Problem set 13 for Cosmology (ns-tp430m)

Problems are due at Thu May 22. In total 9 points

25. The energy density in neutrinos. (*4 points*)

(a) (*3 points*)

Derive the following relation for the energy density of neutrinos,

$$\Omega_\nu h^2 \equiv \frac{\sum_i \rho_{\nu_i}}{\rho_{\rm cr}} h^2 = \frac{\sum_i m_{\nu_i} c^2}{94 \mathrm{eV}}\,, \tag{1}$$

where $\rho_{\rm cr} = (3c^2 H_0^2)/(8\pi G_N)$ denotes the critical density of the Universe, $h = 0.68 \pm 0.01$ is defined in terms of the current Hubble parameter as, $H_0 = h \times 100$ km/s/Mpc and m_{ν_i} $(i = 1, 2, 3)$ denotes the mass of the ith neutrino mass eigenstate. Recall that there are three neutrino flavour states: electron, muon and tau neutrino. Upon diagonalising the neutrino mass matrix one obtains the three neutrino mass eigenstates.

(b) (*1 point*)

What are the current upper bounds from cosmological observations on Ω_ν and $\sum_i m_{\nu_i} c^2$?

26. Nonequilibrium particle dynamics and baryogenesis. (*5 points*)

A particle density n_i of a species i changes close to thermal equilibrium as

$$\frac{dn_i}{dt} \equiv \frac{\partial n_i}{\partial t} + \nabla \cdot \vec{J}_i = -\Gamma_i (n_i - n_{i0}) \tag{2}$$

where n_{i0} denotes the equilibrium density and Γ_i denotes the total decay rate of the species i, $\vec{J}_i = \vec{J}_i^{\rm inj} + \vec{J}_i^{\rm diff}$ is the current, which can be split into the injected current $\vec{J}_i^{\rm inj}$ and diffusion current $\vec{J}_i^{\rm diff} = -D_i \nabla n_i$, where D_i denotes the diffusion constant for the species i, which can be calculated from scattering rates between the species i and other species in the plasma.

Based on Eq. (2) derive the following equation for the evolution of baryon density n_B,

$$D\nabla^2 n_B - \partial_t n_B - \Gamma_B n_B = \nabla \cdot \vec{J}_B^{\rm inj}\,, \tag{3}$$

where Γ_B denotes the baryon decay rate and D the baryon diffusion constant.

Consider a model in which baryons are produced around the surface of expanding (Higgs field) bubbles of the broken phase of the electroweak phase transition. Assume that bubbles are very big and nearly spherical, such that locally their interfaces can be considered as planar walls. Show that in the frame of expanding walls (propagating with a velocity $\vec{v} = v_w \hat{z}$ in the $\hat{z}$ direction) Eq. (3) can be reduced to the following diffusion equation,

$$D n_B'' + v_w n_B' - \Gamma_B n_B = \frac{d}{dz} J_z\,, \tag{4}$$

where $\vec{J}_B^{\rm inj} = \hat{z} J_z$. Assume further that the injected current, which is a consequence of CP violating dynamics at the bubble interface, is of the form, $J_z = J_0 \delta(z)$, $\delta(z)$ is the Dirac δ-function.

Calculate $n_B = n_B(z)$ assuming the following form for the baryon violating processes (sphalerons),

$$\Gamma_B = \begin{cases} \Gamma, & \text{for } z > 0 \\ 0, & \text{for } z < 0. \end{cases} \tag{5}$$

Express your answer in terms of D, Γ, v_w and J_0.

NB: This problem illustrates how a nonzero baryon number can be produced by dynamical processes in the Early Universe if the rate of baryon number violating processes changes; this mechanism works even in the absence of a net baryon number source.