Problem set 15 for Cosmology (ns-tp430m)

Problems are due at Thu Jun 12. In total 8 plus 3 bonus points.

29. Domain walls. (*8 points + 3* bonus points*)

Domain walls may appear in the early Universe if there is a (thermally induced) phase transition of a real scalar field with a potential of the form,

$$V_{\rm dw} = \frac{\lambda}{4}\Big(\phi^2 - \mu^2\Big)^2 \tag{1}$$

A domain wall is the (approximately planar) region between two regions, one of which belongs to the $\phi = \mu$ vacuum, and the other to the $\phi = -\mu$ vacuum (the residual symmetry of the vacuum is Z_2).

Assume that there is a thermally induced phase transition in the early universe, by which (for simplicity) a domain wall forms, which stretches accross the Universe, and which is located at coordinate value $z = 0$, such that $\phi = \mu$ at $z \gg 0$, and $\phi = -\mu$ at $z \ll 0$, and $\phi = 0$ at $z = 0$.

(a) (*2 points*)

Show that the stress energy tensor of a domain wall placed in an expanding Universe ($g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$) is given by,

$$T_{\mu\nu} = (\partial_\mu\phi)(\partial_\nu\phi) - g_{\mu\nu}\Big(\frac{1}{2}g^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - V_{\rm dw}(\phi)\Big) \tag{2}$$

(b) (*2 points*)

Show that, when the Universe expansion is neglected, the equation of motion for a static wall can be integrated to yield,

$$\Big(\frac{d\phi}{dz}\Big)^2 = 2V_{\rm dw}\,. \tag{3}$$

(c) (*4 points*)

Based on Eqs. (2) and (3), show that the stress energy tensor of a static wall placed at $z = 0$ is of the form,

$$T^\mu_{\ \nu} = \begin{pmatrix} \rho_{\rm dw} & 0 & 0 & 0 \\ 0 & -p_{\rm dw} & 0 & 0 \\ 0 & 0 & -p_{\rm dw} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4}$$

and that

$$\rho_{\rm dw} = -p_{\rm dw} = 2V_{\rm dw} = \frac{\lambda}{2}\frac{\mu^4}{\cosh^4\Big(\sqrt{\frac{\lambda}{2}}\,\mu z\Big)}\,. \tag{5}$$

(d*) *(3* bonus points)*

Show that the acceleration of a test particle just outside the wall sheet ($z \approx 0$) is

$$\vec{a} \approx 2\pi G_N \sigma \mathrm{sign[z]} \vec{e}_z \tag{6}$$

where $\vec{e}_z$ denotes the unit vector in the z-direction, σ is the wall tenson, defined as the energy per unit area, $\sigma = \int_{-\infty}^{\infty} dz \rho_{\mathrm{dw}}$. Since $\mathrm{sign[z]} = 1$ for $z > 0$ and $\mathrm{sign[z]} = -1$ for $z < 0$, note that a domain wall is *repulsive*. *Hint:* Show that the line element just outside the sheet is of the form, $ds^2 = (1 - 2\pi G_N \sigma |z|)^2 dt^2 - (1 - 2\pi G_N \sigma |z|)^2 \mathrm{e}^{4\pi G_N \sigma t}(dx^2 + dy^2) - dz^2$, from which you can read off the Newtonian potential $\phi_N \simeq -2\pi G_N \sigma |z|$, and hence the acceleration (6).