Problem set 8 for Cosmology (ns-tp430m)

Problems are due at Thu Apr 10.

15. *Particle Horizon.* (*4 points + 3 bonus points*)

Particle horizon is defined as the distance the light travels from some initial time to today.

(a) (*2 points*)

Show that in the FLRW space-time with the metric in spherical coordinates,

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2/(1-\kappa r^2, -a^2 r^2, -a^2 r^2 \sin^2(\theta)), \tag{1}$$

particle horizon, when expressed in physical units, equals

$$\ell_{\rm phys}(t) = a(t) \int_0^t \frac{c dt'}{a(t')} \,. \tag{2}$$

In comoving coordinates, particle horizon (when expressed in comoving coordinates) is simply, $\ell = \ell_{\rm phys}/a(t) = \int_0^t c dt'/a(t')$.

Show also that, when the metric is expressed in terms of conformal time ($d\eta = dt/a$), the (comoving) particle horizon is simply,

$$\ell(t) = c(\eta(t) - \eta(0)) \,. \tag{3}$$

(b) (*2 points*)

If the particle horizon at the photon-electron decoupling is $\ell = r_s = 147$ Mpc (WMAP result) (the redshift at decoupling equals $z = 1089$), what is particle horizon (a) at the time of the matter-radiation equality, when $z = 3233^{+194}_{-210}$, (b) today (at $z = 0$). For simplicity assume that the Universe is matter dominated, $a \propto t^{2/3}$ and flat ($\kappa = 0$).

(c*) (*3 bonus points*)

Repeat the calculation from **15.(b)** by calculating in a Universe dominated by a cosmological term Λ and nonrelativistic matter. Evaluate particle horizon at $z_{\rm eq} = 3233$ and at $z = 0$. For your numerical estimates, take $\Omega_\Lambda = 0.74$, and $\Omega_{\rm m} = 0.26$ and radiation density, $\Omega_\gamma = \Omega_{\rm m}/(1+z_{\rm eq})$. You may perform approximate calculations, so your answers need not be exact, yet they should represent a good approximation to the correct answers.

16. *Quintessence.* (*8 points*)

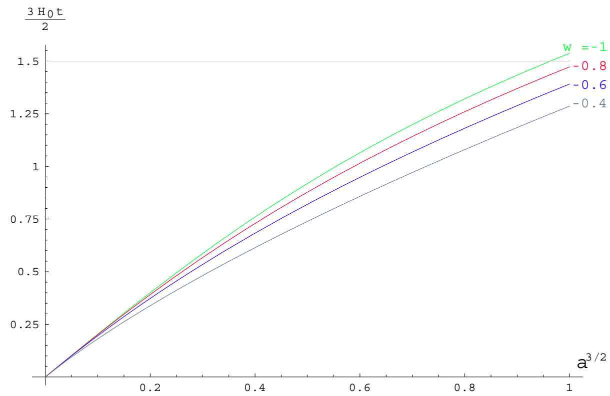

FIG. 1: $(3/2)H_0 t$ as a function of $a^{3/2}$ for $w_Q = \{-1, -0.8, -0.6, -0.4\}$.

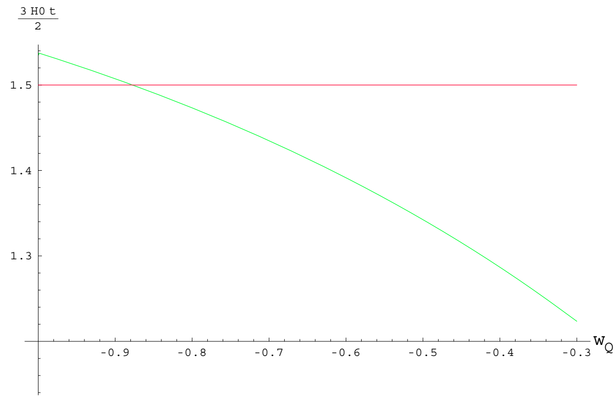

FIG. 2: The age of the Universe in units of $(3/2)H_0 t_0$ as a function of w_Q for $a = a_0 = 1$. Note that for $\Omega_m = 0.24$ and $\Omega_Q = 0.76$, the age, $t_0 = H_0^{-1}$, is obtained for $w_Q \simeq -0.88$.

Assume that the matter content of the Universe consists of nonrelativistic matter with the density, $\Omega_m = \rho_m/\rho_{\rm cr} = 0.24$, and a scalar field matter (quintessence), whose equation of state has the form,

$$p_Q = w_Q \rho_Q \,, \qquad w_Q = \text{const.} \qquad (-1 \leq w_Q \leq -1/3) \,. \tag{4}$$

Take for the density of the Q-matter today to be $\Omega_Q = 0.76$, such that the Universe is spatially flat $\Omega_\kappa = 0$, $\Omega_\Lambda = 0$.

(a) *(2 points)*

Derive the functional dependence of ρ_Q and ρ_m on the scale factor a!

(b) *(3 points)*

Write down an integral expression which relates the scale factor a to the cosmic time t.

(c) *(3 points)*

By making use of `Mathematica`, perform the integral you obtained in (b). Write the resulting transcendental equation which relates the age t to the scale factor a. The result can be expressed in terms of a hypergeometric function. Plot the curve for several values of w_Q. The result is shown in figure 1. Note that the age of the Universe decreases as w_Q increases, which is shown in figure 2.