The Practicability of Situational Signs for QPNs

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Abstract

Qualitative probabilistic networks summarise the probabilistic influences between their variables by means of signs. The non-monotonic influences in a network have associated an ambiguous sign, which tends to lead to ambiguities upon inference. Such an ambiguous sign can be supplemented with a situational sign that summarises the influence in the current state of the network. Using these situational signs can forestall ambiguities upon inference. In this paper, we study the practicability of situational signs in a real-life qualitative network in oncology.

Keywords: qualitative probabilistic inference, ambiguity, real-life application

1 Introduction

Qualitative probabilistic networks (QPNs) were introduced in the early 1990s for probabilistic reasoning in a qualitative way [5]. A qualitative network is a graphical model of the probabilistic influences among a set of statistical variables. It encodes the variables from a domain of application, and the relationships between them, in a directed acyclic graph. An arc $A \rightarrow B$ between two variables A and B in this graph expresses that observing a value for A occasions a shift in the probability distribution for B. The direction of this shift is indicated by a qualitative sign. Inference with the network is performed by propagating and combining these signs [3]. It results, for each variable, in an indication of the direction of the shift in probability distribution occasioned by the available observations.

Qualitative networks capture the influences between the variables of their domain of application at a relatively coarse level of representation detail. One of the consequences is that they do not model in an informative way probabilistic influences that are positive in one state and negative in another state of the network. Such non-monotonic influences are associated with the ambiguous sign '?'. The presence of ambiguous signs in a network typically leads to uninformative results upon inference.

The above observation led to the development of the concept of situational sign [1]. The ambiguous sign of a non-monotonic influence is supplemented with an informative sign that summarises the influence in the current state of the network. Upon inference the situational sign of the influence is used, rather than the original ambiguous sign. As long as the situational sign remains informative, that is, '$+$', '$-$' or '0', it serves to forestall the ambiguities that would arise from using the original sign of the influence upon inference.

So far, the use of situational signs was investigated in a small, artificially constructed, network only. To study the practicability of situational signs, we investigate in this paper the difference in performance between a real-life qualitative network with ambiguous signs for its non-monotonic influences and the same network in which these ambiguous signs are supplemented with situational signs. The two networks under study provide for the staging of oesophageal cancer. We compare their performance using the medical records of 156 real patients diagnosed with cancer of the oesophagus. We find that the use of a situational sign decreases the percentage of ambiguous signs that are propagated from a specific part of the network to one of the key diagnostic variables from 45% to 12%.

The remainder of the paper is organised as follows. Section 2 reviews qualitative probabilistic networks. Section 3 introduces the qualitative oesophageal cancer network. In Section 4, the effect of the introduction of a situational sign into this network is examined. The paper ends with our concluding observations in Section 5.

2 Preliminaries

Qualitative probabilistic networks are generally looked upon as qualitative abstractions of Bayesian networks. Before reviewing qualitative networks, therefore, we briefly address their quantitative counterparts. In the sequel, we use upper-case letters to denote (sets of) variables. We assume each variable A to be binary, taking one of the values a_1 and a_2; we further assume that these values are ordered, where $a_1 > a_2$. We write a to denote $A = a_1$, and $\bar{a}$ to denote $A = a_2$.

2.1 Bayesian Networks

A Bayesian network is a model of a joint probability distribution Pr on a set of statistical variables. The variables are encoded as nodes in a directed acyclic graph and the probabilistic relationships between them are captured by arcs. For each variable A, moreover, a set of conditional probability distributions $\Pr(A \mid \pi(A))$ is specified, where $\pi(A)$ is the set of parents of A in the digraph.

We introduce a small Bayesian network for our running example.

Example 1 The Bayesian network from Figure 1 represents a fragment of fictitious knowledge about the effects of the smell of food and of influenza on appetite. Node S models whether or not one smells spicy food, node I captures whether or not one has influenza, and node A models whether or not one has an appetite. All three variables can take one of the values *true* and *false*, where *true* > *false*. □

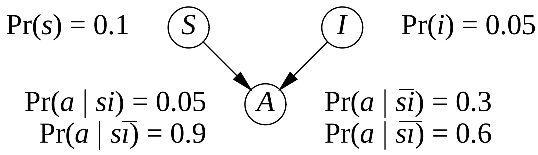

Figure 1: An example Bayesian network, modelling the influences of smelling spicy food (S) and of influenza (I) on appetite (A).

A Bayesian network defines a unique joint probability distribution on its variables. In its initial state, the network captures the prior distribution. As observations are entered, it converts to a new state, that represents the posterior distribution given the observations.

2.2 QPNs

A qualitative probabilistic network models qualitative features of a joint probability distribution on a set of statistical variables. Like a Bayesian network, it comprises a directed acyclic graph. Instead of conditional probability distributions, however, a qualitative probabilistic network associates with its digraph qualitative influences and qualitative synergies [5].

A *qualitative influence* between two variables expresses how the values of the one variable influence the probabilities of the values of the other variable. For example, a *positive qualitative influence* of the variable A on the variable B, denoted $S^+(A,B)$, expresses that observing the higher value for A makes the higher value for B more likely, regardless of any other direct influences on B, that is,

$$\Pr(b \mid ax) - \Pr(b \mid \bar{a}x) \geq 0$$

for any combination of values x for the set $\pi(B) \setminus \{A\}$ of parents of B other than A. A negative influence, denoted S^-, and a zero influence, denoted S^0, are defined analogously. If the sign of the difference $\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)$ varies for the different value combinations x, then the influence is non-monotonic. A non-monotonic or an unknown influence of A on B is denoted by $S^?(A,B)$.

The set of all influences of a qualitative network exhibits various convenient properties [5]. The property of *symmetry* states that, if the network includes the influence $S^\delta(A,B)$, then it also includes $S^{\delta}(B,A)$, $\delta \in \{+,-,0,?\}$. The *transitivity* property asserts that the qualitative influences along a trail that specifies at most one incoming arc for each variable, combine into a net influence whose sign is captured by the $\otimes$-operator from Table 1. The property of *composition* asserts that multiple influences between variables along parallel trails combine into a net influence whose sign is captured by the $\oplus$-operator.

Table 1: The $\otimes$- and $\oplus$-operators.

$\otimes$	$+$	$-$	0	?
$+$	$+$	$-$	0	?
$-$	$-$	$+$	0	?
0	0	0	0	0
?	?	?	0	?

$\oplus$	$+$	$-$	0	?
$+$	$+$	?	$+$	?
$-$	?	$-$	$-$	?
0	$+$	$-$	0	?
?	?	?	?	?

In addition to influences, a qualitative probabilistic network includes *additive synergies*. An additive synergy expresses how two variables interact in their influence on a third variable. For example, a *positive additive synergy* of the variables A and B on their common child C, denoted $Y^{+}(\{A,B\},C)$, expresses that A and B serve to strengthen each other's influence on C, regardless of any other direct influences on C, that is,

$$\Pr(c|abx) + \Pr(c|\bar{a}\bar{b}x) \geq \Pr(c|a\bar{b}x) + \Pr(c|\bar{a}bx)$$

for any combination of values x for the set $\pi(C) \setminus \{A,B\}$ of parents of C other than A and B. A negative additive synergy, denoted Y^{-}, and a zero additive synergy, denoted Y^{0}, are defined analogously. A non-monotonic or an unknown additive synergy of A and B on C is denoted by $Y^{?}(\{A,B\},C)$.

Example 2 We consider the qualitative abstraction of the Bayesian network from Figure 1. We have that $\Pr(a \mid si) - \Pr(a \mid s\bar{\imath}) \leq 0$ and $\Pr(a \mid \bar{s}i) - \Pr(a \mid \bar{s}\bar{\imath}) \leq 0$, and therefore that $S^{-}(I,A)$: influenza decreases the probability of having an appetite, regardless of the smell of food. We further have that $\Pr(a \mid si) - \Pr(a \mid \bar{s}i) < 0$ and $\Pr(a \mid s\bar{\imath}) - \Pr(a \mid \bar{s}\bar{\imath}) > 0$, and therefore that $S^{?}(S,A)$: the effect of the smell of spicy food on having an appetite depends on whether or not one is suffering from influenza. From $\Pr(a \mid si) + \Pr(a \mid \bar{s}\bar{\imath}) \leq \Pr(a \mid s\bar{\imath}) + \Pr(a \mid \bar{s}i)$, to conclude, we find that $Y^{-}(\{S,I\},A)$. The resulting qualitative network is shown in Figure 2; the signs of the influences are shown along the digraph's arcs, and the sign of the additive synergy is indicated over the curve over node A. $\square$

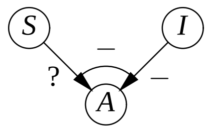

Figure 2: The qualitative abstraction of the Bayesian network from Figure 1.

Note that, although in Example 2 the signs of the qualitative relationships are computed from the conditional probabilities of the corresponding quantitative network, in real-life applications these signs are elicited directly from experts. Experience shows that qualitative signs are more easily given by experts than numerical probabilities [3].

Inference with a qualitative probabilistic network amounts to determining, for each variable V, a *node sign* 'sign[V]' [3]. This node sign indicates the direction of the shift in the probability of $V = v_1$ that is occasioned by an observation entered into the network. The observation a_1 for a variable A results in a '$+$' for the node sign of A and the observation a_2 results in a '$-$'. A variable with a change in node sign can occasion a change in the node signs of its neighbours, provided that these neighbours are dependent on the observed variable. The sign of the influence that the variable exerts, equals the sign-product of its own node sign and the sign of the arc with its neighbour. Influences on a variable from different neighbours combine with the $\oplus$-operator into an overall influence. The joint effect of multiple observations is computed as the sign-sum of the influences of each of the observations separately [2].

2.3 QPNs with Situational Signs

Qualitative probabilistic networks capture the influences between their variables at a relatively coarse level of representation detail. One of the consequences is that they model only monotonic influences in an informative way. We recall that a qualitative influence of a variable A on a variable B is monotonic if the difference $\Pr(b \mid ax) - \Pr(b \mid$

$\bar{a}x$) has the same sign for *all* combinations of values x for the set $\pi(B) \setminus \{A\}$ of parents of B other than A. The sign of the influence then is valid for any distribution $\Pr(X)$ on X. If the difference $\Pr(b \mid ax) - \Pr(b \mid \bar{a}x)$ yields contradictory signs for different combinations of values x, however, the influence of A on B is non-monotonic and is associated with the uninformative ambiguous sign '?'. The presence of such ambiguous signs is likely to give rise to ambiguous inference results throughout the network. Yet, in each specific state of the network, associated with a particular probability distribution $\Pr(X)$, the influence of A on B is unambiguous, that is, it is either positive, negative or zero.

The above observation led to the introduction of *situational signs* in qualitative networks [1]. The ambiguous sign of a non-monotonic influence is supplemented with a situational sign that expresses the current sign of the influence, associated with the current state of the network. For example, a *positive situational sign* for the non-monotonic influence of the variable A on the variable B indicates that

$$[\Pr(b \mid a) - \Pr(b \mid \bar{a})]_{\Pr(X)} \geq 0$$

where $[\Pr(b \mid a) - \Pr(b \mid \bar{a})]_{\Pr(X)}$ denotes the difference between $\Pr(b \mid a)$ and $\Pr(b \mid \bar{a})$ in the state of the network associated with $\Pr(X)$. Negative, zero and unknown situational signs have analogous meanings. An influence with a situational sign 'δ' is called a *situational influence*; the sign of the situational influence is denoted '$?(\delta)$'. A qualitative network with situational signs is termed a *situational network*.

Example 3 We consider once again the example Bayesian network from Figure 1 and its qualitative abstraction shown in Figure 2. In the prior state of the network we have that $\Pr(a \mid s) - \Pr(a \mid \bar{s}) \approx 0.27$ from which we conclude that the situational sign of the influence of S on A is positive, that is, $S^{?(+)}(S,A)$. The situational qualitative network for the prior state is shown in Figure 3. □

Once again we note that although in the previous example the prior situational sign was computed from the corresponding quantitative network, in real-life applications it would be elicited directly from a domain expert. In the remainder of the paper, we assume that an expert specifies situational signs for just the prior state of the network.

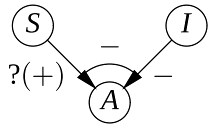

Figure 3: The network from Figure 2, now with the prior situational influence of S on A.

Inference with a situational network in essence is the same as inference with a regular qualitative network. The main difference is that, with a situational network, the situational signs of non-monotonic influences are used rather than the original ambiguous signs. Moreover, while the signs of regular qualitative influences have general validity, situational signs are dynamic in nature and pertain to a specific state of the network. After an observation has been entered and the state of the network has changed, therefore, the situational signs need to be updated. For the sign '$?(\delta)$' along an arc between a variable A and a variable B as in Figure 4, for example, after a change in the probability distribution of C, the situational sign 'δ' is updated to

$$\delta \oplus (\mathrm{sign}[C] \otimes \delta_1)$$

We note that the updating of the situational signs is incorporated into the inference algorithm and does not require any re-assessment by the expert.

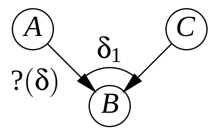

Figure 4: A situational network with $S^{?(\delta)}(S,A)$ and $Y^{\delta_1}(\{S,I\},A)$.

Example 4 We consider Figure 3, showing the situational qualitative abstraction of the Bayesian network from Figure 1, and assume that it is a fragment of a larger network. If the sign of I changes to '$-$', then the situational sign is updated to $+ \oplus (- \otimes -) = +$. The situational sign then retains its validity. From the quantified network we observe indeed that if $\Pr(i)$ decreases, then $\Pr(a \mid s) - \Pr(a \mid \bar{s})$ increases and remains positive. If the sign of I becomes '$+$', however, then the situational sign is updated to $+ \oplus (+ \otimes -) = ?$. From the quantified network, we note that an increase of $\Pr(i)$ results in a decrease of $\Pr(a \mid s) - \Pr(a \mid \bar{s})$; for $\Pr(i) \geq 0.55$, in fact, the difference becomes negative. The qualitative network, however, does not provide for determining the sign of the difference and the situational sign loses its informativeness. □

3 The Oesophageal Cancer Network

To investigate the practicability of situational signs, we study the effects of their introduction into a real-life qualitative network in the field of oesophageal cancer. In this section, we provide some background information on this network.

A chronic lesion of the inner wall of the oesophagus may develop into a malignant tumour. The tumour invades the oesophageal wall and upon further growth may invade adjacent organs. In time, the tumour may give rise to metastases in lymph nodes and to haematogenous metastases in the lungs and the liver. The depth of invasion and extent of metastasis determine the stage of the cancer. To establish these factors in a patient, various diagnostic tests are performed.

The state-of-the-art knowledge about oesophageal cancer has been captured in a quantified Bayesian network with the help of two gastro-enterologists from the Netherlands Cancer Institute, Antoni van Leeuwenhoekhuis [4]. The network currently includes 42 statistical variables for which the experts assessed some thousand conditional probabilities. Its main diagnostic variable is the variable *Stage*, classifying a patient's cancer in one of six possible stages of disease. The leaves of the network represent the diagnostic tests.

We abstracted the quantified oesophageal cancer network to a qualitative probabilistic network. To this end, we first summarised all variables into binary variables, building upon our knowledge of the domain. The original six-valued variable *Stage*, for example, was translated into the binary variable *Stage* with the values *early* and *late*. We then defined an ordering on the values of the resulting binary variables. For example, we assumed *late* $>$ *early*. To conclude, we computed the signs for the influences and the additive synergies in the qualitative network from the quantified network. We deleted the arcs that had associated zero or negligible influences.

Figure 5 shows the binary quantitative oesophageal cancer network as well as its qualitative abstraction. For each variable, its name, its values, and its prior probability distribution are shown; with each arc, the sign of the associated qualitative influence is indicated. For readability, the figure shows only the additive synergy that involves a non-monotonic influence.

The qualitative oesophageal cancer network includes a single non-monotonic influence, between the variables *Lymph-metas* and *Metas-cervix*. The variable *Lymph-metas* models whether or not distant lymphatic metastases of the primary tumour are present. The variable *Metas-cervix* models whether or not the lymph nodes in the neck are affected by the cancer. The sign of the influence between the two variables depends on the value of the variable *Location*. This variable models whether the primary tumour resides in the upper one-third of the oesophagus, or in the lower two-thirds. The lymph nodes in the neck are considered local for a primary tumour in the upper one-third of the oesophagus, and distant otherwise. For a primary tumour located in the upper one-third of the oesophagus, the presence of metastases in distant lymph nodes has a negative effect on the probability that there are metastases in the neck, and vice versa. If, however, the primary tumour is located in the lower two-thirds of the oesophagus, the presence of distant lymphatic metastases has a positive effect on the probability of metastases in the neck, and vice versa. In the initial state of the network, the probability that the tumour is located in the lower two-thirds of the oesophagus is quite high, and the situational sign for the non-monotonic influence accordingly is '$+$'.

The non-monotonic influence resides at a pivotal location in the network. For establishing the stage of a patient's oesophageal cancer, the presence or absence of distant lymphatic metastases is of primary importance. The presence or absence of metastases in the neck and their classification as local or distant, therefore, play an important role in the staging. The non-monotonic influence now forms the bridge between the part of the network

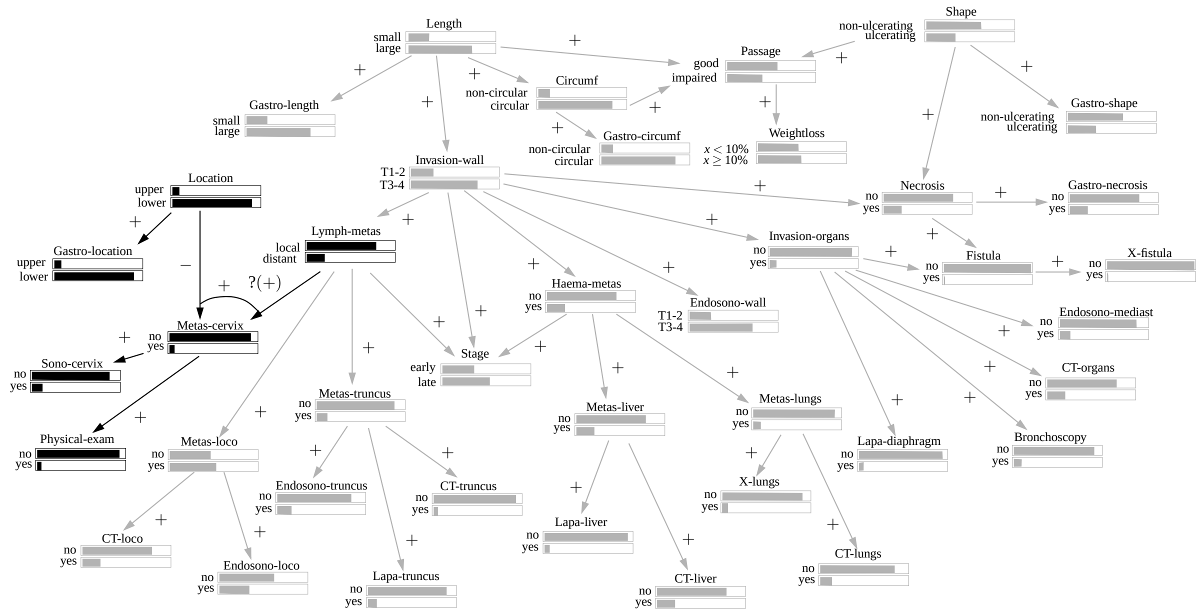

Figure 5: The combined binary and qualitative oesophageal cancer networks.

pertaining to metastases in the neck and the variable *Lymph-metas*.

The variables *Physical-exam* and *Sono-cervix* model the diagnostic tests that are generally performed to establish the presence or absence of lymphatic metastases in the neck; they represent the findings from a physical examination of the neck and from a sonography of the neck, respectively. The location of the primary tumour is established through a gastroscopic examination of the oesophagus; the result is captured by the variable *Gastro-location*. The variables *Physical-exam* and *Sono-cervix* upon observation influence the node sign of *Lymph-metas*. An observation for the variable *Gastro-location* does not influence the node sign of *Lymph-metas*, because in the prior state of the network *Gastro-location* is independent of *Lymph-metas*. All three variables upon observation influence the node sign of *Location*. We observe that the node sign of the latter variable is instrumental in the updating of the situational sign of the non-monotonic influence between *Metas-cervix* and *Lymph-metas* after observations have caused the network's state to change.

4 An Experimental Study

To gain insight into the practicability of situational signs, we study the performance of the qualitative oesophageal cancer network, before and after the introduction of the situational sign. In doing so, we focus on the part of the network that serves for interpreting the findings with regard to metastases in the neck. We investigate whether useful information from this part of the network is propagated towards the variable *Lymph-metas* upon inference. The part of the network under study is indicated in black in Figure 5. In our study, we use the data of 156 real patients diagnosed with oesophageal cancer. In Section 4.1 we demonstrate, as an example, the effect of introducing the situational sign for a single patient. We summarise the effects for all patients from our data collection in Section 4.2.

4.1 An Example Patient

For patient 90-1042, a physical examination did not reveal enlarged lymph nodes in the neck; a gastroscopic examination showed a primary tumour in the lower two-thirds of the oesophagus. These observations are entered into the network as a '$-$' for the node sign of *Physical-exam* and a '$+$' for the node sign of *Gastro-location*, respectively. In the qualitative network without the situational sign, the variable *Lymph-metas* receives a $- \otimes + \otimes ? = ?$ from *Pysical-exam*. Since the observation of *Gastro-location* has no influence on the node sign of *Lymph-metas*, we find an overall influence of '?' on *Lymph-metas*.

In the situational qualitative network, the situational sign of the non-monotonic influence is used upon inference. Because the available observations change the node sign of the variable *Location*, however, the situational sign needs to be updated before it can be used. The node sign of *Location* captures the combined effect of the two observations: since both observations have a positive effect on *Location*, its node sign is '$+$'. The additive synergy of *Location* and *Lymph-metas* on *Metas-cervix* also is '$+$'. Updating the situational sign of the influence between *Metas-cervix* and *Lymph-metas* now gives $+ \oplus (+ \otimes +) = +$, that is, the situational sign retains its validity and hence its informativeness. The variable *Lymph-metas* now receives a $- \otimes + \otimes + = -$ from the part of the network that pertains to metastases in the neck.

Note that, if the node sign of *Location* would have changed to '$-$', then the situational sign would have been updated to '?'. The observation for the variable *Physical-exam* would then have exerted an ambiguous influence on *Lymph-metas*. A similar observation holds if the node sign of *Location* would have changed to '?'. Such a change would occur if the available observations would exert discordant influences on *Location*, like *Physical exam* = *yes* and *Gastro-location* = *lower*.

4.2 Results

The data collection that we had available for our study includes the medical records of 156 patients diagnosed with oesophageal cancer. For 4 of these patients we have that *Sono-cervix* = *yes* and *Physical-exam* = *yes* or that one of these observations is *yes* and the other one is unknown. In the sequel we call such observations consistently positive; negative consistency has an analogous meaning. For these 4 patients we further have that *Gastro-location* = *upper*. For 7 patients we have that the observations for *Sono-cervix* and *Physical-exam* are consistently positive, and *Gastro-location* = *lower*. For another 7 patients we have that the observations for *Sono-cervix* and *Physical-exam* are consistently negative and *Gastro-location* = *upper*. For 52 patients, we have that the observations for *Sono-cervix* and *Physical-exam* are consistently negative, and *Gastro-location* = *lower*. For one patient, contradictory results were found from the sonography and the physical examination. For the remaining 85 patients, no observation was available from a sonography of the neck nor from a physical examination. For 2 of these patients we have that *Gastro-location* = *lower* and for 83 of these patients we have that *Gastro-location* = *upper*. These statistics are summarised in Table 2.

Table 2: The availability of observations for the relevant variables for 156 patients.

Sono-cervix and	*Gastro-location*	
Physical-exam	*upper*	*lower*
cons. positive	4	7
cons. negative	7	52
↯	-	1
not observed	2	83

For the 85 (55%) patients for whom no observations are available for *Sono-cervix* and *Physical-exam*, the part of the network under consideration does not partake in establishing the node sign of *Lymph-metas*. The non-monotonic influence, therefore, is not used upon inference with these patients. For the remaining 71 (45%) patients, inference with the regular qualitative network results in an unknown influence on *Lymph-metas*.

We now address inference with the situational network. For the 85 patients without any observations for *Sono-cervix* and *Physical-exam*, the availability of the situational sign makes no difference. For the other 71 patients, the situational sign for the non-monotonic influence between the variables *Lymph-metas* and *Metas-cervix* is used, rather than the original '?'. For all these patients, however, the available observations result in a change of the node sign of *Location*, thereby enforcing the situational sign to be updated.

For 19 (12% of all patients) of the 71 patients for whom an observation is available for *Sono-cervix* or *Physical-exam*, the node sign of *Location* changes to a '$-$' or a '?'. As for these patients the situational sign is updated to '?', inference still results in an unknown effect on the variable *Lymph-metas*. For the other 52 (33% of all patients) of these 71 patients, however, the node sign of *Location* changes to a '+' and the situational sign remains a '+'. For these patients, inference yields a negative influence on *Lymph-metas* and, hence, an informative result. The results obtained with the regular and situational qualitative networks are summarised in Table 3.

Table 3: The signs propagated from the part of the network under consideration to the variable *Lymph-metas*.

	+	−	?	0
reg.	-	-	71 (45%)	85 (55%)
sit.	-	52 (33%)	19 (12%)	85 (55%)

For the 85 patients for whom no observations are available for *Sono-cervix* and *Physical-exam*, the node sign of *Location* changes as a result of the observation for *Gastro-location*. For these patients, therefore, the situational sign is also updated even though it is not used upon inference. For 2 of these patients, the situational sign changes to a '?', and for 83 of these patients the situational sign remains a '+'. We find that for a total of 135 (87%) of the patients the situational sign remains a '+', and thus retains its validity. This robustness of the situational sign is not coincidental. The situational sign depends on the prior probability of the tumour being located in the lower two-thirds of the oesophagus. This prior probability is rather high and in fact causes the prior situational sign to be positive. Because of this high prior probability, moreover, we are more likely to find observations that lead to a change of the node sign of *Location* to a '+', which serves to preserve the validity of the situational sign.

5 Conclusions

Recently, situational signs were proposed for qualitative probabilistic networks to provide for a more informative way of capturing non-monotonic influences. We investigated the practicability of these signs in a real-life qualitative network for the staging of oesophageal cancer. To this end, we compared the performance of the network before and after the introduction of a situational sign, using the data of 156 real patients. We found that, before the situational sign was introduced, for 45% of the patients ambiguous information was propagated from the part of the network under consideration. After the introduction of the situational sign this percentage was reduced to 12%. We conclude that the situational sign served to considerably increase the expressive power of the qualitative oesophageal cancer network. As this network is in no aspect exceptional, we expect similar results for other real-life qualitative networks in a variety of problem domains.

Aknowledgement

This research was supported by the Netherlands Organisation for Scientific Research (NWO).

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