Modified Newtonian Dynamics, a possible alternative to the dark matter hypothesis

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Abstract

A review of the MOND theory is presented in this paper. The theory is a modification of Newtonian dynamics at low acceleration scales aimed at solving the rotation curve problem without invoking dark matter. We compute some rotation curves for a class of model galaxies and compare these with the experimental data. It is suggested that MOND might also solve the Pioneer anomaly. Some other research topics in MOND are suggested as well.

Contents

1 Introduction

The invention/discovery of General Relativity (GR) is one of the chief scientific accomplishments of the 20th century. The theory describes and explains a wide variety of physical phenomena. In recent years experimental facts have cast doubts upon the theory. Rotation curves of galaxies are not well described by GR. To solve the apparent problem people invented the dark matter hypothesis: "There exists an enormous amount of matter which is not luminous." Dark matter is usually thought of to be comprised of several different constituents. One part is understood to be ordinary matter emitting light in a very low frequency band, unobservable by current technology, and another part is understood to be of an exotic nature. The exotic matter does not (or extremely weakly) couple to electromagnetism.

I believe that we have not discovered all fundamental particles; in this view the standard model of elementary particles (SM) is an effective theory of some underlying fundamental theory. This view provides room in which particles can exists which do not couple to electromagnetism. Recent dark model theories suggest that all energy content of the universe is made up of 4% ordinary baryonic matter, 33% dark matter and 63% of the even more elusive dark energy [10]. I will not discuss dark energy in this article.

In 1983 Milgrom advocated[7] a modification of the laws of motion, for many physicists an even more radical idea than the introduction of dark matter. He suggested the introduction of a new fundamental constant a_0, called the acceleration constant, at which acceleration scale Newton's second law fails to hold[1]. Some predictions Milgrom made where later found to fit observed phenomena perfectly. This encourages a further study of his theory. His theory has been called Modified Newtonian Dynamics(MOND).

There is another phenomenon which apparently cannot be solved using Einsteins theory of gravity. The predicted orbits of two spaceships, the Pioneer 10 and Pioneer 11, are not completely overlapping the orbits they travel. If one takes into account all known gravitational sources in the solar system, one can of course

[1]The speed of the rotating particles at the rim of a galaxy is highly non-relativistic. It is therefore natural to first study this problem non-relativistically. After this paper people started to generalise the idea to the relativistic regime. A recent attempt by Bekenstein, a theory called TeVeS[3], has correct relativistic, Newtonian and MOND limits. This is discussed in the appendix.

compute the acceleration the spaceships should experience. This does not equal the acceleration they experience however; the spaceships undergo an unmodelled acceleration. The size of the acceleration is of the same magnitude as the acceleration scale introduced by MOND. This was not known the time MOND was written down. Is this a further test of the theory?

2 The observational discrepancies

There are two main observational discrepancies which we would like MOND to fix. One concerns the rotational curves of galaxies, and the other the pioneer anomaly.

2.1 Rotation speeds of galaxies

The first discrepancy is well known and is concerned with the rotational velocities of galaxies. The gravitational mass of a galaxy makes stars roughly travel in circular orbits around the centre of the galaxy. One can measure the galaxies mass content via luminosity. Using a generalised Kepler law one can plot the expected velocity versus the radial distance. Using the Doppler shift of the 21cm line one can actually measure the velocities of the stars rotating around the centre. These plots completely fail to correlate!

This phenomenon made Zwicky postulate[16] the existence of invisible mass. It is possible to fix the rotation curves using the dark matter hypothesis. This is not strange since one can completely arrange the distribution of dark matter for each galaxy separately. The parameter space in which one can fix the rotation curve is huge. We will see that MOND solves the rotation curve problem for all galaxies, without a large parameter space. For each galaxy there is only one parameter to fix in this theory (that is the mass to luminosity ratio). The errors in the final results are smaller, when compared to the measured velocity curves.

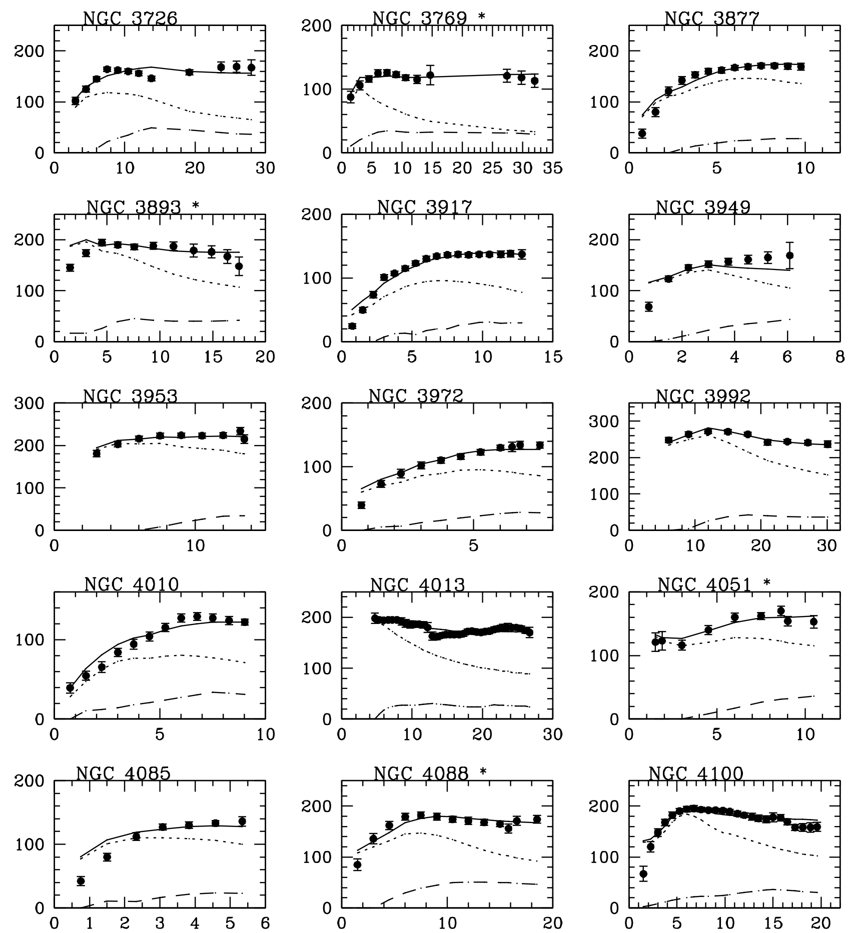

Figure 1: Rotation curves for some galaxies. The dots are the measured velocities. The dotted line is the curve Newtonian dynamics predicts. The solid line is the line MOND predicts. We see that the experimental data is fitted very well in MOND[13].

2.2 The Pioneer anomaly

The second problem is less well known. It is more down to earth than the previous problem. NASA has send out several deep space missions. In 1972 the spaceship Pioneer 10 was launched[1]. In 1973 this was followed by a second one, called Pioneer 11. The objective of these missions was to study the outer solar system, Jupiter, and Saturn[14]. The observations were highly valued by the astrophysical community. Pioneer 10 was the first spaceship to make close up pictures of Jupiter.

One observation was unexpected, and is still insufficiently explained. It is known as the Pioneer anomaly. Both spacecrafts experience an unmodelled acceleration of around $a_p = (8 \pm 3)\, 10^{-10} \frac{m}{s^2}$ directed towards the sun. Note that this acceleration is extremely small. Unmodelled in this context means that after all the gravitational influences of all known stellar objects have been taken into account, one can compute an acceleration of the spaceship. Upon measurement the predicted and modelled acceleration were found to differ by an amount of a_p. Several possible explanations have been considered by Anderson et al.[1]. These include

1. Gravity influence of the Kuiper Belt
2. Spacecraft gas leaks
3. Errors in accepted physical values of the Earth's orientation, precession, and nutation.

Accelerations produced from these sources were found to be several orders too small. In their thorough search for explanations of this effect they also considered

1. Momentum loss due to signals sent back to Earth
2. Unknown systematic errors in the spacecraft measuring devices.
3. Unknown viscous drag force
4. Thermal radiation of the spacecraft itself
5. Gravity breaks down.

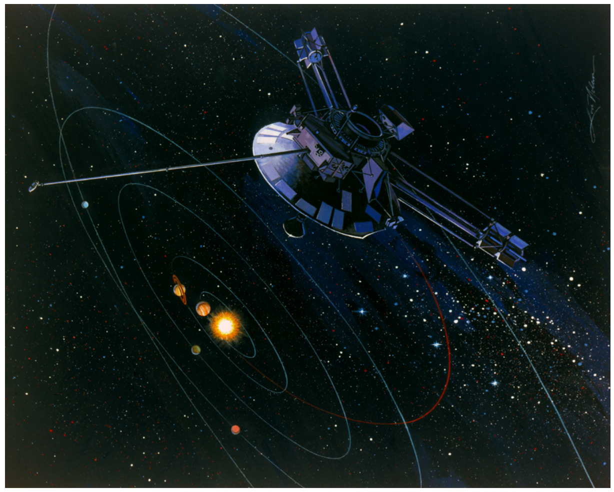

Figure 2: The spaceship Pioneer 10, in an artist impression[11].

The power emitted from the antenna of Pioneer 10 is about $8W$. Taking into account the mass of Pioneer, we can compute that this maximally can cause an acceleration of 9% of a_p. More importantly, it would be directed away from the sun. Thus this cannot possibly explain a_p either (note that this might call for a positive contribution to the unknown acceleration a_p). The second possibility is not likely either. The same deceleration is measured in both spacecraft, and was later measured in spacecraft of very different design. The fourth of these causes was rejected by Anderson. One expects that the spaceship does not have a preferred direction in which it radiates heat, thus cannot generate a force. The study was finally done in detail by Bertolami et al.[5]. They found, by precise moddeling of the spacecraft, that thermal radiation could explain up to 70% of a_p. We see it does not completely explain a_p. Thus one is left with the last two probable reasons. The unknown drag force should be studied by considering orbits of other spaceships. The breakdown of gravity is what we study in this article.

3 Effective versus a physical theory

In this section we will consider the difference between an effective and a physical theory. A physical theory tries to explain a physical phenomenon, while an effective theory merely tries to describe it.

A good example of a physical theory is general relativity. One of the main attractive features of Einsteins General Relativity is the physical nature of this theory. From a few underlying principles, general covariance, strong equivalence principle etc, a beautiful theory is set up. The theory explains how acceleration and gravity are basically the same phenomenon. In physics theoretical explanation is not enough, the theory also has to pass experimental verification. General relativity does this very well, except for the problems mentioned in section 2.

Effective theories merely describe physical phenomena. Hooke's law is a good example. The amount a spring stretches is proportional to the force experienced by the spring. In modern physics we can "explain" this force by considering the molecular structure of the spring. When this law was formulated however, it merely stated what force one can expect; it does not explain why it is so. This did not damage the usefulness of Hooke's law in any way.

MOND, as the theory is understood now, is effective. One needs to search for underlying principles for which a law like formula (5) can be derived. Only then people will start taking the experimental verification, which is abound, with any sense of seriousness.

4 What Newton's second law really says

Newtons second law is usually formulated along the lines of:

Observed in an inertial frame, the net force on a particle equals the rate of change of the linear momentum. Momentum is the product of the velocity and the mass.

In a formula this is stated as

$$F = \dot{p} = m\, a \tag{1}$$

It is not often realised that this can be derived from a more mundane principle. The observation is as follows. We want to describe a physical system with a mathematical model. The physical system can consist of balls, planets, and whatnot. We call the entities in all cases particles. The properties of these particles (their locations, masses, charges, etc.) can be measured with measuring devices. These measuring devices have some limited domain in which they work (our ruler on earth cannot be used to measure the diameter of mars); one can patch results from these measuring devices together. It is therefore natural to take for the configuration space (all parameters that specify the particles) a *manifold*. Since all measuring apparatuses return real numbers, one takes the manifold to be a manifold over $\mathbb{R}^n$, lets dub it $\mathcal{Q}$. The n equals the number of parameters needed to specify the system. As an example we have two pointlike particles, moving in three dimensions. In this case $n = 6$.

One defines the velocity of the particles as the time derivative of the coordinate functions on the manifold (these objects thus live in the tangent bundle of $\mathcal{Q}$, denoted $T\mathcal{Q}$. The derivative of the velocity vector field is defined to be the acceleration[2].

[2]Naively one expects the acceleration to live in $TT\mathcal{Q}$. It actually lives in a manifold which is a strict submanifold of the previous one. One can see this as follows. The original manifold $\mathcal{Q}$ is n dimensional. The velocity is n dimensional as well. The tangent bundle is $2n$ dimensional. This is correct. The tangent bundle of $T\mathcal{Q}$ is $4n$ dimensional. The acceleration is expected to be a n dimensional object as well. One introduces $4n - 2n - n = n$ coordinates to many. This is because the tangent bundle of the tangent bundle introduces new velocity vectors, which where present on the original manifold. One should identify these. On the quotient manifold we can do physics. These subtleties are not the main point of this discussion, so we let them rest here. More information can for example be found in[2]. Lets denote this quotient manifold by $T^2\mathcal{Q}$.

The main point of the discussion is the following. Suppose we do an experiment multiple times. If we fix all initial coordinates and velocities of our particles to be the same, we get the same time evolution of the system. Thus we conclude that the acceleration is a *function* of the coordinates and velocities (and any other parameters, such as the mass and charges of the particles). In equation this can be written as

$$a = f(x, v, t, m, ...). \tag{2}$$

Since the acceleration is the second time derivative of the coordinates, we conclude that nature is governed by a second order differential equation. For general functions f this equation has no solutions. One needs some regularity on f such that this equation has solutions. Since we derived this equation in the hope of describing the evolution of a system uniquely we also need uniqueness of this equation. Further regularity (Lipschitz continuity in the first two variables is sufficient) ensures that our model has this property. Implicit in our model are these regularity conditions on f.

Newton realised that the function on the right hand side of 2 has a peculiar structure for most systems he considered (in fact all non-gravitationally interacting systems). For each particle the function f was inversely proportionally to a constant, called the mass.

$$f(x, v, t, m, ..) = m^{-1} F(x, v, t, ..). \tag{3}$$

Since we have an n dimensional vector on the right hand side, and the masses of the particles can be different, m is a diagonal matrix. The only exception to this rule was the gravitational force. Thus it seemed natural to modify (2) to

$$m\, a = F(x, v, t, ..). \tag{4}$$

And this is what people remember. We will see that MOND changes this law to

$$m\, a\, \mu(\frac{|a|}{a_0}) = F \tag{5}$$

Here μ is some unknown scalar function, which is hoped to be derivable from some deeper theory and a_0 is a fundamental acceleration constant, in the same way

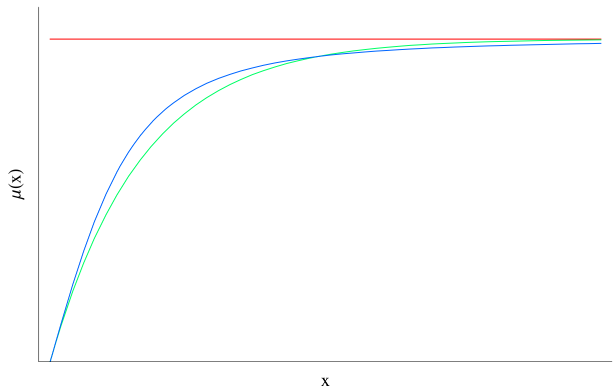

Figure 3: Some possible candidates for the function μ are plotted. In blue $\mu(x) = \frac{x}{x^2+1}$ is graphed, in green $\mu(x) = 1 - e^{-x}$. For comparison the classical case $\mu(x) = 1$ is shown in red.

c is a fundamental velocity constant. The function μ should satisfy the following asymptotics

$$\mu(x) \approx x \quad \text{when } x \ll 1 \text{ and } \quad \mu(x) \approx 1 \quad \text{when } x \gg 1. \tag{6}$$

This is the simplest asymptotic behaviour such that the Newtonian limit is recovered when $a \gg a_0$, and which explains the rotation curves of galaxies asymptotically. μ satisfies some regularity conditions such that 5 yields unique time evolution.

This equation of course differs from Newtons second law (4). It does not violate the principles from which this is "derived" however. MOND's equation 5 *is* a second order differential equation, albeit not in explicit form

$$\phi(a, v, x, t, m, ..) = 0. \tag{7}$$

If the function ϕ satisfies some regularity conditions this equation has a unique solution given initial velocities and positions. It is enough to demand that ϕ is (at least locally) invertible, differentiable, and its inverse (with respect to the first variable) is Lipschitz continuous, in v and x. This puts some mathematical constraints on μ.

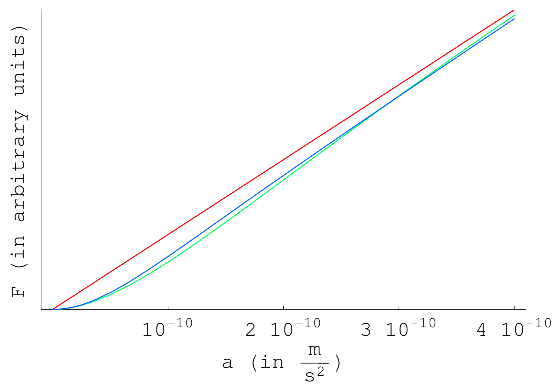

Figure 4: In this graph the acceleration versus the force is graphed for MOND's force law. The blue line denotes the force law for $\mu(x) = \frac{x}{x^2+1}$, the green line for $\mu(x) = 1 - e^{-x}$. The red line corresponds to Newtonian mechanics; that is, $\mu(x) = 1$.

5 MOND and its applicability to galaxies

As explained in the previous section MOND modifies Newton's second law by replacing equation (4) with equation (5). We repeat this equation here for convenience

$$m\, a\, \mu(\frac{|a|}{a_0}) = F. \tag{8}$$

Where μ satisfies the following asymptotics

$$\mu(x) \approx x \quad \text{when } x \ll 1 \text{ and } \quad \mu(x) \approx 1 \quad \text{when } x \gg 1. \tag{9}$$

MOND was designed to make the rotation curves flat. This can be shown with some easy asymptotics, which we do here. We assume a galaxy, which is circular at the rim, where the speed at the rim is constant. This is applicable to most galaxies. Some kinematics (Newtons second law is unnecessary for its derivation) shows that a particle moving in a circular motion with constant speed has acceleration a[3] given by

$$a = \frac{v^2}{|r|}. \tag{10}$$

[3]From here one we take a to be the magnitude of the acceleration. It is a scalar. The speed v is scalar as well.

Here v is the speed of the particle, and r is the radius of the circle. Under the assumption that the acceleration is small compared to a_0 Milgrom's law yields

$$\frac{a^2}{a_0} = g_N(r). \tag{11}$$

Where $g_N(r)$ is the acceleration experienced by the body due to gravitational forces. For a point particle, located at $r = 0$ this is the familiar $g_N = \frac{G\,M}{r^2}$. For a galaxy we have to integrate over the density profile ρ of the galaxy

$$g_N(r) = G \int dr' \frac{\rho(r')}{|r - r'|^2}. \tag{12}$$

If the particle is at the rim of the galaxy (there is no considerable mass outside r) we can approximate the above expression by

$$g_N(r) = \frac{G\,M_{eff}(r)}{r^2} \tag{13}$$

$M_{eff}(r)$ depends on the geometry of the mass of the galaxy. For a spherical body like a star we have $M_{eff} = M$ if $r > R$ the size of the spherical body, as Newton laboriously proved in the Principia. For any other distribution we have that $M_{eff} \to M$ as $r \to \infty$. We can plug the above result in 5 assuming the acceleration is small, therefore using the $\mu(x) = x$ asymptotics of 6. This yields the equation

$$v^4(r) = G\,M\,a_0 \qquad \text{as} \qquad r \to \infty. \tag{14}$$

Which, wondrously, does not depend on the distance r. The velocity for particles rotating at the rim of the galaxy approaches a constant velocity $v_\infty := (G\,M\,a_0)^{\frac{1}{4}}$. If we can measure the mass of a galaxy by other means (we can, by luminosity), we can determine a_0 by determining the speed of the particles at the rim of the galaxy. Milgrom has done this, and found for a_0 the value

$$a_0 = (2.9 \pm 2) \times 10^{-8} \frac{m}{s^2}. \tag{15}$$

6 A thorough derivation of galactic rotation curves

One of the main issues MOND tries to fix are the rotation curves. We compute them here. We will do this in arbitrary units. We will model a galaxy as follows

- The galaxy consists of a disk, and a spherical bulge with radius smaller than the disk.
- The motion of the disk is completely circular.

According to Milgrom's law, the speed of the circular motion in the plane of the disk is given by

$$\frac{|v|^2}{|r|}\mu(\frac{|v|^2}{|r|a_0}) = g_N(r). \tag{16}$$

We now rewrite g_N, and define $\gamma(r, t_1, .., t_n)$ as

$$g_N(r) = \frac{GM}{|r|^2}\int dr' \frac{|r|^2\rho(r')}{M|r-r'|^2} =: \frac{GM}{|r|^2}\gamma(r, t_1, .., t_n).$$

The parameters $t_1, .., t_n$ capture the geometry of the mass distribution. In our toy model a parameter is needed for the fraction of masses in both component. Another parameter for the typical distance ratio is also needed. The following limit holds

$$\lim_{r\to\infty}\gamma(r, t_1, .., t_n) = 1.$$

This is easily computed. Write $r = |r|\frac{r}{|r|}$. Then we have

$$\begin{aligned}
\lim_{r\to\infty}\gamma(r, t_1, .., t_n) &= \lim_{r\to\infty}\frac{1}{M}\int dr' \frac{|r|^2\rho(r')}{||r|\frac{r}{|r|} - r'|^2} \\
&= \frac{1}{M}\lim_{r\to\infty}\int dr' \frac{\rho(r')}{|\frac{r}{|r|} - \frac{r'}{|r|}|^2} \\
&= \frac{1}{M}\int dr'\rho(r') = \frac{M}{M} = 1.
\end{aligned}$$

In the second last step we used $\lim_{r\to\infty}|\frac{r}{|r|} - \frac{r'}{|r|}| = 1$ and in the last step we used that the space integral over ρ equals the total mass of the galaxy. We will not fit the experimental data to the rotation curves. We want to study the phenomenological structure of the rotation. It is therefore natural to introduce new units, which scale with the size of the galaxy. We introduce

$$s := \frac{|r|}{h} \qquad , \qquad V(s) := \frac{|v(sh)|}{v_\infty}, \qquad \text{and} \qquad \xi := \frac{v_\infty^2}{a_0\, h}. \tag{17}$$

Of course s is our new length scale which is measured with respect to h, a typical distance in the galaxy (the size of the disk for example), $V(s)$ a new velocity scale, and ξ is a measure how "spread out" our mass distribution is. In these variables equation (16) becomes

$$\frac{V^2 \xi \mu(\frac{V^2 \xi}{s})}{s} = \frac{\xi^2 \gamma(s, t_1, .., t_n)}{s^2}. \tag{18}$$

Since MOND is an incomplete theory, and does not specify μ, we need to invent one. It is easy to come up with candidate functions which specify the asymptotics (6). We will see that the predictions are relatively insensitive to the specification of μ. We will use

$$\mu(x) = \frac{x}{\sqrt{x^2+1}}.$$

After some high school algebra one finds the implicit (in v) equation

$$\frac{V^4}{\sqrt{1+\frac{V^4 \xi^2}{s^2}}} = \gamma(s\, h, t_1, .., t_n). \tag{19}$$

If we now specify $\gamma(s, t_1, .., t_n)$, we can solve the above equations. In practice we measure $\gamma(s, t_1, .., t_n)$ via luminosity. For our discussion we assume an exponential decay profile of the disk, which yields to a $\gamma(s)$ of

$$\gamma_d(s) = \frac{s^3}{2}(I_0(\frac{s}{2})K_0(\frac{s}{2}) - I_1(\frac{s}{2})K_1(\frac{s}{2})).$$

The exponential decay profile is measured in galaxies. The functions I_k and K_k are modified Bessel functions of the first and second kind respectively. And the sphere has an even less friendly analytic expression. Define

$$\rho(s) = \int_1^\infty dt \frac{e^{-b\, s^{-\frac{1}{4}}\, t}}{\sqrt{t^8+1}} \qquad \text{with} \qquad b = 7.66924944$$

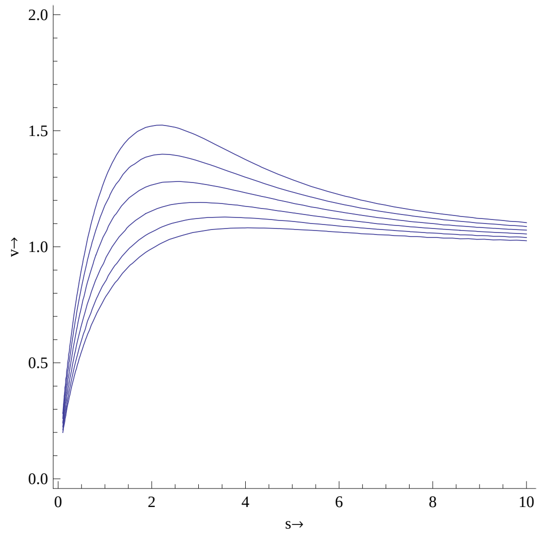

Figure 5: A plot for v for a pure disklike galaxy. The compactness parameter ξ runs from $\xi = 0$ to $\xi = 6$. The lowest function is for $\xi = 0$. Notice the typical flattening for $s \to \infty$.

Then, in our coordinate system

$$\gamma_s(s\,h) = \frac{\int_0^s dt\, t^2 \rho(t)}{\int_0^\infty dt\, t^2 \rho(t)}$$

These formulas can be computed by considering the experimental de Vaucouleurs luminosity relation, and deprojecting it to a sphere. The procedure can be found in [15] and [12]. Using numerical simulations we can plot rotation curves of various model galaxies. These plots all show the typical flattening at large distances. We also notice that we can both have bumps and holes in our rotation curves, depending on the exact structure of our galaxy. All these phenomena are found in experimental data as well.

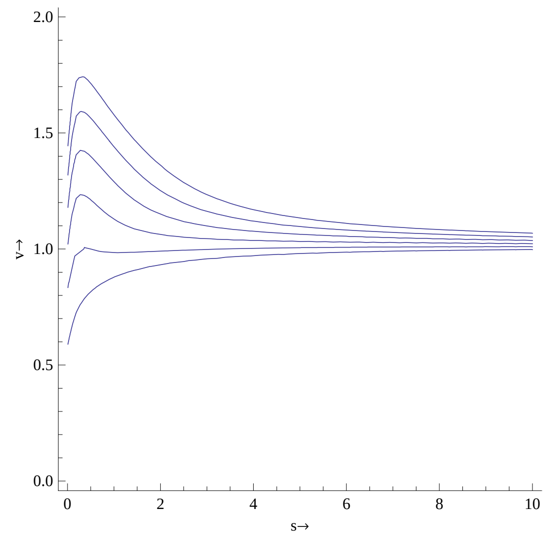

Figure 6: A plot for v for a pure spherelike galaxy. The compactness parameter ξ runs from $\xi = 0$ to $\xi = 6$. The lowest function is for $\xi = 0$. Notice the typical flattening for $s \to \infty$.

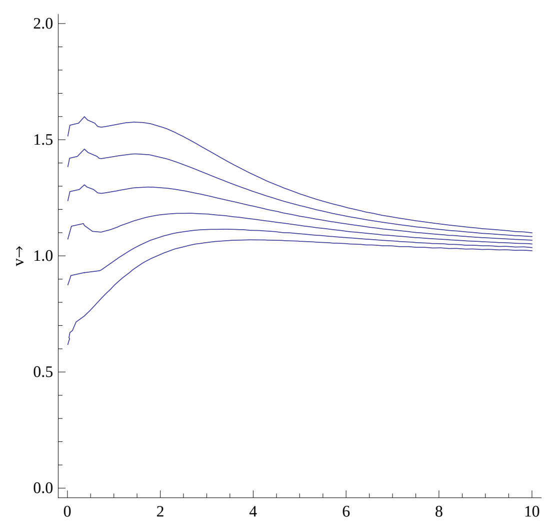

Figure 7: A plot for v for a mixed galaxy. The mass in the disklike component is trice that of the mass in the spheroidal component. The typical distance of the sphere is $\frac{1}{4}-$th of the disk component. The compactness parameter ξ runs from $\xi = 0$ to $\xi = 6$. The lowest function is for $\xi = 0$. Notice the typical flattening for $s \to \infty$.

7 Cosmology

It is interesting to try to couple the new fundamental constant a_0 to old physics. From our collection of fundamental constants (the speed of light, Planck's constant, the charge of an electron etc.) we can try to form a new constant with the dimensions of acceleration. Milgrom has noted that the value of a_0 within errorbars equals the speed of light divided by the lifetime of the universe. The WMAP has as a value for the lifetime of the universe

$$t = 1.4\ \ 10^{10} y$$

. And the speed of light is fixed at

$$c = 299,792,458 \frac{m}{s},$$

therefore

$$\frac{c}{t} = 6.8 \times 10^{-10} \frac{m}{s^2}.$$

This discovery is reminiscent of the discovery that light is an electromagnetic phenomenon, made by Maxwell. Thus cosmological effects might influence the inertia of slowly accelerating objects. This observation might indicate a route of research.

8 Problems

MOND, as discussed in this article, cannot be complete. It is not even a theoretical consistent theory. There are several theoretical en experimental problems to face.

8.1 Relativity

One of the biggest problems of MOND is that it is strongly non-relativistic. The mond equation (5) is assumed to hold in some fundamental frame of reference. One cannot easily write a formula like (5) without resorting to a fundamental frame. It is unclear how the centre of mass acceleration of a galaxy influences a_0. In recent years Bekenstein[3] proposed a field theory with correct relativistic and MOND limits. This can be used to study gravitational lensing in the MOND paradigm. This is useful, since dark matter distributions are measured using weak gravitational lensing methods. This theory might also be used to do cosmology, since the universe is one of the few systems where both a small acceleration scale, and large velocity scale play a role.

8.2 Bullet Cluster

For most clusters and galaxies the rotation curves predicted by MOND fit the experimental data very well. One cluster, called the Bullet Cluster, does not seem to have this property. Even though non-relativistic MOND has no gravitational lensing effects, we expect a relativistic version of the theory will. The problem in the bullet cluster is the following. One can compute (in ordinary GR) centre of mass of a galaxy by considering all the gravitational lensing enforced by this mass. For the bulletcluster this lensing centre of mass is not in the same place as the centre of mass of the visible mass. This is the most direct evidence we have for the dark matter hypothesis. It is not clear why MOND should generate asymmetric lensing effects. It has been speculated that this can be fixed in MOND if one assumes some dark matter. This is theoretically very wrong. The theory was invented to predict rotation curves *without* invoking dark matter. To salvage it we seem to be in need of dark matter. It is better to realise that the theory fails, than to salvage it by invoking dark matter. One other route one can go is to consider a non-rotationally invariant MOND theory.

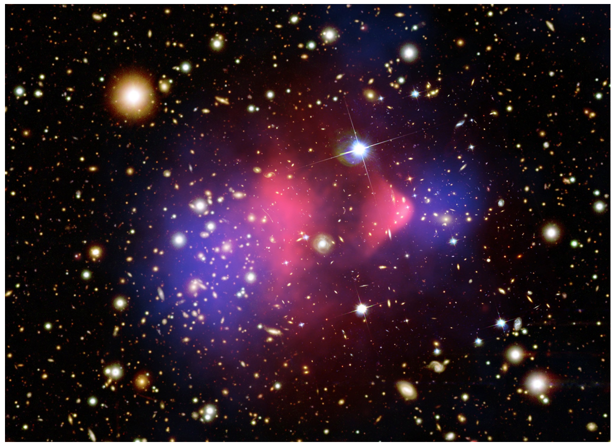

Figure 8: The Bullet cluster[11]. The red blobs are the hot gas measured directly. The blue blobs are dark matter, computed via weak gravitational lensing methods. One sees that the centre of masses of these blobs are not in the same spot. This is a mystery in view of MOND.

8.3 The function μ and non-rotational invariant problems

The function μ is a scalar function. It is actually a function of the norm of the acceleration divided by the acceleration scale a_0. This seems a reasonable assumption in problems with a rotational symmetry. How does this work in non-symmetric problems? We expect that the function μ becomes a two indexed tensor. The exact form of even less known than the scalar μ. Some symmetry arguments might be applied to reduce this freedom of mu.

9 Conclusion

In this article we have discussed the MOND paradigm. Rotation curves for model galaxies were studied extensively. Some comparison with experimental data have been made. This fitted very well. We have seen that MOND is not a complete theory. Further study in MOND theory should at least involve the following questions.

- Can we compute μ?
- What does relativistic MOND theory look like?
- How can one practise MOND in problems without rotational symmetry?
- How can we understand the bullet cluster in MOND theory?
- Can the Pioneer anomaly be understood in view of MOND?
- Can we derive MOND as an effective dark matter theory?
- What are the fundamental physical principles from which we can derive MOND?
- Is the numerical correspondence between the Hubble parameter, the speed of light and the acceleration constant a coincidence?
- What does MOND do to cosmology?

If we can answer all these questions I think we can say that MOND is a good theory. The experimental data do encourage further studies in MOND.

10 Appendix: TeVeS

In 2004 Bekenstein proposed a relativistic field theory which has MOND as a limit. We give a small introduction to this theory. This closely follows [4]. The fields involved in the theory are two tensorial fields $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, one vector field $\mathcal{U}_\mu$ and one scalar field ϕ. These fields are related by

$$\tilde{g}_{\mu\nu} = e^{-2\phi}\left(g_{\mu\nu} + \mathcal{U}_\mu \mathcal{U}_\nu\right) - e^{2\phi}\mathcal{U}_\mu \mathcal{U}_\nu. \tag{20}$$

In this equation one should think of $\tilde{g}_{\mu\nu}$ as the physical metric. It is the metric which couples to the matter fields. The geometrical metric $g_{\mu\nu}$ is the metric used to make dual corrospondence; it raises and lowers indices. The vector field is dynamically normalized to be a lightlike vector field. That is

$$\mathcal{U}^\mu \mathcal{U}_\mu = -1. \tag{21}$$

Please remember that we use $g_{\mu\nu}$ for raising and lowering purposes i.e. $\mathcal{U}^\mu := g^{\mu\nu}\mathcal{U}_\nu$. All fields introduced here should have an action which is added to the Einstein Hilbert action of GR. Without further ado, we state these components of the action. The vectorial action is dictated by

$$S_v := \frac{-K}{32\pi G}\int (g^{\mu\nu}g^{\tau\rho}(\partial_{[\mu}\mathcal{U}_{\nu]})(\partial_{[\tau}\mathcal{U}_{\rho]}) - \frac{2\lambda}{K}(\mathcal{U}^\mu\mathcal{U}_\mu + 1))\sqrt{\det(-g)}dx, \tag{22}$$

the scalar part is

$$S_s := \frac{-1}{2}\int (\sigma^2(g^{\mu\nu} - \mathcal{U}^\mu\mathcal{U}^\nu)(\partial_\mu\phi)(\partial_\nu\phi) + \frac{1G\sigma^4}{2l^2}F(k\,G\,\sigma^2))\sqrt{\det(-g)}dx. \tag{23}$$

The metric $g_{\mu\nu}$ satisfies the Einstein Hilbert action

$$S_{\mathcal{H}} = \frac{1}{16\pi G}\int R\,\sqrt{-\det(g)}dx. \tag{24}$$

While writing these actions we introduced some new constants, variables and fields. The λ is a Lagrange multiplier field. This field is introduced to dynamically satisfy the constraint (21). The field σ is a non-dynamical field, which allows us to write the action in a nice form. One first computes σ in terms of $\partial_\mu\phi$ by varying the action with respect to σ^2. Then one rexpresses the action S_s in terms of $\partial_\mu\phi$ alone. This action is much harder to work with. The σ field does not contain any real physics however. The constant K is a coupling parameter. The constant l is a typical length scale. The function F is in this theory unspecified. It fulfills the same role as μ in ordinary MOND.

It is not hard to see that this theory has GR limit as $K \to 0$ and $l \to \infty$. This limit forces $\mathcal{U} = 0$ and $\phi = 0$, which by (20) shows that $\tilde{g}_{\mu\nu}$ is proportional to $g_{\mu\nu}$. Showing that the matter coupling is as ordinary GR.

The MOND limit is much harder to study. One has to make a proper choice for F. This limit is in detail done in the paper of Bekenstein[3]. One then takes the "Newtonian limit", which amounts to linearizing the metric $\tilde{g}_{\mu\nu}$. One obtains a limiting expression for the physical metric $\tilde{g}_{\mu\nu}$, in proper coordinates

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)dx^2. \tag{25}$$

Here Φ is the mondian substitute for the Newtonian potential. It is related to the Newtonian potential Φ_N via

$$\Phi = \Xi\, \Phi_N + \phi. \tag{26}$$

In this formula Ξ is a constant depending on the particular form of F and the value of K. One has to determine ϕ from the TeVeS analog of the Poisson equation, taking in consideration the mass distribution. One then obtains, assuming spherical symmetry

$$\mu \bigtriangledown \Phi = \bigtriangledown \Phi_N \qquad \text{with} \qquad \mu = \frac{-1 + \sqrt{1 + \frac{4|\bigtriangledown\Phi|}{a_0}}}{1 + \sqrt{1 + \frac{4|\bigtriangledown\Phi|}{a_0}}}, \qquad \text{and} \qquad a_0 = \frac{\sqrt{3\,k}}{4\pi\, l}. \tag{27}$$

This is the MOND theory we wanted to have as a limit. It has been noted that this μ does not fit experimental data of some rotation curves [6]. It is unclear if a better choice of F fixes this problem.

We have discussed only one possible relativistic generalization to MOND theory. It is clear that one has the freedom to introduce many more fields. One can also write the couplings in S_s and S_v to the fields ϕ and $\mathcal{U}_\mu$ not via $g_{\mu\nu}$ but via $\tilde{g}_{\mu\nu}$. One can also add a kinetic term for the σ field. Theoretically one can do so much. Careful astronomical observations should enlighten the path to follow.

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Unraveling the mysteries of black holes using holography

Black holes physics has been at the forefront of theoretical physics for over a quarter of century. Their properties have led to deep questions about the nature of quantum gravity. In this talk we review the black holes puzzles and discuss a recent idea that emergent from string theory and holographic dualities, the fuzzball proposal, that has the potential to resolve all black puzzles.F. Fransisco, P.J.S. Gil, and J. Paramos, Thermal Analysis of the Pioneer Anomaly: A method to estimate radiative momentum transfer, Phys.Rev.D78:103001, 2008

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