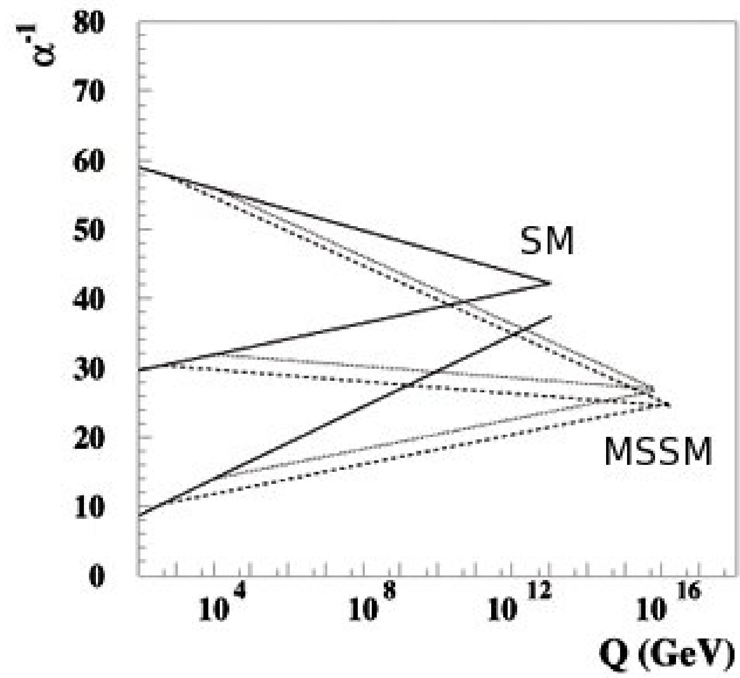

Figure 4: Evolution of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings for SM (solid) and MSSM (dashed) (From Braz[14]

generators are then written as

$$[P_\mu, P_\nu] = 0, \tag{26}$$
$$[M_{\mu\nu}, P_\lambda] = i(\eta_{\nu\lambda}P_\mu - \eta_{\mu\lambda}P_\nu), \tag{27}$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}), \tag{28}$$

forming a *Lie algebra* kown as the Poincaré algebra, with all the generators satsifying the Jacobi identity.

The Lorentz group generators may also be rewritten as $A_i = \frac{1}{2}(J_i + iK_i)$ and $B_i = \frac{1}{2}(J_i - iK_i)$, in which case the algebra decomposes into two sub-algebras

$$[A_i, A_j] = i\epsilon_{ijk}A_k, \quad [B_i, B_j] = i\epsilon_{ijk}B_k, \quad [A_i, B_j] = 0, \tag{29}$$

with each isomorphic to the Lie group $SU(2)$. We can therefore find representations of the Lorentz group by taking those of the product group $SU(2) \times SU(2)$. The Casimir operators[2] of this group are A^2 and B^2, and have as their (angular momentum) eigenvalues $j(j+1)$ and $j'(j'+1)$ respectively. A representation of the Lorentz group can thus be labelled as (j, j'). A Lorentz scalar transforms as the representation $(0, 0)$ and a four-vector as $(\frac{1}{2}, \frac{1}{2})$. The representations $(\frac{1}{2}, 0) \equiv \psi_L$ and $(0, \frac{1}{2}) \equiv \chi_R$ are equivalent to two component *Weyl* spinors, which transform independently under the action of

[2]Operators that commute with every generator

the group $SL(2,C)$[7];

$$\begin{aligned} \psi_L &\rightarrow exp(i\sigma \cdot (\theta - i\phi))\psi_L = M\psi_L, & (30)\\ \chi_R &\rightarrow exp(i\sigma \cdot (\theta + i\phi))\chi_R = N\chi_R, & (31)\\ & \text{for } M, N \in SL(2,C), & (32) \end{aligned}$$

where θ and ϕ are the rotation and boost parameters of $\mathbf{J}$ and $\mathbf{K}$ respectively. The four-component *Dirac* spinor transforms as the direct sum of these two representations $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$ and may be written as

$$\psi_D = \begin{pmatrix} \psi_L \\ \chi_R \end{pmatrix}. \tag{33}$$

Similarly, by noting that the spinor $-i\sigma_2\psi_L^*$ transforms as $(0,\frac{1}{2})$, a four-component *Majorana* spinor satisfying the condition $\psi = \psi^c = C\bar{\psi}^T$ (see appendix A.1) is written as

$$\psi_M = \begin{pmatrix} \psi_L \\ -i\sigma_2\psi_L^* \end{pmatrix}. \tag{34}$$

It is therefore clear that a Weyl spinor may be written as a four-component Majorana spinor (and vice versa), as we will use in the next section.

3.2.2 Extension to a Graded Lie Algebra

A no-go theorem by Coleman and Mandula[10] states that the most general Lie algebra for symmetries of an S-matrix (for a local relativistic quantum field theory in 4D spacetime) can only have as generators those of the Poincaré group along with a finite number of Lorentz scalar generators belonging to the Lie algebra of a compact Lie group (for example the generators of the standard model internal symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$). This restriction on the spacetime generators can however be relaxed if the Lie algebra of the symmetries is generalized to a *graded Lie algebra*.

A graded Lie algebra consists of commuting *even* generators X and anti-commuting *odd* generators Q, satisfying

$$\{Q, Q'\} = X, \quad [X, X'] = X", \quad [Q, X] = Q'. \tag{35}$$

The even generators X are therefore those of the original Poincaré algebra, namely P_μ and $M_{\mu\nu}$, which satisfy the (even) commutation relations given in (28). To find valid odd generators we consider irreducible representations of the Lorentz group, (j, j'), with spin $j + j'$. Such representations may be written with spinor components as linear combinations of $Q_{\alpha_1...\alpha_{2j};\dot{\beta}_1...\dot{\beta}_{2j'}}$, where the undotted and dotted spinor indices denote transformation as left and right Weyl spinors, respectively. Consider the anti-commutator of a Q with its hermitian conjugate $Q^\dagger$

$$\{Q_{\alpha_1...\alpha_{2j};\dot{\beta}_1...\dot{\beta}_{2j'}}, Q^\dagger_{\dot{\gamma}_1...\dot{\gamma}_{2j};\delta_1...\delta_{2j'}}\}. \tag{36}$$

By choosing all the spinor indices equal $\alpha = \dot{\beta} = \gamma = \dot{\delta} = 1$ (to simplify the Glebsch Gordon coefficients), the resulting components of the commutator become

$$\{Q_{\underbrace{1...1}_{2j};\underbrace{\dot{1}...\dot{1}}_{2j'}}, Q^\dagger_{\underbrace{\dot{1}...\dot{1}}_{2j};\underbrace{1...1}_{2j'}}\} = X_{\underbrace{1...1}_{2(j+j')};\underbrace{\dot{1}...\dot{1}}_{2(j+j')}} \tag{37}$$

and therefore transform as a $(j+j', j+j')$ representation. Because Q and $Q^\dagger$ are anti-commuting (odd) generators, their spin $j+j'$ must be half integer. Therefore the spin of the resulting $2(j+j')$ representation must be integer, and we can conclude that it belongs to the commuting (even) generators X.

From the no-go theorem we know that the only valid non-scalar generators of the even Lie algebra are those of the Poincaré group, such that

$$\{Q, Q^\dagger\} = P + M, \tag{38}$$

where indices and prefactors have been suppressed for simplicity. P_μ as a four vector transforms as the representation $(\frac{1}{2}, \frac{1}{2})$ and $M_{\mu\nu}$ as an anti-symmetric second rank tensor transforms as a combination of $(1,0)$ and $(0,1)$.

Considering first only the odd generators Q that commute with translations

$$[Q, P_\mu] = 0, \tag{39}$$

we find that

$$[P, \{Q, Q^\dagger\}] = [P, P] + [P, M], \tag{40}$$

$$\Rightarrow [P, M] = 0 \tag{41}$$

in contradiction with (27), from which we conclude that equation (38) can not have M dependence. Hence the only valid candidate for $(j+j', j+j')$ is $(\frac{1}{2}, \frac{1}{2})$ representing the energy-momentum generator of the Poincaré group. This implies Q must be a Weyl spinor, and we may associate

$$Q_\alpha \equiv (\frac{1}{2}, 0), \quad Q^\dagger_{\dot{\alpha}} \equiv (0, \tfrac{1}{2}). \tag{42}$$

With a suitable choice of normalization, we arrive at the anti-commutation relation

$$\{Q_\alpha, (Q_\beta)^\dagger\} = P_{\alpha\dot{\beta}} = \sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{43}$$

where σ^i are the standard Pauli matrices and $\sigma^0 = -\mathbf{1}_{2\times 2}$ and P_μ the energy-momentum four vector of the Poincaré group.

The possibility of having odd generators Q that do not commute with translations is considered in the original paper of Haag *et al.*[8], which concludes that there are in general no such new generators.

In four-component spinor notation, the supersymmetry extension to the Poincaré algebra, known as the *super-Poincaré* algebra, is given by

$$[P_\mu, Q_a] = 0, \tag{44}$$

$$[M_{\mu\nu}, Q_a] = -(\frac{1}{2}\sigma_{\mu\nu})_{ab} Q_b, \tag{45}$$

$$\{Q_a, \bar{Q}_b\} = 2(\gamma^\mu)_{ab} P_\mu. \tag{46}$$

Using the property that the charge is a Majorana spinor satisfying the condition $Q = Q^c = C\bar{Q}^T$, the following (anti-)commutators also follow

$$[P_\mu, \bar{Q}_a] = 0, \tag{47}$$

$$[M_{\mu\nu}, \bar{Q}_a] = \bar{Q}_b(\frac{1}{2}\sigma_{\mu\nu})_{ba}, \tag{48}$$

$$\{Q_a, Q_b\} = -2(\gamma^\mu C)_{ab} P_\mu, \tag{49}$$

$$\{\bar{Q}_a, \bar{Q}_b\} = 2(C^{-1}\gamma^\mu)_{ab} P_\mu. \tag{50}$$

From relation (44) we see that P^2 is a Casimir of the algebra. This means that a state ψ and its superpartner state $Q\psi$ must have the same mass

$$Q(P^2\psi) = Q(m_\psi^2\psi) \quad \Rightarrow \quad P^2(Q\psi) = m_\psi^2(Q\psi). \tag{51}$$

The W^2 Casimir of the original Poincaré algebra constructed from the Pauli-Lubanski four-vector $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$ is no longer a Casimir of the super-Poincaré algebra, as is evident from (45). This means that a state and its superpartner state will have different spin.

3.3 Superfields

3.3.1 Superfield Formalism

In the previous section it was found that a supersymmetry transformation on a field alters its spin. It would thus be useful to combine fields with different spin (bosonic and fermionic), which transform into one another under supersymmetry, into one all encompassing *superfield*. This superfield would then conveniently transform non-trivially into itself under supersymmetry transformations. However, we cannot simply add bosonic and fermionic fields together as they differ in their commutation and Lorentz transformation properties. To be able to add a fermionic field ψ to scalar (bosonic) fields, we need to contract the spinor into a scalar. We introduce a Majorana spinor θ whose components θ_1, θ_2, θ_3 and θ_4 are anti-commuting Grassmann numbers, satisfying

$$\{\theta_a, \theta_b\} = 0 \tag{52}$$

By requiring that

$$\{\theta_a, \psi_b\} = 0 \tag{53}$$

we can construct scalar terms for a spinor ψ, such as $\bar{\theta}\psi$. These scalar contractions behave identically to ordinary bosonic scalar fields $\mathcal{S}$ and hence expressions such as $\bar{\theta}\psi + \mathcal{S}$ are now possible. For a more detailed discussion see [6][5].

Superfields are thus defined to exist in an extension of ordinary four-dimensional spacetime known as *superspace*, which is labeled by the spacetime coordinates x^μ and the four spinor coordinates θ_a. Because of the anti-commuting nature of the θ components, it is possible to expand a superfield into a finite number of linearly independent θ-terms. A convenient basis for this expansion is given by Baer *et al.*[5]

$$\begin{aligned}\hat{\Phi}(x,\theta) = \mathcal{S} - i\sqrt{2}\bar{\theta}\gamma_5\psi - \frac{i}{2}(\bar{\theta}\gamma_5\theta)\mathcal{M} + \frac{1}{2}(\bar{\theta}\theta)\mathcal{N} + \frac{1}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)V^\mu \\ + i(\bar{\theta}\gamma_5\theta)[\bar{\theta}(\lambda + \frac{1}{\sqrt{2}}\partial\!\!\!/\psi)] - \frac{1}{4}(\bar{\theta}\gamma_5\theta)^2[\mathcal{D} - \frac{1}{2}\Box\mathcal{S}]\end{aligned} \tag{54}$$

giving 8 fermionic and 8 bosonic complex component fields $\{\psi_a, \lambda_b\}$ and $\{\mathcal{S}, \mathcal{M}, \mathcal{N}, V^\mu, \mathcal{D}\}$ respectively. We will refer to this basis as the canonical basis. Certain operations, such as the product of two superfields or a symmetry transformation, may give a superfield with different θ terms but it will always be possible to rewrite these in the canonical basis.

3.3.2 SUSY Transformations

Now that we have a general superfield we consider how it transforms under an infinitismal global supersymmetry transformation. In order to construct a unitary transformation operator we require a scalar operator. The supersymmetry generator Q however is a Majorana spinor, therefore the infinitesimal transformation parameter α is chosen to also be a Majorana spinor, so that the contraction $\bar{\alpha}Q$ gives a scalar. The transformation is then given by

$$\hat{\Phi}' = e^{i\bar{\alpha}Q}\hat{\Phi}e^{-i\bar{\alpha}Q} = \hat{\Phi} + i[\bar{\alpha}Q, \hat{\Phi}] \tag{55}$$

Recall that the spacetime generator P_μ generates infinitismal spacetime translations

$$\delta_{\mathrm{a}}\phi = a^\mu[iP_\mu, \phi] = a^\mu\partial_\mu\phi, \tag{56}$$

such that it may be represented by a differential operator $P_\mu \equiv -i\partial_\mu$ in spacetime. Similarly, we can expect Q to be represented by a differential operator in superspace. To derive this representation, first observe that Q as a spinorial operator will change the Lorentz transformation properties of the superfield by either removing or adding a θ, so that

$$[Q_m, \hat{\Phi}(x,\theta)] = \left(M_{mn}\frac{\partial}{\partial\bar{\theta}} + N_{mn}\theta\right)\hat{\Phi}(x,\theta) \tag{57}$$

where the matrices M_{mn} and N_{mn} must still be determined. Next consider two successive SUSY transformations

$$[[\bar{\alpha}_1 Q, \bar{\alpha}_2 Q], \hat{\Phi}] = [\bar{\alpha}_1 Q, [\bar{\alpha}_2 Q, \hat{\Phi}]] - [\bar{\alpha}_2 Q, [\bar{\alpha}_1 Q, \hat{\Phi}]]. \tag{58}$$

The right hand side is found to reduce to an expression involving the unknown matrices, whereas the left hand side contains the commutation relation

$$\{Q, Q\} = -2\gamma^\mu C P_\mu \equiv 2i\gamma^\mu C\partial_\mu, \tag{59}$$

and therefore contributes a spacetime derivative of the superfield. The equation is solved by setting $M = 1$ and $N = i\not{\partial}$, thereby giving the supersymmetric transformation of the superfield

$$\delta_\alpha\hat{\Phi} = i[\bar{\alpha}Q, \hat{\Phi}] = (-\bar{\alpha}\frac{\partial}{\partial\bar{\theta}} - i\bar{\alpha}\not{\partial}\theta)\hat{\Phi}. \tag{60}$$

By rewriting the transformed superfield $\delta_\alpha\hat{\Phi}$ in the canonical basis, the components are found to transform supersymmetrically as

$$\begin{aligned}
\delta\mathcal{S} &= i\sqrt{2}\bar{\alpha}\gamma_5\psi && (61)\\
\delta\psi &= -\frac{\alpha\mathcal{M}}{\sqrt{2}} - i\frac{\gamma_5\alpha\mathcal{N}}{\sqrt{2}} - i\frac{\gamma_\mu\alpha V^\mu}{\sqrt{2}} - \frac{\gamma_5\not{\partial}\mathcal{S}\alpha}{\sqrt{2}} && (62)\\
\delta\mathcal{M} &= \bar{\alpha}(\lambda + i\sqrt{2}\not{\partial}\psi) && (63)\\
\delta\mathcal{N} &= i\bar{\alpha}\gamma_5(\lambda + i\sqrt{2}\not{\partial}\psi) && (64)\\
\delta V^\mu &= -i\bar{\alpha}\gamma^\mu\lambda + \sqrt{2}\bar{\alpha}\partial^\mu\psi && (65)\\
\delta\lambda &= -i\gamma_5\alpha\mathcal{D} - \frac{1}{2}[\not{\partial}, \gamma_\mu]V^\mu\alpha && (66)\\
\delta\mathcal{D} &= \bar{\alpha}\partial_\mu(\gamma^\mu\gamma_5\lambda) && (67)
\end{aligned}$$

3.3.3 Irreducible Representations of SUSY

Irreducible representations (irreps) of the superfield formulated above are the smallest possible collections of component fields that transform only into themselves. This effectively allows us to set the component fields not in this collection permanently to zero as they will not be regenerated by a supersymmetry transformation. Here we present the irreps of global supersymmetry that are needed to construct the MSSM.

We construct our first irrep by observing that the component fields λ and $\mathcal{D}$ transform into one another up to the term $[\not{\partial}, \gamma_\mu]V^\mu$ in the λ transformation. As this term is anti-symmetric, we may take $V_\mu = \partial_\mu \zeta$ to set it to zero, and subsequently also set $\lambda = \mathcal{D} = 0$. The remaining components can be further reduced into two distinct irreps. One of these is obtained by setting $V^\mu = i\partial^\mu \mathcal{S}$, $\psi_R = 0$ and $\mathcal{M} = -i\mathcal{N} =: \mathcal{F}$. The components $\hat{\mathcal{S}} := \{\mathcal{S}, \psi_L, \mathcal{F}\}$ are then collectively known as the chiral scalar superfield and transform as

$$\delta\mathcal{S} = -i\sqrt{2}\bar{\alpha}\psi_L \tag{68}$$

$$\delta\psi_L = -\sqrt{2}\mathcal{F}\alpha_L s + \sqrt{2}\not{\partial}\mathcal{S}\alpha_R \tag{69}$$

$$\delta\mathcal{F} = i\sqrt{2}\bar{\alpha}\partial_\mu(\gamma^\mu\psi_L) \tag{70}$$

The other irrep is obtained by setting $V^\mu = -i\partial^\mu\mathcal{S}$, $\psi_L = 0$ and $\mathcal{M} = i\mathcal{N} =: i\mathcal{F}$ and is known as the anti-chiral scalar superfield. Conjugating a chiral scalar superfield gives an anti-chiral scalar superfield

$$\begin{aligned}\hat{\mathcal{S}}^\dagger &= \left(\mathcal{S} + i\sqrt{2}\bar{\theta}\psi_L + i\bar{\theta}\theta_L\mathcal{F} + \frac{i}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)\partial^\mu\mathcal{S} - \frac{1}{\sqrt{2}}(\bar{\theta}\gamma_5\theta)\cdot\bar{\theta}\not{\partial}\psi_L + \frac{1}{8}(\bar{\theta}\gamma_5\theta)^2\Box\mathcal{S}\right)^\dagger \\ &= \mathcal{S}^\dagger - i\sqrt{2}\bar{\theta}\psi_R - i\bar{\theta}\theta_R\mathcal{F}^\dagger - \frac{i}{2}(\bar{\theta}\gamma_5\gamma_\mu\theta)\partial^\mu\mathcal{S}^\dagger - \frac{1}{\sqrt{2}}(\bar{\theta}\gamma_5\theta)\cdot\bar{\theta}\not{\partial}\psi_R + \frac{1}{8}(\bar{\theta}\gamma_5\theta)^2\Box\mathcal{S}^\dagger.\end{aligned} \tag{71}$$

The product of two chiral scalar superfields gives another scalar superfield, whereas the product of a chiral scalar superfield with an anti-chiral scalar superfield returns a general superfield

$$\hat{\mathcal{S}}\hat{\mathcal{S}}' = \hat{\mathcal{S}}'', \qquad \hat{\mathcal{S}}\hat{\mathcal{S}}'^\dagger = \hat{\Phi}. \tag{72}$$

By setting $F^{\mu\nu} := \partial^\mu V^\nu - \partial^\nu V^\mu$ we find that

$$\delta F^{\mu\nu} = -i\bar{\alpha}[\gamma^\nu\partial^\mu - \gamma^\mu\partial^\nu]\lambda \tag{73}$$

$$\delta\lambda = -i\gamma_5\alpha\mathcal{D} + \frac{1}{4}[\gamma\nu, \gamma_\mu]F^{\mu\nu}\alpha \tag{74}$$

$$\delta\mathcal{D} = \bar{\alpha}\not{\partial}\gamma_5\lambda. \tag{75}$$

However, we cannot in general set the component fields $\mathcal{S}$, ψ, $\mathcal{M}$ and $\mathcal{N}$ to zero, as they will be regenerated by supersymmetry transformations. In order to successfully form an irrep from these components we require the presence of gauge transformations for the superfields. These gauge transformations are dependent on a real superfield for which the components $\mathcal{S}$, ψ, $\mathcal{M}$ and $\mathcal{N}$ can be set to zero by a suitable gauge choice. This gives the irrep $\hat{V} := \{V^\mu, \lambda, \mathcal{D}\}$ known as the real vector superfield.

3.3.4 Building a SUSY Lagrangian

The Lagrangian of a supersymmetric theory can be formulated as a density in spacetime or superspace with the action integral following suite. Here we develop the former formulation as an ordinary spacetime density. We assume our supersymmetric theory has provided us with a collection of irreps, in our case chiral scalar superfields $\hat{\mathcal{S}}_i$ and real vector superfields $\hat{V}_j$ where i and j denote the number of each such superfields present respectively. The Lagrangian will then be made up of component fields (to remove the θ dependence) of combinations of these irrep superfields. In what follows, only examples of how this is done for chiral scalar superfields are given, for a full discussion including gauge superfields please refer to Baer *et al.*[5].

Recall from (72) that only the product of two scalar chiral superfields gives back a scalar chiral superfield. A mixture of scalar chiral and anti-scalar chiral superfields will give a general superfield. From (67) and (70) we see that none of the component fields are invariant under a supersymmetry transformation, which means it will be impossible to build a Lagrangian invariant under supersymmetry (at least by the method prescribed above). Fortunately, it is not the Lagrangian density but rather the action that must be invariant under supersymmetry transformations for supersymmetry to be realized as a symmetry of nature. Therefore component fields transforming as total derivatives are valid candidates from which to build a Lagrangian:

$$\delta S = \int d^4x \; \delta\mathcal{L} = 0 \Rightarrow \delta\mathcal{L} = 0 \text{ or } \partial_\mu(\ldots). \tag{76}$$

For a general superfield $\hat{\Phi}$, the only component field transforming as a total derivative is the $\mathcal{D}$ field: $\delta\mathcal{D} = \not{\partial}(\ldots)$. Therefore for some function of superfields $f(\hat{\Phi}_i)$, the $\mathcal{D}$ field component (i.e. the coefficient of the $-\frac{1}{4}(\bar{\theta}\gamma_5\theta)^2$ term) is a valid candidate for the Lagrangian (granted it is also renormalizable)

$$f\Big|_{\mathcal{D}\text{-term}} \in \mathcal{L}. \tag{77}$$

An an example, consider the Kähler potential $K(\hat{\mathcal{S}}, \hat{\mathcal{S}}^\dagger) = \hat{\mathcal{S}}^\dagger\hat{\mathcal{S}}$, which after some calculation, yields the kinetic terms of the component fields for the chiral scalar superfield

$$\hat{\mathcal{S}}^\dagger\hat{\mathcal{S}}\Big|_{\mathcal{D}\text{-term}} = \partial_\mu\mathcal{S}^\dagger\partial^\mu\mathcal{S} + \frac{i}{2}\bar{\psi}\not{\partial}\psi + \mathcal{F}^\dagger\mathcal{F} \in \mathcal{L} \tag{78}$$

Note that the $\mathcal{F}$ field here has no derivative terms, which means it is an auxillary field with algebraic equations of motion (giving constraints on the system).

For a chiral scalar superfield $\hat{\mathcal{S}}$, the only component field transforming as a total derivative is the $\mathcal{F}$ field: $\delta\mathcal{F} = \not{\partial}(\ldots)$. We define a function made up of only chiral scalar superfields (to insure the $\mathcal{F}$ component yields a total derivative) as the superpotential $\hat{W}(\hat{\mathcal{S}})$, such that

$$\hat{W}\Big|_{\mathcal{F}\text{-term}} \in \mathcal{L} \tag{79}$$

The master supersymmetric Lagrangian of a theory with chiral scalar superfields $\hat{\mathcal{S}}_i$ and real vector superfields $\hat{V}_\alpha$, obtained by including all possible renormalizable contributions from these superfields, is

$$
\begin{aligned}
\mathcal{L} = & \sum_i (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i) + \frac{i}{2}\sum_i \bar{\psi}_i \not{D} \psi_i + \sum_\alpha \left[\frac{i}{2}\bar{\lambda}_\alpha (\not{D}\lambda)_\alpha - \frac{1}{4} F_{\mu\nu\alpha} F^{\mu\nu}_\alpha\right] \\
& - \sqrt{2} \sum_{i,\alpha} \left(\mathcal{S}_i^\dagger g_\alpha t_\alpha \bar{\lambda}_\alpha \psi_{Li} + \text{h.c.}\right) \\
& - \frac{1}{2}\sum_\alpha \left[\sum_i \mathcal{S}_i^\dagger g_\alpha t_\alpha \mathcal{S}_i + \xi_\alpha\right]^2 - \sum_i \left|\frac{\partial \hat{W}}{\partial \hat{\mathcal{S}}_i}\right|^2_{\hat{\mathcal{S}}=\mathcal{S}} \\
& - \frac{1}{2}\sum_{i,j} \bar{\psi}_i \left[\left(\frac{\partial^2 \hat{W}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}} P_L + \left(\frac{\partial^2 \hat{W}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)^\dagger_{\hat{\mathcal{S}}=\mathcal{S}} P_R\right] \psi_j .
\end{aligned}
\tag{80}
$$

Notice that the $\mathcal{F}$ and $\mathcal{D}$ fields have been substituted by their algebraic equations of motion. The only remaining freedom any supersymmetric theory of this type has after specifying the irreps is a choice for the superpotential $\hat{W}$. The term in the second line gives interactions between a particle, its superpartner and a gauge fermion.

4 The Minimal Supersymmetric Standard Model

4.1 The MSSM

With the master supersymmetric Lagrangian template given in (80) it is straightforward to construct a supersymmetric extension to the Standard Model. We keep the same internal symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$, and promote each Standard Model gauge field to a real vector superfield:

$$
\begin{aligned}
B_\mu &\to \hat{B} \ni (B_\mu, \lambda_0, \mathcal{D}_B); && \\
W_{A\mu} &\to \hat{W}_A \ni (W_{A\mu}, \lambda_A, \mathcal{D}_{WA}), && A = 1,2,3; \\
g_{A\mu} &\to \hat{g}_A \ni (G_{A\mu}, \tilde{g}_A, \mathcal{D}_{gA}), && A = 1,\ldots,8.
\end{aligned}
\tag{81}
$$

Every fermion field in the Standard Model is likewise promoted to a chiral scalar superfield

$$
\begin{aligned}
\begin{pmatrix}\nu_{iL}\\ e_{iL}\end{pmatrix} &\to \begin{pmatrix}\hat{\nu}_i\\ \hat{e}_i\end{pmatrix} \equiv \hat{L}_i, & \begin{pmatrix}u_{iL}\\ d_{iL}\end{pmatrix} &\to \begin{pmatrix}\hat{u}_i\\ \hat{d}_i\end{pmatrix} \equiv \hat{Q}_i, \\
(e_{iR})^c &\to \quad \hat{E}^c_i, & (u_{iR})^c &\to \quad \hat{U}^c_i, \\
& & (d_{iR})^c &\to \quad \hat{D}^c_i,
\end{aligned}
\tag{82}
$$

where $\hat{e} \ni (\tilde{e}_L, \psi_{eL}, \mathcal{F}_e)$ etc.

The Higgs potential must enter via the superpotential $\hat{W}(\hat{\mathcal{S}})$, as this is the only freedom we have for adding new terms into the Lagrangian. Therefore the Higgs fields are promoted to be chiral scalar superfields

$$
\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \to \hat{H}_u = \begin{pmatrix}\hat{h}_u^+\\ \hat{h}_u^0\end{pmatrix}. \tag{83}
$$

If we let $\hat{H}_u$ have hypercharge $Y = 1$ so that it may couple to up-type quarks in the superpotential, then we also need a Higgs field with hypercharge $Y = -1$ to couple to the down-type quarks. In the Standard Model this was achieved simply by taking the conjugate of the Higgs field, but now taking the conjugate would give us an anti-chiral scalar field, which is not allowed to enter in the superpotential. We are thus forced to introduce a second Higgs doublet superfield with hypercharge $Y = -1$:

$$\hat{H}_d = \begin{pmatrix} \hat{h}_d^- \\ \hat{h}_d^0 \end{pmatrix} \tag{84}$$

The minimal superpotential of the Minimal Supersymmetric Standard Model (MSSM) is then

$$\hat{W} = \mu \hat{H}_u \hat{H}_d + \mathbf{f}_u \epsilon \underbrace{\hat{Q}}_{\frac{1}{3}} \underbrace{\hat{H}_u}_{1} \underbrace{\hat{U}}_{-\frac{4}{3}} + \mathbf{f}_d \underbrace{\hat{Q}}_{\frac{1}{3}} \underbrace{\hat{H}_d}_{-1} \underbrace{\hat{D}}_{\frac{2}{3}} + \mathbf{f}_e \hat{L} \hat{H}_d \hat{E} \tag{85}$$

with hypercharge conservation indicated. The $\mathbf{f}$ matrices here are analagous to the Yukawa coupling matrices from the Standard Model. A list of all the MSSM particle fields before electroweak breaking is given in table 1.

SM Particles		**Superpartners**	
Fermions		Scalar Fermions	
Quarks	$u\ c\ t$ $d\ s\ b$	Squarks	$\tilde{u}\ \tilde{c}\ \tilde{t}$ $\tilde{d}\ \tilde{s}\ \tilde{b}$
Leptons	$e\ \mu\ \tau$ $\nu_e\ \nu_\mu\ \nu_\tau$	Sleptons	$\tilde{e}\ \tilde{\mu}\ \tilde{\tau}$ $\tilde{\nu_e}\ \tilde{\nu_\mu}\ \tilde{\nu_\tau}$
Gauge Bosons		Gauginos	
Photon	A_μ	Photino	$\sin\theta_w \lambda_3 + \cos\theta_w \lambda_0$
W,Z Bosons	${W^\pm}_\mu$ Z_μ	W-ino Z-ino	$\frac{1}{\sqrt{2}}(\lambda_1 \mp i\lambda_2)$ $-\cos\theta_w \lambda_3 + \sin\theta_w \lambda_0$
Gluon	$g_{A\mu}$	Gluino	$\tilde{g}_A$
Higgs Bosons		Higgsinos	
$h_u^+\ h_u^0\ h_d^-\ h_d^0$		$\tilde{h}_u^+\ \tilde{h}_u^0\ \tilde{h}_d^-\ \tilde{h}_d^0$	

Table 1: The particle fields of the MSSM before electroweak breaking.

4.1.1 R-Parity

Unlike the Standard Model, the MSSM does not naturally conserve baryon or lepton numbers. This is easily illustrated by the following superpotential term

$$\text{e.g } \hat{W}_{\not{L}} = \epsilon \hat{L}_e \hat{H}_u$$

which is gauge invariant and renormalizable but violates lepton number. To insure that the deliberate absence of these baryon and lepton violating terms in the MSSM superpotential is well motivated, a new symmetry named R-parity is often introduced. R-parity is defined as

$$R = (-1)^{3(B-L)+2s}, \tag{86}$$

where B and L are the baryon and lepton numbers respectively and s is the particles spin. Conservation of R-parity implies conservation of baryon and lepton numbers. It turns out that all superpartners have odd R-parity whereas the Standard Model particles and the Higgs doublets have even R parity. The MSSM with R-parity therefore requires all superpartners to occur in pairs. This leads to an interesting implication for cosmology: the lightest superpartner cannot decay and is therefore stable. If we can further show that this particle is also weakly interacting, we have a WIMP candidate.

4.1.2 Breaking of the MSSM

From relation (44) it is clear that particles and their superpartners must have the same mass. However, we do not observe particles such as scalar electrons in nature, we in fact do not observe any of the superpartners. Therefore, if supersymmetry truely is a symmetry of nature, it must be broken. How it must be broken is not known: it could be broken spontaneously (like the electroweak symmetry), explicitly, dynamically, etc. Breaking of supersymmetry will in general add additional non-supersymmetric *breaking terms* to the supersymmetric Lagrangian. Some of these breaking terms could re-introduce the quadratic divergences that supersymmetry so vitally helped eliminate, this is known as hard breaking. To protect the scalar masses of the theory, which was one of the primary motivations for extending the Standard Model with supersymmetry, we assume that supersymmetry is broken softly, i.e. that the breaking terms do not re-introduce quadratic divergences. Due to our ignorance of how supersymmetry is broken, we must add all possible soft breaking terms to our MSSM Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &\left[\tilde{L}_i^\dagger \mathbf{m}^2_{Lij}\tilde{L}_j + \ldots + m^2_{H_u}|H_u|^2 + \ldots\right] \\ &- \frac{1}{2}\left[M_1\bar{\lambda_0}\lambda_0 + \ldots\right] + \left[(\mathbf{a_e})_{ij}\epsilon_{ab}\tilde{L}_i H_d \tilde{e}^\dagger_{Rj} + \ldots\right] \\ &+ \left[(\mathbf{c_e})_{ij}\epsilon_{ab}\tilde{L}_i H_d^* \tilde{e}^\dagger_{Rj} + \ldots\right] + \left[bH_uH_d + h.c\right]\end{aligned} \tag{87}$$

The terms on the first line are the mass terms for the scalar fields in the theory. The terms in the square brackets on the second line are the mass terms of the gauginos. The **a** and **c** matrices describe trilinear scalar interactions and the terms in the last square brackets give the mixing of the scalar Higgs fields. The total number of free parameters of the MSSM including the soft breaking terms is 178.

A model with such a large parameter space is clearly quite unmanageable and has almost no predictive power. To make the MSSM a more reasonable theory to work with in practice, a series of phenomenological assumption are made to simplify the parameter space. These assumptions include removing sources of CP violation and flavour mixing and considering only the heaviest family of Standard Model particles. For a detailed discussion of these assumptions and SUSY breaking in general please refer to Baer *et al.*[5]. A model of this type usually has between $5-10$ free parameters and is often referred to as the Constrained MSSM (CMSSM).

Besides from the CMSSM, another popular supersymmetric extension to the Standard Model with only four free parameters is minimal supergravity (mSUGRA). This model has local rather than global supersymmetry, and devolves its small set of parameters from the GUT scale via the renormalization group equations to fix parameters at the electroweak breaking scale.

4.2 The Lightest Supersymmetric Particle

4.2.1 LSP Candidates

As discussed in section 4.1.1, the lightest supersymmetric particle (LSP) in the MSSM with R-parity will be stable. We combine arguments given by Ellis *et al.*[15] and experimental results to determine which particle is most likely the LSP, in a similar fashion to Jungman *et al.*[2].

If the LSP were a charged gaugino or slepton it would be expected to have a moderate relic density[16] and subsequently be expected to mix with ordinary matter due to its charge. Such particles have been ruled out by searches for anomalously heavy terrestrial protons[16].

If the LSP were a gluino or squark it would be expected to form hadrons, for which charged hadrons would show up in heavy protons searches. It may be possible however for them to form only evasive neutral hadrons. In most GUT models involving supersymmetry, however, it is found that gluinos are heavier than neutralinos and squarks heavier than sleptons.

If the LSP was a sneutrino it would be a good WIMP candidate as it would have weak scale interactions and importantly not interact electromagneticly. However, most of the parameter space where the sneutrino is the LSP has been ruled out by direct-detection experiments.

In terms of supersymmetric particles of the MSSM this leaves only the Neutralino (which we will derive shortly) as both the LSP and a good WIMP candidate.

Beyond the MSSM, other possible LSP candidates are the gravitino and axino. The gravitino arises from theories with local supersymmetry and the axino in models such as the MSSM extended with the Peccei-Quinn mechanism to solve the strong CP problem. The reader is referred to the brief review of Steffen[3] for a summary of how these particles would behave if they were the LSP.

4.2.2 The Neutralino

Recall from the master lagrangian (80) that the first term on the second line gave a trilinear interaction between a particle, its superpartner and a gaugino. When the MSSM undergoes electroweak breaking, similar to that of the Standard Model, the trilinear terms involving a Higgs scalar and its superpartner higgsino will be reduced to bilinear terms as the Higgs scalar becomes a constant vacuum expectation value:

$$\begin{aligned} \mathcal{L} &\ni -\sqrt{2} g \mathcal{S}_i^\dagger g t_A \bar{\lambda}_A \psi_{Li} \\ \xrightarrow{\cancel{\text{EW}}} &\ni g \underbrace{\langle h_u^0 \rangle}_{v_u} \bar{\lambda}_3 \tilde{h}_u^0 + \dots . \end{aligned} \tag{88}$$

These bilinear terms mix the higgsino and gaugino states. The true physical particles of the higgsino and gaugino states after electroweak breaking will then be the eigenstates of the total mass matrix.

As the name neutralino suggests, we are interested in the mass matrix of the neutral higgsino and gaugino fermion states. Besides from the mixing terms of the electroweak breaking, mass term contributions also come from the Higgs superfield mixing in the superpotential $\hat{W} \ni \mu \hat{H}_u \hat{H}_d$ and from the soft breaking gaugino mass terms

$$\mathcal{L}_{\text{soft}} \ni -\frac{1}{2} M_1 \bar{\lambda}_0 \lambda_0 - \frac{1}{2} M_2 \bar{\lambda}_3 \lambda_3. \tag{89}$$

All mass term contributions of the neutral fermions may thus be written as

$$-\frac{1}{2}\begin{pmatrix}\bar{\tilde{h}}_u^0 & \bar{\tilde{h}}_d^0 & \bar{\lambda}_3 & \bar{\lambda}_0\end{pmatrix}\begin{pmatrix} 0 & \mu & -\frac{gv_u}{\sqrt{2}} & \frac{g'v_u}{\sqrt{2}} \\ \mu & 0 & \frac{gv_d}{\sqrt{2}} & -\frac{g'v_d}{\sqrt{2}} \\ -\frac{gv_u}{\sqrt{2}} & \frac{gv_d}{\sqrt{2}} & M_2 & 0 \\ \frac{g'v_u}{\sqrt{2}} & -\frac{g'v_d}{\sqrt{2}} & 0 & M_1 \end{pmatrix}\begin{pmatrix}\tilde{h}_u^0 \\ \tilde{h}_d^0 \\ \lambda_3 \\ \lambda_0\end{pmatrix}. \tag{90}$$

The mass matrix can be diagonalized by a unitary matrix $\mathcal{M}_D = V^\dagger \mathcal{M}_{\text{neutral}} V$, giving the neutralino eigenstates

$$\begin{pmatrix}\chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4\end{pmatrix} = V^\dagger \begin{pmatrix}\tilde{h}_u^0 \\ \tilde{h}_d^0 \\ \lambda_3 \equiv \tilde{W}^3 \\ \lambda_0 \equiv \tilde{B}\end{pmatrix}. \tag{91}$$

We call the lightest neutralino eigenstate *the* neutralino and denote it by χ_0 (i.e. the index i for which χ_i is the lightest is replaced by 0). The neutralino may be written as a linear combination of higgsinos and gauginos

$$\chi_0 = V_{10}^* \tilde{h}_u^0 + V_{20}^* \tilde{h}_d^0 + V_{30}^* \tilde{W}^3 + V_{40}^* \tilde{B}. \tag{92}$$

It is common to define a quantity such as the gaugino fraction $f_g := |V_{30}|^2 + |V_{40}|^2$ to measure if the neutralino is primarily gaugino $f_g > 0.5$ or higgsino < 0.5.

A common relation from GUTs can be used to relate the gaugino masses $M_1 = \frac{5}{3} M_2 \tan^2\theta_w$, thereby leaving only two parameters μ and M_2 on which the neutralino mass and gaugino fraction depend (assuming the Higgs VEVs and MSSM coupling constants are fixed). Figure 5 gives a contour plot of these quantities with respect to the free parameters μ and M_2. It is clear that for a large mass (~ 3 TeV) the gaugino fraction is arbitrary, whereas for lower masses it is quite sensitive to the parameters μ and M_2.

4.2.3 Neutralino Interactions

The neutralino is a linear combination of the third wino $\tilde{W}^3$, the bino $\tilde{B}$ and the neutral higgsinos $\tilde{h}_u^0$ and $\tilde{h}_d^0$. From the master supersymmetric lagrangian given in equation (80), we see that the gaugino components (the wino and bino, denoted here by λ) couple electroweakly to matter (chiral) superfield pairs through the term

$$-\sqrt{2} g \mathcal{S}_i^\dagger g t_A \bar{\lambda}_A \psi_{Li} \in \mathcal{L}. \tag{93}$$

Therefore quark-neutralino scattering is possible via the exchange of a scalar quark. By setting the chiral superfield pairs to be the Higgs superfield, the higgsino nature of the neutralino gives an interaction between a pair of neutralinos and a scalar Higgs field. The higgsino components can also couple to fermion-scalar fermion pairs via the Yukawa couplings present in the term

$$-\frac{1}{2}\sum_{i,j}\bar{\psi}_i \left(\frac{\partial^2 \hat{W}}{\partial \hat{\mathcal{S}}_i \partial \hat{\mathcal{S}}_j}\right)_{\hat{\mathcal{S}}=\mathcal{S}} P_L \psi_j + h.c. \in \mathcal{L} \tag{94}$$

Finally, the term

$$\frac{i}{2}\sum_i \bar{\psi}_i \not{D} \psi_i \in \mathcal{L} \tag{95}$$

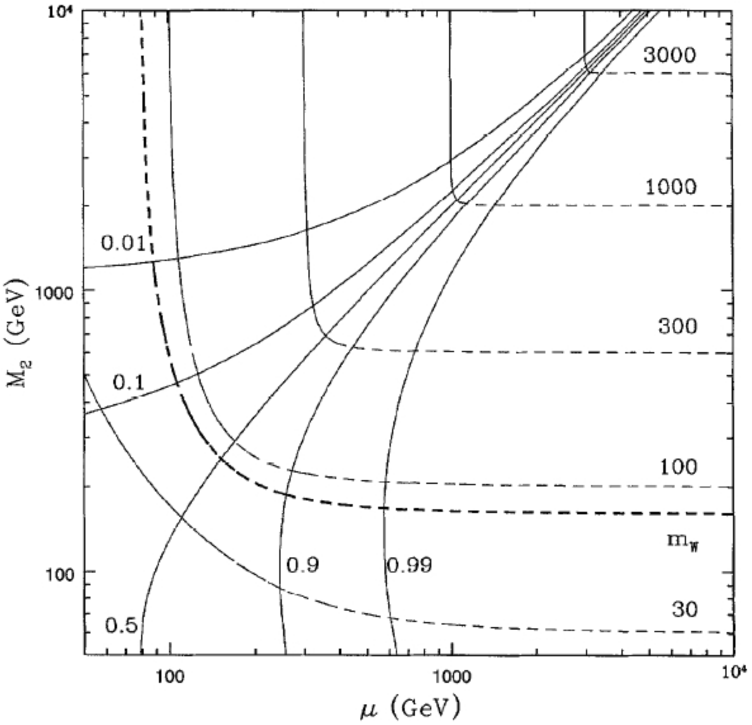

Figure 5: Contour plot of the neutralino mass (in GeV) and the gaugino fraction for parameters μ and M_2. Broken curves are m_χ (GeV), solid curves f_g (From G. Jungman *et al.*[2])

gives couplings between a pair of neutralinos and electroweak gauge bosons. These interaction vertices are summarized in figure 6. It is important to note that the neutralino does not couple strongly (no vertex carries a factor of g_S), so that our classification of the neutralino as a WIMP is still justified. Whether the neutralino has a weak enough annihilation cross section to give the correct relic density is dependent on which parameters we choose. Fortunately, regions of the MSSM parameter space where this is the case exist and also coincide with phenomenologically constrained MSSM models.

5 WIMP Detection

5.1 Direct Detection

If there is a spherical dark matter halo centered around our galaxy, our solar system should be moving directly through it, and consequencently, we would expect a flux of dark matter particles passing through the earth's surface of about $100 - 1000\text{cm}^{-2}s^{-1}$[2]. Direct detection of WIMPs refers to observing the recoil of a nuclei after WIMP-nucleus scattering. Unfortunately the weakly interacting nature of WIMPs mean such events occur very infrequently. Nonetheless, the odds are increased by building large sensitive detectors and waiting long periods of time, and over the past twenty years there have been at least a dozen such detectors running at any given time [17].

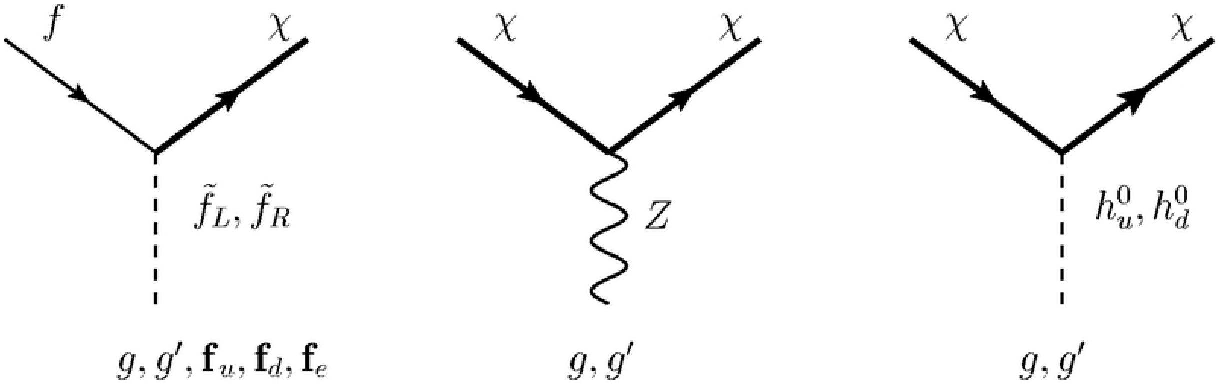

Figure 6: Summary of the possible neutralino interaction vertices. The neutralino χ interacts with fermions-scalar fermions (left), neutral gauge bosons (middle) and neutral Higgs scalar fields before electroweak breaking (right). Dependence on the various coupling constants are also shown (note the absence of strong interactions g_S).

The qualitative event rate is

$$R \approx \frac{n_\chi \sigma_{\text{scatt.}} \langle v \rangle}{m_N} \text{kg}^{-1}/\text{day} \tag{96}$$

where $n = \rho_0/m_\chi$ is the WIMP number density, σ is the elastic-scattering cross section, $\langle v \rangle$ the average speed of the WIMP relative to the target and m_N the mass of the nucleus. For the derivation of a more accurate event rate that takes into account the velocity distribution, the cross sections dependence on this distribution and the detectors threshold energy, please refer to Jungman *et al.*[2]. A typical WIMP with a mass $m_\chi \approx 20 - 400$ GeV and velocity $v \approx 270$kms^{-1} hitting a typical nucleus with mass $m_N \approx 1 - 200$ GeV will deposit an energy in the range of $1 - 100$ keV at a rate of $10^{-4} - 1$ events per kg per day. In comparison, a cosmic ray with energies in the keV-MeV range occur at a rate > 100 events per kg per day. Therefore cosmic rays give huge event backgrounds for WIMP direct detection, and force WIMP detectors deep underground where they are more shielded.

The two leading direct detection experiments in operation today are the CDMS-II detector, located in an old iron mine in Minnesota, and the XENON detector in Gran Sasso, Italy. These detectors use different methods to detect nuclear recoil. The CDMS-II detector uses an array of germanium and silicon crystals as its target. One side of these crystals measures ionization as the crystal band structure is disrupted by an interaction, while the other side detects thermal phonons (vibrations in the crystal) produced by an interaction using superconducting transition edge sensors. These two different detection methods for one event allow for good discrimination from background. The XENON detector used a tank of liquid xenon as its target. The xenon is placed in a vertical electric field with arrays of photomultiplier tubes positioned above and below to detect the scintillation produced from scattering events. Discrimination from background is achieved by taking the ratio of direct scintillation with scintillation occuring in the xenon gas layer at the top of the tank, as this ratio is different for nucleus and electron events[18]. The advantage the XENON method has over that of CDMS-II is that it is more straightforward to scale up the target size.

No direct detection experiment has yet to detect a WIMP signal. This places strong upper

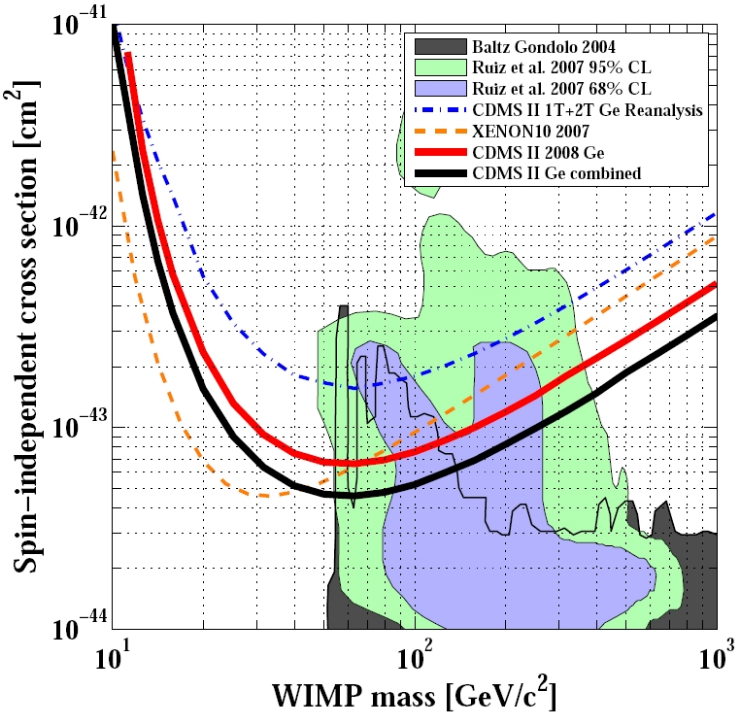

Figure 7: Spin-independent cross section 90% CL upper limits versus WIMP mass [13]

limits on the nucleur scattering cross section of such a particle. Figure 7 gives 90% confidence level upper limits of the spin-independent cross sections versus the WIMP mass found by the CDMS-II and XENON experiments[13]. Also shown in the figure are the parameter spaces of two constrained MSSM models. This illustrates how direct detection experiments can be employed to cut down the vast parameter space of the MSSM, bringing us closer to answering the question of how supersymmetry may be realized in nature.

5.2 Indirect Detection

Due to the massive nature of WIMP dark matter, there should be regions in our universe where it becomes gravitationally trapped, such as in the centre of stars, galaxies etc. In these dense trapped regions we would expect annihilation to still occur, even though it has in general long frozen out elsewhere in the universe. Products from this WIMP annihilation would then form all sorts of energetic cosmic rays which should be detectable to us on earth. Detection of these cosmic rays and their decay products is classed as indirect WIMP detection.

One important source for the indirect detection of WIMPs is the sun. A WIMP particle passing through the sun scatters off a nucleus such that its resulting velocity is less than the escape velocity of the sun, and it becomes trapped. It subsequently continues scattering, eventually drifting to the centre of the sun due to the gravitational attraction. At the centre it meets other WIMP particles, and if the WIMP is its own anti particle (such as the Neutralino, which is a Majorana spinor) it annihilates. The key product particle resulting from this annihilation is an energetic neutrino with $E_\nu \sim \frac{1}{3}m_\chi \gg E_{\mathrm{solar}-\nu}$. Thus detection of a highly energetic neutrino would be suggestive evidence of WIMP dark matter. There are currently a number of neutrino detectors under construction; including IceCube, ANTARES and Km3net; that hope to detect such energetic dark matter neutrinos.

Another detectable product of WIMP annihilation that we expect to observe are cosmic gamma rays. From 1991 to 1996 the EGRET detector on the NASA CGRO satellite measured gamma ray point sources with energies 30 MeV to 30 GeV. Figure 8 gives the third EGRET catalog consisting of 271 point sources, of which 170 were unidentified [19]. Some authors believe these signals are evidence of WIMP annihilation, however this analysis contradicts the limits placed by direct detection experiments [20].

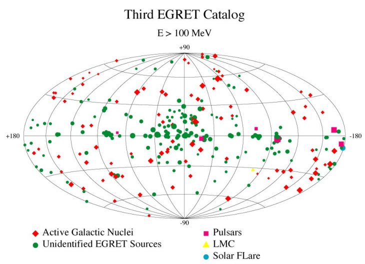

Figure 8: Third EGRET catalog with 271 found point sources (From the CGRO Science Support Center[19])

5.3 Colliders

WIMPs are weakly interacting, and so by their very nature, they are difficult to detect directly at colliders. To detect a massive weakly interacting particle such as a WIMP, collider experiments search for missing transverse energy and spin that would arise if one of these particles were created during a collision process and escaped undetected. Most hope for the detection of a WIMP in this way currently lies with CERN's Large Hadron Collider (LHC), which will probe nature at energies an order of magnitude higher than today's current limits. The LHC is scheduled to come back online in 2009. Although the LHC could confirm the existence of supersymmetry, we may have to wait for the next generation of colliders such as the International Linear Collider (ILC) to identify the LSP as the Neutralino or perhaps a gravitino, axino etc. [3].

6 Conclusion

We have seen that supersymmetry is a welcome extension to the Standard Model. It solves the fine-tuning problem of the Higgs mass, gives in-roads to buildings GUTs, and, most importantly for the realm of cosmology, it delivers a WIMP candidate. For the MSSM to deliver a WIMP candidate, we must assume R-parity symmetry both exists and is conserved, so that we have a stable particle: the LSP. Because the MSSM is broken in an unknown way, it has a large parameter space, meaning that the LSP could be many different things depending on which parameters are chosen. Fortunately, in most phenomenologically viable parameter spaces the LSP remains the same particle, the Neutralino, which interacts weakly. Therefore the Neutralino is a strong WIMP candidate. However, these parameter spaces remain large, severely limiting the predictive power of this model for experiments. Conversely, experiments are currently used to narrow down the MSSM parameter space. Direct detection experiments for example have put strong upper limits on the possible scattering cross sections of a WIMP for its likely mass range. The strongest evidence for the existence of supersymmetric dark matter in the near future would be the discovery of supersymmetry at the LHC together with positive results from direct detection experiments.

A Notation and Conventions

Minkowski metric signature is $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, giving the four momentum relation

$$p^2 = E^2 - |\mathbf{p}|^2 = m^2 \tag{97}$$

The fourier expansions of canonically quantized complex and spinor fields respectively are

$$\phi(x) = \sum_s \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\mathbf{k}}} (a_{\mathbf{k}} e^{-ik\cdot x} + b^\dagger_{\mathbf{k}} e^{ik\cdot x}) \tag{98}$$

and

$$\psi(x) = \sum_s \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{\mathbf{k}}} (a_{\mathbf{k},s} u(k)_s e^{-ik\cdot x} + b^\dagger_{\mathbf{k},s} v(k)_s e^{ik\cdot x}) \tag{99}$$

where the field operators obey

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{p}}]_{\pm} = (2\pi)^3 2E_{\mathbf{k}} \delta^{(3)}(\mathbf{k} - \mathbf{p}) \tag{100}$$

A.1 Spinor Algebra

Four component Dirac spinor notation is used, as in Baer *et al.*[5]. Gamma matrices are chosen in the chiral representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{101}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \quad , \quad \sigma^{\mu\nu} := \frac{i}{2}[\gamma^\mu, \gamma^\nu] \tag{102}$$

$$\{\gamma_5, \gamma^\mu\} = 0 \quad , \quad (\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0 \tag{103}$$

Left and right chiral projectors are defined as

$$P_L = \frac{1 - \gamma_5}{2} \quad , \quad P_R = \frac{1 + \gamma_5}{2} \tag{104}$$

The generic lagrangian term for a massive fermion is written as

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi = 0 \tag{105}$$

where the conjugate spinor is defined as

$$\bar{\psi} = \psi^\dagger \gamma^0 \tag{106}$$

Plane-wave solutions to the Dirac equation of motion are

$$\psi = u(p)e^{-ip\cdot x} \quad , \quad \psi = v(p)e^{ip\cdot x} \tag{107}$$

for $p^0 > 0$, with the momentum space spinors satisfying

$$(\not{p} - m)u(p) = 0 \quad , \quad (\not{p} + m)v(p) = 0 \tag{108}$$

$$\sum_s u(p)^a_s \bar{u}(p)^b_s = (\not{p} + m)^{ab} \quad , \quad \sum_s v(p)^a_s \bar{v}(p)^b_s = (\not{p} - m)^{ab} \tag{109}$$

Parity is defined as taking $a_{\mathbf{k}} \to a_{-\mathbf{k}}$ and $b_{\mathbf{k}} \to -b_{-\mathbf{k}}$. By defining the variables $\tilde{p} = (p^0, -\mathbf{p})$ and $\tilde{x} = (x^0, -\mathbf{x})$ and using the spinor relations $u(p) = \gamma^0 u(\tilde{p})$ and $v(p) = -\gamma^0 v(\tilde{p})$, parity acts on a Dirac fermion as

$$\psi(x) \to \gamma^0 \psi(\tilde{x}) \quad , \quad \bar{\psi}(x) \to \bar{\psi}(\tilde{x})\gamma^0 \tag{110}$$

Charge conjugation is defined as taking $a_{\mathbf{k}} \to a^c_{\mathbf{k}} = b_{\mathbf{k}}$ and $b_{\mathbf{k}} \to b^c_{\mathbf{k}} = a_{\mathbf{k}}$. Using the spinor relations $u(p) = -i\gamma^2 v(p)^*$ and $v(p) = -i\gamma^2 u(p)^*$, charge conjugation acts on a Dirac fermion as

$$\psi \to \psi^c = C\bar{\psi}^T \quad \text{for} \quad C = -i\gamma^2\gamma^0 \tag{111}$$

where we can also rewrite the spinor relations as $u = C\bar{v}^T$ and $v = C\bar{u}^T$. The charge conjugation matrix C satisfies

$$C\gamma^T_\mu C^{-1} = -\gamma_\mu \quad , \quad C^T = C^{-1} = -C \tag{112}$$

and

$$[C, \gamma_5] = 0 \tag{113}$$

A Majorana fermion is a Dirac fermion that satisfies $\psi = \psi^c$. In the chiral representation this implies that the right chiral component of the Majorana fermion is completely determined by the left chiral component

$$\psi_R = P_R\psi^c = \frac{1+\gamma_5}{2}C\bar{\psi}^T = C\gamma^0\psi^*_L \tag{114}$$

B The Boltzmann Equation for Thermal Relics

We begin with the Boltzmann equation

$$L[f] := \frac{d}{dt}f = \text{Coll}[f], \tag{115}$$

for the distribution function $f(\mathbf{x}, \mathbf{p}, t)$, where L is the Liouville equation and Coll the collision functional which describes interactions. The distribution function f_χ gives the average density of particle species χ on phase space, and hence the particle number density of this species is given as

$$n_\chi(\mathbf{x}, t) = g \int \frac{d^3p}{(2\pi\hbar)^3} f_\chi(\mathbf{x}, \mathbf{p}, t), \tag{116}$$

where g is the number of internal degrees of freedom. Assuming an expanding universe, the physical momentum scales with the scale factor as $\mathbf{p} \propto a^{-1}$ and we find

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_\chi) + \bigtriangledown \cdot (\mathbf{u}_\chi n_\chi) = \int \frac{d^3p}{(2\pi\hbar)^3}\mathrm{Coll}[f_\chi], \tag{117}$$

where $\mathbf{u}_\chi$ denotes the fluid velocity and can be taken to be zero in a homogeneous expanding universe. Taking Coll to describe annihilation and creation of a particle species χ into a lighter particle l

$$\chi + \bar{\chi} \rightleftharpoons l + \bar{l}, \tag{118}$$

we may write

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_\chi) = -\langle\sigma_A v\rangle n_\chi^2 - \psi, \tag{119}$$

where $\langle\sigma_A v\rangle$ is the thermally averaged total annihilation cross section of χ with v the relative velocity. Therefore $\langle\sigma_A v\rangle n_\chi^2$ denotes the rate of annhilations of the particle species χ and likewise ψ is a place holder for the rate of creation. As we expect the *Coll* functional to be zero when the annihilation and creation rates are equal (when the particle is in equilibrium), we write

$$\frac{d}{dt}n_\chi + 3Hn_\chi = -\langle\sigma_A v\rangle(n_\chi^2 - (n_\chi^{eq})^2), \tag{120}$$

where n_χ^{eq} denotes the equilibrium number density of χ, and the left hand side has been expanded to reveal the Hubble rate $H = \dot{a}a^{-1}$.

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