Using Kappas as Indicators of Strength in Qualitative Probabilistic Networks

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Abstract. Qualitative probabilistic networks are designed for probabilistic inference in a qualitative way. They capture qualitative influences between variables, but do not provide for indicating the strengths of these influences. As a result, trade-offs between conflicting influences remain unresolved upon inference. In this paper, we investigate the use of order-of-magnitude kappa values to capture strengths of influences in a qualitative network. We detail the use of these kappas upon inference, thereby providing for trade-off resolution.

1 Introduction

Qualitative probabilistic networks [1] and the kappa calculus [2] both provide for probabilistic reasoning in a qualitative way. A qualitative probabilistic network is basically a qualitative abstraction of a probabilistic network and similarly encodes variables and the probabilistic relationships between them in a directed acyclic graph. The encoded relationships represent influences on the probability distributions of variables and are summarised by a sign indicating the direction of *change* or shift (positive, negative, zero, or unknown) in the distribution of one variable occasioned by another. The kappa calculus offers a framework for reasoning with defeasible beliefs, where belief states are given by a ranking function that maps propositions into non-negative integers called kappa values. Kappa values, by means of a probabilistic interpretation [3], were previously used to abstract probabilistic network into so-called Kappa networks, where a network's probabilities are abstracted into kappa values, which are easier to assess than precise probabilities and lead to more robust inference results [4,5].

Inference in Kappa networks is based on the use of kappa calculus and is in general of the same order of complexity as inference in probabilistic networks (NP-hard). In contrast, inference with a qualitative probabilistic network can be done efficiently by propagating and combining signs [6]. However, qualitative probabilistic networks, due to the high level of abstraction, do not provide for weighing influences with conflicting signs and, hence, do not provide for resolving

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such *trade-offs*. Inference with a qualitative probabilistic network therefore often results in ambiguous signs that will spread throughout most of the network.

Preventing ambiguous inference results is essential as qualitative networks can play an important role in the construction of quantitative probabilistic networks for realistic applications [7]. Assessing the numerous required point probabilities for a probabilistic network is a hard task and typically performed only when the network's digraph is considered robust. By first assessing signs for the modelled relationships, a qualitative network is obtained that allows for studying the inference behaviour of the projected quantitative network, prior to probability assessment. Ambiguous inference results in a qualitative network can to some extent be averted by, for example, introducing a notion of strength of influences. To this end, previous work partitions the set of qualitative influences into strong and weak influences [8]. In this paper, we investigate the combination of qualitative probabilistic networks and kappa values. A novel approach to using kappa values allows us to distinguish *several* levels of strength of qualitative influences, thereby enabling the resolution of more trade-offs.

This paper is organised as follows. Section 2 provides preliminaries concerning qualitative probabilistic networks; Sect. 3 details our use of kappa values to indicate strengths of influences. Section 4 presents an inference procedure for our kappa enhanced networks. The paper ends with some conclusions in Sect. 5.

2 Qualitative Probabilistic Networks

A probabilistic network, or Bayesian network, uniquely encodes a joint probability distribution Pr over a set of statistical variables. A *qualitative probabilistic network* (QPN) can be viewed as a qualitative abstraction of such a network, similarly encoding statistical variables and probabilistic relationships between them in an acyclic directed graph $G = (V(G), A(G))$ [1]. Each node $A \in V(G)$ represents a variable, which, for ease of exposition, we assume to be binary, writing a for $A = \mathit{true}$ and $\bar{a}$ for $A = \mathit{false}$. The set $A(G)$ of arcs captures probabilistic independence between the variables. Where a quantitative probabilistic network associates conditional probability distributions with its digraph, a qualitative probabilistic network specifies qualitative influences and synergies that capture shifts in the existing, but as of yet unknown (conditional) probability distributions. A *qualitative influence* between two nodes expresses how the values of one node influence the probabilities of the values of the other node. For example, a *positive qualitative influence along arc* $A \rightarrow B$ of node A on node B, denoted $S^+(A, B)$, expresses that observing a high value for A makes the higher value for B more likely, regardless of any other direct influences on B, that is, for $a > \bar{a}$ and any combination of values x for the set $\pi(B) \setminus \{A\}$ of (direct) predecessors of B other than A:

$$\Pr(b \mid ax) - \Pr(b \mid \bar{a}x) \geq 0.$$

A negative qualitative influence S^- and a zero qualitative influence S^0 are defined analogously; if an influence is not monotonic or if it is unknown, it is called

Table 1. The $\otimes$- and $\oplus$-operators for combining signs

$\otimes$	$+$	$-$	0	$?$
$+$	$+$	$-$	0	$?$
$-$	$-$	$+$	0	$?$
0	0	0	0	0
$?$	$?$	$?$	0	$?$

$\oplus$	$+$	$-$	0	$?$
$+$	$+$	$?$	$+$	$?$
$-$	$?$	$-$	$-$	$?$
0	$+$	$-$	0	$?$
$?$	$?$	$?$	$?$	$?$

ambiguous, denoted $S^?$. The definition of qualitative influence can be straightforwardly generalised to an influence along a *chain* of nodes in G.

A qualitative probabilistic network also includes *product synergies* [6], that capture the sign of the (*intercausal*) qualitative influence induced between the predecessors A and B of a node C upon its observation; an induced intercausal influence behaves as a regular qualitative influence.

The set of qualitative influences exhibits various properties. The property of *symmetry* states that, if the network includes the influence $S^\delta(A, B)$, then it also includes $S^\delta(B, A)$, $\delta \in \{+, -, 0, ?\}$. The *transitivity* property asserts that the signs of qualitative influences along a chain with no head-to-head nodes combine into a sign for a net influence with the $\otimes$-operator from Table 1. The property of *composition* asserts that the signs of multiple influences between nodes along parallel chains combine into a sign for a net influence with the $\oplus$-operator. Note that composition of two influences with conflicting signs, modelling a *trade-off*, results in an ambiguous sign, indicating that the trade-off cannot be *resolved*.

For inference with a qualitative network an efficient algorithm, that builds on the properties of symmetry, transitivity, and composition of influences, is available [6] and summarised in Fig. 1. The algorithm traces the effect of observing a value for one node on the other nodes by message-passing between neighbours. For each node, a *node sign* is determined, indicating the direction of change in its probability distribution occasioned by the new observation. Initial node signs equal '0', and observations are entered as a '+' for the observed value *true* or a '−' for the value *false*. Each node receiving a message updates its sign with the $\oplus$-operator and subsequently sends a message to each (induced) neighbour that is not independent of the observed node. The sign of this message is the $\otimes$-product of the node's (new) sign and the sign of the influence it traverses.

procedure PropagateSign(*from*,*to*,*messagesign*):

sign[*to*] $\leftarrow$ sign[*to*] $\oplus$ *messagesign*;
for each (induced) neighbour V_i of *to*
do *linksign* $\leftarrow$ sign of (induced) influence between *to* and V_i;
    messagesign $\leftarrow$ sign[*to*] $\otimes$ *linksign*;
    if $V_i \neq$ *from* **and** $V_i \notin$ *Observed* **and** sign[V_i] $\neq$ sign[V_i] $\oplus$ *messagesign*
    then PropagateSign(*to*,V_i,*messagesign*)

Fig. 1. The sign-propagation algorithm

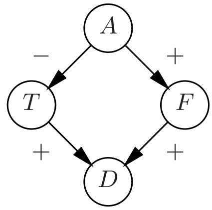

Fig. 2. The qualitative *Antibiotics* network

This process is repeated throughout the network, until each node has changed its sign at most twice (once from '0' to '+', '−', or '?', then only to '?').

Example 1. Consider the qualitative network from Fig. 2, representing a fragment of fictitious medical knowledge which pertains to the effects of taking antibiotics on a patient. Node A represents whether or not the patient takes antibiotics. Node T models whether or not the patient has typhoid fever and node D represents presence or absence of diarrhoea in the patient. Node F describes whether or not the patient's bacterial flora composition has changed. Typhoid fever and bacterial flora change can both cause diarrhoea: $S^+(T, D)$ and $S^+(F, D)$. Antibiotics can cure typhoid fever, $S^-(A, T)$, but may also change the bacterial flora composition, $S^+(A, F)$.

We observe that a patient has taken antibiotics and enter the sign '+' for node A. Node A propagates this sign to T, which receives '$+ \otimes - = -$' and sends this to node D. Node D in turn receives '$- \otimes + = -$' and does not pass on any sign. Node A also sends its sign to F, which receives '$+ \otimes + = +$' and passes this on to node D. Node D then receives the additional sign '$+ \otimes + = +$'. The two signs for D are combined, resulting in the ambiguous '$- \oplus + = ?$'; the modelled trade-off thus remains unresolved. □

3 Introducing a Notion of Strength into QPNs

To provide for trade-off resolution in qualitative probabilistic networks, we introduce a notion of strength of qualitative influences using kappa values.

3.1 Kappa Rankings and Their Interpretations

The *kappa calculus* provides for a semi-qualitative approach to reasoning with uncertainty [2][3]. In the kappa calculus, degrees of (un)certainty are expressed by a ranking κ that maps propositions into non-negative integers such that $\kappa(true) = 0$ and $\kappa(a \vee b) = \min\{\kappa(a), \kappa(b)\}$. For reasoning within the kappa calculus simple combination rules for manipulation of κ-values exist.

Kappa rankings can be interpreted as order-of-magnitude approximations of probabilities [3], allowing, for example, to compute posterior probabilities using kappa calculus. A probability $\Pr(x)$ can be approximated by a polynomial written in terms of a (infinitesimal) *base* number $0 < \epsilon < 1$. $\kappa(x)$ now represents the order of magnitude of this polynomial. More formally,

$$\kappa(x) = n \text{ iff } \epsilon^{n+1} < \Pr(x) \leq \epsilon^n \ . \tag{1}$$

Note that higher probabilities are associated with lower κ-values; for example, $\kappa(x) = 0$ if $\Pr(x) = 1$, and $\kappa(x) = \infty$ iff $\Pr(x) = 0$.

3.2 Using Kappas as Indicators of Strength

We consider a qualitative probabilistic network with nodes A, B and X such that $\pi(B) = X \cup \{A\}$. Let $I_x(A, B)$ denote the influence of A on B for a certain combination of values x for the set X. We recall that the sign δ of the qualitative influence of A on B is defined as the sign of $\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)$, for all x; the absolute values of these differences lie in the interval $[0, 1]$. Analogous to equivalence (1), we define the κ-value of an influence of A on B for a certain x:

$$\kappa(I_x(A, B)) = n \text{ iff } \epsilon^{n+1} < |\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)| \leq \epsilon^n.$$

We then define the *strength factor* associated with the influence of A on B to be an *interval* $[p, q]$ such that

$$p \geq \max_x \ \kappa(I_x(A, B)) \ \text{ and } \ \ 0 \leq q \leq \min_x \ \kappa(I_x(A, B)),$$

and each κ expresses an order of magnitude in terms of the same base. We associate strength factors with positive and negative influences; zero and ambiguous influences are treated as in regular qualitative probabilistic networks. The above definitions extend to chains of influences as well. The resulting network will be termed a *kappa-enhanced* qualitative probabilistic network and we write $S^{\delta[p,q]}(A, B)$ to denote a qualitative influence of node A on node B with sign δ and strength factor $[p, q]$ in such a network.

Note that for a strength factor $[p, q]$ we always have that $p \geq q$, where p is greater than or equal to the kappa value of the *weakest* possible influence and q is less than or equal to the kappa value of the *strongest* possible influence. The reason for allowing influences to pretend to be stronger or weaker than they are will become apparent. Note that for each influence $[\infty, 0]$ is a valid strength factor, but not a very informative one.

We can now express strength of influences in a kappa-enhanced network in terms of the base ϵ chosen for the network: the influence of node A on node B has strength factor $[p, q]$ iff for all x

$$\epsilon^{p+1} < |\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)| \leq \epsilon^q \ .$$

Instead of capturing the influences between variables by using kappa values for probabilities, as is done in Kappa networks, we capture influences by associating kappa values with the arcs. A Kappa network requires a number of kappa values that is exponential in the number of parents for each node; our kappa-enhanced networks require only a number of kappa values that is linear in the number of parents for each node.

4 Inference in Kappa-Enhanced Networks

Probabilistic inference in qualitative probabilistic networks builds on the properties of symmetry, transitivity and composition of influences. In order to exploit the strength of influences upon inference in a kappa-enhanced network, we define new $\otimes$- and $\oplus$-operators.

4.1 Kappa-Enhanced Transitive Combination

To address the effect of multiplying two signs with strength factors in a kappa-enhanced network, we consider the *left* network fragment from Fig. 4. The fragment includes the chain of nodes A, B, C, with two qualitative influences between them; in addition, we take $X = \pi(B) \setminus \{A\}$, and $Y = \pi(C) \setminus \{B\}$. For the net influence of A on C, we now find by conditioning on B that

$$\Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) = (\Pr(c \mid by) - \Pr(c \mid \bar{b}y)) \cdot (\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)) \quad (2)$$

for any combination of values x for the set X and y for Y. Similar equations are found given other arc directions, as long as node B has at least one outgoing arc. Other influences of A on C than those shown are taken into account by the $\oplus$-operator.

$\otimes$	$+[r,s]$	$-[r,s]$	0	$?$
$+[p,q]$	$+[p+r+1,q+s]$	$-[p+r+1,q+s]$	0	$?$
$-[p,q]$	$-[p+r+1,q+s]$	$+[p+r+1,q+s]$	0	$?$
0	0	0	0	0
$?$	$?$	$?$	0	$?$

Fig. 3. The new $\otimes$-operator for combining signs and strength factors

Transitively combining influences amounts to multiplying differences in probability, resulting in differences that are smaller than those multiplied; transitive combination therefore causes weakening of influences. This is also apparent from Fig. 3 which shows the table for the new $\otimes$-operator: upon transitive combination, the strength factor shifts to higher kappa values, corresponding to weaker influences. From the table it is also readily seen that signs combine as in a regular qualitative probabilistic network; the difference is just in the handling of the strength factors. We illustrate the combination of two positive influences; similar observations apply to other combinations.

Proposition 1. *Let A, B and C be as in the left fragment of Fig. 4, then*

$$S^{+[p,q]}(A,B) \;\wedge\; S^{+[r,s]}(B,C) \;\Rightarrow\; S^{+[p+r+1,q+s]}(A,C)\ .$$

Proof: Let X and Y be as in the left fragment of Fig. 4. Suppose $S^{+[p,q]}(A,B)$ and $S^{+[r,s]}(B,C)$. We now have for the network-associated base ϵ that

$$\epsilon^{p+1} < \Pr(b \mid ax) - \Pr(b \mid \bar{a}x) \leq \epsilon^{q} \ \text{ and}$$
$$\epsilon^{r+1} < \Pr(c \mid by) - \Pr(c \mid \bar{b}y) \leq \epsilon^{s} \ .$$

From Equation (2) for the net influences of node A on node C, we now find that

$$\epsilon^{(p+r+1)+1} = \epsilon^{p+1} \cdot \epsilon^{r+1} < \Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) \leq \epsilon^{q} \cdot \epsilon^{s} = \epsilon^{q+s}$$

for any combination of values xy for the set $X \cup Y$. As $\epsilon \geq 0$, we find that the resulting net influence of A on C is positive with strength $[p+r+1, q+s]$. $\square$

4.2 Kappa-Enhanced Parallel Composition

For combining multiple qualitative influences between nodes along parallel chains, we provide the new $\oplus$-operator in Fig. 5, which takes strength factors into account. In addressing parallel composition we first assume that ϵ is infinitesimal; the effect of a non-infinitesimal ϵ on the $\oplus$-operator is discussed at the end of this section. We consider the *right* network fragment from Fig. 4, which includes the parallel chains A, C, and A, B, C, respectively, between the nodes A and C, and various qualitative influences; in addition, we take $X = \pi(B) \setminus \{A\}$ and $Y = \pi(C) \setminus \{A, B\}$. For the net influence of A on C along the two parallel chains, conditioning on B gives the following equation for any combination of values x for X and y for Y

$$\begin{aligned} \Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) = \quad & (\Pr(c \mid aby) - \Pr(c \mid a\bar{b}y)) \cdot \Pr(b \mid ax) \\ - \ & (\Pr(c \mid \bar{a}by) - \Pr(c \mid \bar{a}\bar{b}y)) \cdot \Pr(b \mid \bar{a}x) \\ + \ & \Pr(c \mid a\bar{b}y) - \Pr(c \mid \bar{a}\bar{b}y) \ . \end{aligned} \tag{3}$$

Similar equations are found if arc directions are changed, as long as the fragment remains acyclic and B has at least one outgoing arc.

Parallel composition of influences may result in a net influence of larger magnitude: the result of adding two positive or two negative influences is at least as strong as the strongest of the influences added. This observation is also apparent from Fig. 5: the minimum of kappa values represents a stronger net influence. On the other hand, adding conflicting influences may result in a net influence of smaller magnitude, which is also apparent from Fig. 5.

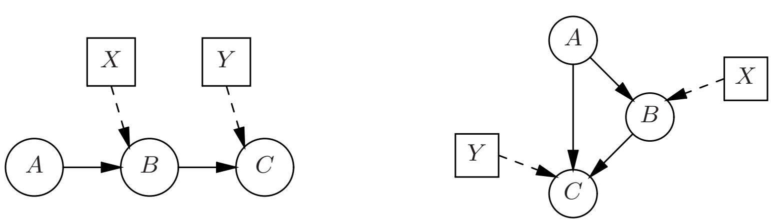

Fig. 4. Two illustrative network fragments

As examples, we now illustrate the parallel composition of two positive influences, and — more interesting in the light of resolving trade-offs — the composition of a positive and a negative influence. Similar observations with respect to the sign and strength factor of a net influence apply to situations in which the signs of the influences are different from those discussed.

Proposition 2. *Let A and C be as in the right fragment of Fig. 4, then*

$$S_1^{+[p,q]}(A,C) \ \wedge \ S_2^{+[r,s]}(A,C) \ \Rightarrow \ S_{net}^{+[\min\{p,r\},\min\{q,s\}]}(A,C) \ .$$

Proof: Let B, X and Y be as in the right fragment of Fig. 4. Suppose that $S_1^{+[p,q]}(A,C)$ and $S_2^{+[r,s]}(A,C)$, and that the positive influence $S_2^{+[r,s]}(A,C)$ is composed of the influences $S^{+[r',s']}(A,B)$ and $S^{+[r'',s'']}(B,C)$ such that $r = r' + r'' + 1$ and $s = s' + s''$. Similar observations apply when these latter two influences are negative. We now have for the network-associated base ϵ that

$$\begin{aligned} \epsilon^{p+1} &< \Pr(c \mid ab_iy) - \Pr(c \mid \bar{a}b_ix) \leq \epsilon^q, \text{ for all values } b_i \text{ of } B, \\ \epsilon^{r'+1} &< \Pr(b \mid ax) - \Pr(b \mid \bar{a}x) \leq \epsilon^{s'}, \quad \text{and} \\ \epsilon^{r''+1} &< \Pr(c \mid a_iby) - \Pr(c \mid a_i\bar{b}y) \leq \epsilon^{s''}, \text{ for all values } a_i \text{ of } A. \end{aligned}$$

From Equation (3) for the net influence of node A on node C, we now find that

$$\begin{aligned} &\Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) > \epsilon^{r'+r''+2} + \epsilon^{p+1} = \epsilon^{r+1} + \epsilon^{p+1} \geq \epsilon^{min\{r,p\}+1}, \text{ and} \\ &\Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) \leq \epsilon^q + \epsilon^{s'+s''} = \epsilon^q + \epsilon^s, \end{aligned}$$

for any combination of values xy for the set $X \cup Y$. The lower-bound for this difference is, for example, attained for $\Pr(b \mid \bar{a}x) = 0$ and $\Pr(b \mid ax) = \epsilon^{r'+1}$. The upper-bound is attained, for example, for $\Pr(b \mid ax) = 1$ and $\Pr(b \mid \bar{a}x) = 1 - \epsilon^{s'}$. In computing these bounds, we have exploited the available information with regard to the signs and strengths of the influences involved.

For infinitesimal ϵ the upper-bound $\epsilon^q + \epsilon^s$ is approximated by $\epsilon^{\min\{q,s\}}$; the net influence is thus positive with strength factor $[\min\{p,r\},\min\{q,s\}]$. □

If two influences have conflicting signs, then one 'outweighs' the other if its weakest effect is stronger than the other influence's strongest effect. We adapt the

$\oplus$	$+[r,s]$	$-[r,s]$	0	$?$
$+[p,q]$	$+[u,v]$	a)	$+[p,q]$	$?$
$-[p,q]$	b)	$-[u,v]$	$-[p,q]$	$?$
0	$+[r,s]$	$-[r,s]$	0	$?$
$?$	$?$	$?$	$?$	$?$

$[u,v] = [\min\{p,r\},\min\{q,s\}]$

a) $+[p,q]$, if $p+1 < s$;
$+[\infty,q]$, if $p < s$;
$-[r,s]$, if $r+1 < q$;
$-[\infty,s]$, if $r < q$;
$?$, otherwise

b) see a) with $+$ and $-$ reversed

Fig. 5. The new $\oplus$-operator for combining signs and strength factors (ϵ infinitesimal)

safest and most conservative approach to combining conflicting influences, that is, by comparing the lower bound of the one influence with the upper bound of the other. Other interval comparison methods are however possible (see e.g. [9]).

Proposition 3. *Let A and C be as in the right fragment of Fig. 4, then*

$$S_1^{+[p,q]}(A,C) \ \wedge \ S_2^{-[r,s]}(A,C) \Rightarrow \begin{array}{ll} S_{net}^{+[p,q]}(A,C) & \text{if } p+1<s; \\ S_{net}^{+[\infty,q]}(A,C) & \text{if } p<s; \\ S_{net}^{-[r,s]}(A,C) & \text{if } r+1<q; \\ S_{net}^{-[\infty,s]}(A,C) & \text{if } r<q; \\ S_{net}^{?}(A,C) & \text{otherwise.} \end{array}$$

Proof: Let B, X and Y be as in the right fragment of Fig. 4. Suppose $S_1^{+[p,q]}(A,C)$ and $S_2^{-[r,s]}(A,C)$, and let the negative influence $S_2^{-[r,s]}(A,C)$ be composed of $S^{-[r',s']}(A,B)$ and $S^{+[r'',s'']}(B,C)$ such that $r = r' + r'' + 1$ and $s = s' + s''$. Similar observations apply when these latter signs are switched. From Equation (3), we now have for the network-associated base ϵ that

$$\begin{aligned} \Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) &> \epsilon^{p+1} - \epsilon^{s''} \cdot \epsilon^{s'} = \epsilon^{p+1} - \epsilon^{s}, \text{ and} \\ \Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) &< \epsilon^{q} - \epsilon^{r''+1} \cdot \epsilon^{r'+1} = \epsilon^{q} - \epsilon^{r+1}, \end{aligned}$$

for any combination of values xy for $X \cup Y$. The lower-bound for the difference (not distance!) is attained, for example, for $\Pr(b \mid ax) = 0$ and $\Pr(b \mid \bar{a}x) = \epsilon^{s'}$; the upper-bound for the difference is attained, for example, for $\Pr(b \mid \bar{a}x) = 1$ which enforces $\Pr(b \mid ax) < 1 - \epsilon^{r'+1}$. In computing these bounds, we have once again exploited the available information with regard to the signs and strengths of the influences involved.

Now, if $\epsilon^{p+1} \geq \epsilon^{s}$ then $\Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) > 0 = \epsilon^{\infty}$. Given infinitesimal ϵ the lower-bound $\epsilon^{p+1} - \epsilon^{s}$ is approximated by ϵ^{p+1} under the tighter constraint $p+1 < s$. The constraint $p+1 \leq s$ also implies $q < r+1$, giving an upper-bound of $\epsilon^{q} - \epsilon^{r+1} \leq \epsilon^{q}$. We conclude that the resulting influence is positive with strength factor $[p,q]$ if $p+1 < s$ and strength factor $[\infty, q]$ if $p < s$.

On the other hand, if $\epsilon^{r+1} \geq \epsilon^{q}$ then $\Pr(c \mid axy) - \Pr(c \mid \bar{a}xy) < 0 = \epsilon^{\infty}$. Given infinitesimal ϵ the (negative!) upper-bound $-\epsilon^{r+1} + \epsilon^{q}$ is approximated by $-\epsilon^{r+1}$ under the tighter constraint $r+1 < q$. The constraint $r+1 \leq q$ also implies $p+1 > s$, so we find a (negative) lower-bound of $-\epsilon^{s} + \epsilon^{p+1} \geq -\epsilon^{s}$. We conclude that the resulting influence is negative. Taking the absolute values of the given bounds, we find a strength factor $[r,s]$ if $r+1 < q$ and $[\infty, s]$ if $r < q$. □

The Non-infinitesimal Case. The $\oplus$-operator defined above explicitly uses the fact that our kappa values are order of magnitude *approximations* of differences in probability by just taking into account the most significant ϵ-term in determining the strength factor of the net influence. Such approximations are valid as long as ϵ indeed adheres to the assumption that it is infinitesimal. In a realistic problem domain, however, probabilities and even differences in probability are hardly ever all very close to zero or one, and a non-infinitesimal ϵ is required to distinguish different levels of strength.

Although the inference algorithm sums only two signs with strength factors at a time, ultimately a sign and strength factor can be the result of a larger summation. If $1/\epsilon$ parallel chains to a single node are combined upon inference, the approximation used by the $\oplus$-operator will be an order of magnitude off, affecting not only the strength factor of the net influence (the interval becomes too 'tight': the influence can be stronger or weaker than captured by the interval), but possibly its sign as well. For inference in a kappa-enhanced network in which the assumption of an infinitesimal ϵ is violated, therefore, we have to perform an additional operation. We have a choice between two types of operation, depending on whether or not the actual value of ϵ is known. If ϵ is unknown, this operation consists of 'broadening' the interval an extra order of magnitude upon each sign addition: when composing two influences with the same sign, the occurrences of $\min\{q, s\}$ in Fig. 5 should be replaced by $\max\{0, \min\{q, s\} - 1\}$ to obtain a true upper-bound, assuming that $\epsilon \leq 0.5$. Under this same assumption, when adding a positive and a negative influence, we find true lower-bounds by replacing in Fig. 5 each p and r in a) and b) by $p + 1$ and $r + 1$, respectively. If the actual value of ϵ is known, the additional operation consists of performing the discussed correction only when necessary, that is, if a sign is composed (a multiple of) $1/\epsilon$ times. For this option, each sign needs to record how often it is summed during sign-propagation.

The adaption of parallel composition for non-infinitesimal ϵ leads to weaker, but at least correct, results. In correcting the upper- and lower-bounds of the strength factor, we have assumed that $\epsilon \leq 0.5$. This assumption seems reasonable, as each probability distribution has at most one probability larger than 0.5, and differences between probabilities are therefore likely to be less than 0.5.

4.3 Applying the Inference Algorithm

The properties of transitivity and parallel composition of influences can, as argued, be applied in a kappa-enhanced network. The property of symmetry holds for qualitative influences with respect to their sign, but not with respect to their strength. For an influence against the direction of an arc, we must therefore either use the default interval $[\infty, 0]$, or an explicitly specified strength factor.

Using the new $\otimes$- and $\oplus$-operators, the sign-propagation algorithm for regular qualitative probabilistic networks can now be applied to kappa-enhanced networks. Node-signs are again initialised to '0'; observations are once again entered as a '+' or '$-$'. The strength factor associated with an observation is either a *dummy* interval $[-1, 0]$ (so as to cause no loss of information upon the first operations), or an actual interval of kappa values to capture the strength of the observation. We illustrate the application of the algorithm.

Example 2. Consider the network from Fig. 6, with strength factors provided by domain experts. We again observe that a patient has taken antibiotics and enter this observation as '$+[-1,0]$' for node A. Node A propagates this 'sign' to T, which receives '$+[-1,0] \otimes - [1,0] = -[1,0]$' and sends this to D. Node D in turn receives '$-[1,0] \otimes + [2,0] = -[4,0]$' and does not pass on any sign. Node A also sends its sign to F, which receives '$+[-1,0] \otimes + [4,3] = +[4,3]$' and passes this on to D. Node D receives the additional sign '$+[4,3] \otimes + [5,3] = +[10,6]$'. The net influence of node A on D therefore is '$-[4,0] \oplus + [10,6]$' which equals '$-[4,0]$' if ϵ is infinitesimal, and '$-[5,0]$' otherwise. Taking antibiotics thus decreases the chance of suffering from diarrhoea. Note that we are now able to resolve the represented trade-off. □

Inference in a kappa-enhanced network may become less efficient than in a regular qualitative network, because strength factors change more often than signs. In theory, a strength factor could change upon each sign-addition enforcing propagation to take time polynomial in the number of *chains* to a single node in the digraph. Kappa-enhanced networks, however, allow for resolving trade-offs which qualitative networks do not. A polynomial bound on inference in kappa-enhanced networks can be ensured by limiting the number of sign-additions performed and reverting to the use of default intervals once this limit is reached. The use of default intervals may again lead to weaker results, but never to incorrect ones. Another option is to isolate the area in which trade-offs occur, use kappa-enhanced inference in that area and regular qualitative inference in the remaining network [10].

5 Conclusions and Further Research

A drawback of qualitative probabilistic networks is their coarse level of detail. Although sufficient for some problem domains, this coarseness may lead to unresolved trade-offs during inference in other domains. In this paper, we combined and extended qualitative probabilistic networks and kappa values. We introduced the use of kappa values to provide for levels of strength within the qualitative probabilistic network framework, thereby allowing for trade-off resolution. The kappa-enhanced networks are very suitable for domains in which all differences in probability are close to zero. Previous research has shown that Kappa networks can give good results even for non-infinitesimal ϵ [5,11]. For our purpose, however, we feel that the little information we are depending on better be reli-

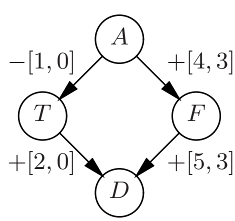

Fig. 6. The kappa-enhanced *Antibiotics* network.

able. If ϵ is not infinitesimal, a minor adaption upon sign-addition ensures that inference still leads to correct, though possibly weaker, results.

This paper presents a possible way of combining qualitative probabilistic networks with elements from the kappa calculus. Other combinations may of course be possible. We adapted the basic sign-propagation algorithm for regular qualitative probabilistic networks, with new operators for propagating signs and strength factors in kappa-enhanced networks; the resulting algorithm may, however, become less efficient. We already mentioned two possible solutions to this problem. Another possibility may be to exploit more elements from the kappa calculus: although NP-hard in general, under certain conditions, reasoning with kappa values can be tractable [12]; further research should indicate if strength factors may and can be propagated more efficiently using combination rules from the kappa calculus.

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