Laboratory Exercises in Astronomy — Variable Stars in M15

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LIKE many other globular clusters, Messier 15 in Pegasus is a rich mine of variable stars. Of its more than 100 known variables, most are of the RR Lyrae type, characterized by periodic changes of brightness with cycles shorter than a day. As we shall verify here, the distance of a globular cluster can be determined quite simply if it contains RR Lyrae variables. This fact makes these stars powerful tools for studying the structure of our Milky Way system.

The abundance of variable stars in M15 was discovered about 70 years ago by Solon I. Bailey at Harvard Observatory. In 1919 he published a study of them based upon 75 photographs taken with Harvard's 13-inch refractor in Peru, the 24-inch reflector in Massachusetts, and the 60-inch Mount Wilson reflector in California. For 61 variables he found periods ranging from 0.30 to 1.44 days, the most common value being about 0.6 day. The average variation from maximum to minimum light was about 0.8 magnitude.

From his work on several globular clusters, Bailey found that the RR Lyrae stars of shortest period had nearly symmetrical light curves of small amplitude; those of longer period had large amplitudes and asymmetrical curves, rapid brightening being followed by gradual fading. But he also discovered that the mean magnitudes (average of the maximum and minimum of the same variable) were nearly the same for all RR Lyrae stars in any one cluster, irrespective of period length.

This conclusion led to an important generalization: All RR Lyrae stars have nearly the same mean intrinsic luminosity or absolute magnitude *(M)*. Hence, if *M* is known, the distance *(d)* of a globular cluster can be calculated from the mean apparent magnitude *(m)* of an RR Lyrae star observed in it, using the formula,

$$M = m + 5 - 5 \log d.$$

But before this method could be applied, it was necessary to establish the mean absolute magnitude of at least one such variable. Although the prototype, 7th-magnitude RR Lyrae itself, is the nearest of these "cluster-type variables," it is still too far away for its distance to be measured directly by the trigonometric parallax method.

However, an indirect method was applied by Harlow Shapley, while working at Mount Wilson Observatory from 1914 to 1921. In these pioneering studies, he carefully selected a small sample of Cepheid and RR Lyrae variables whose proper motions on the celestial sphere were known. By statistical means he derived an average absolute magnitude of the RR Lyrae stars on the assumption that they were related to the ordinary Cepheids.

Since all RR Lyrae stars were considered to have the same intrinsic brightness, as soon as the mean absolute magnitudes for a few were found, those of all the others became known immediately. Then, using the formula above, Shapley was able to derive the distance to every globular cluster in which RR Lyrae stars had been observed.

He then showed that the system of globular clusters was centered tens of thousands of light-years from the sun, and that our Milky Way galaxy was vastly larger than previously believed. Over the years many revisions of his distances have been made, but none of them has altered the basic picture of the sun's noncentral location in the vast Milky Way pinwheel.

In 1951, Walter Baade showed that the RR Lyrae stars were not as closely related to the Cepheids as had been supposed. In fact, the faintest classical Cepheids were found to be about a magnitude brighter than the RR Lyrae stars, and this meant that other indirect means would be needed to give a better calibration of the RR Lyrae variables.

The most satisfactory method now used involves matching the main sequence on Hertzsprung-Russell diagrams for clusters with a fundamental H-R diagram for nearby stars having accurately known distances. Hence, the absolute magnitudes of RR Lyrae stars can be read off the calibrated diagram for nearby stars after the two have been fitted together.

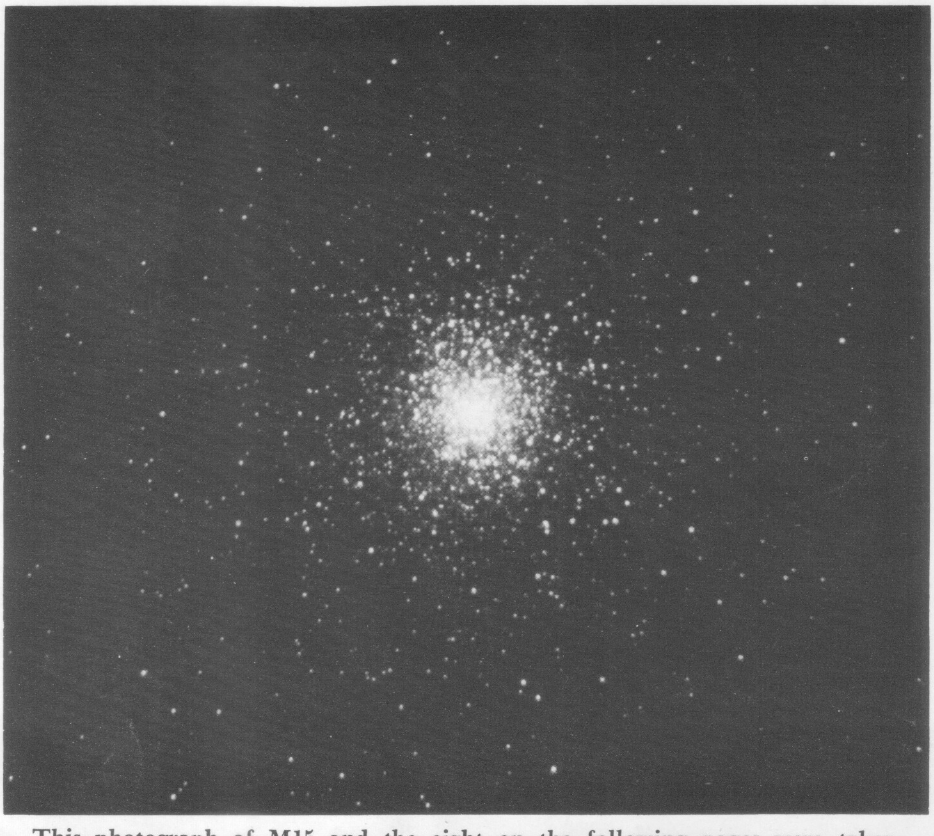
This procedure has yielded a mean absolute magnitude for RR Lyrae stars close to +0.5, which is the average of results from several clusters. It is quite possible that the intrinsic luminosities of these variables differ slightly from cluster to cluster.

MEASURING THE MAGNITUDES

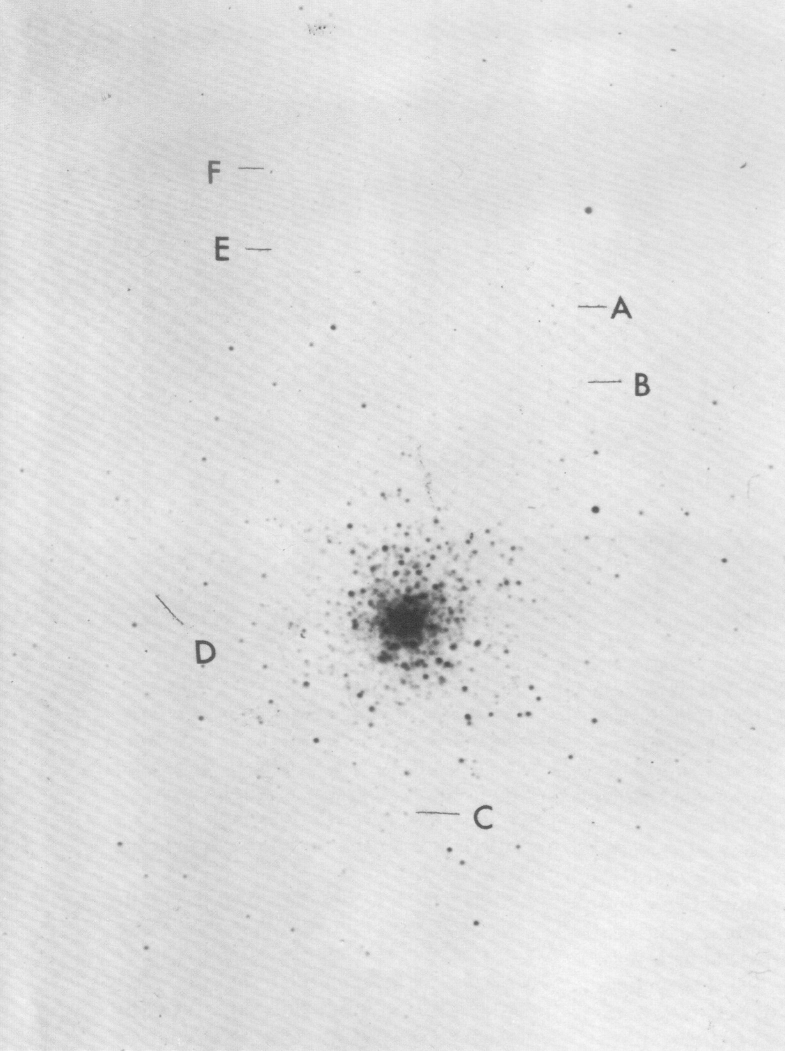
On the following two pages are eight photographs of Messier 15. They were taken with the 100-inch reflector at Mount Wilson Observatory on a single night in July, 1932, by John C. Duncan, who has permitted the writer to adapt his original exercise to this form.

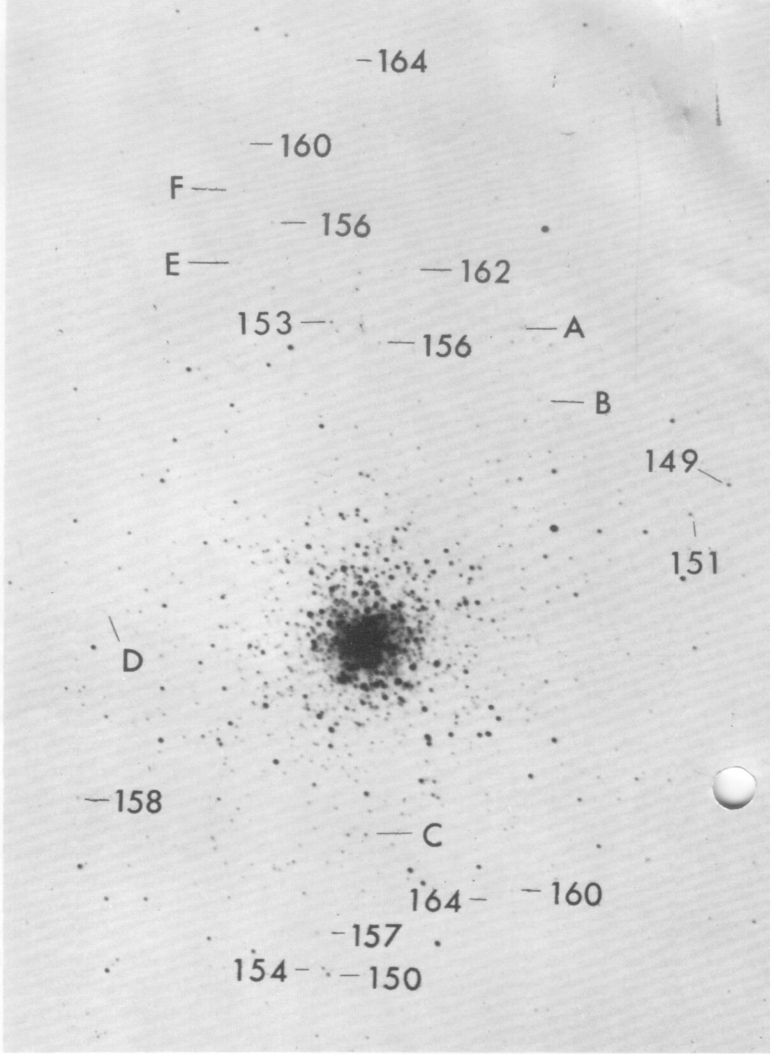
Letters on the photographs designate six of Bailey's RR Lyrae stars; his magnitudes for several nonvariable comparison stars are also marked on some pictures. By estimating magnitudes on these plates, light curves of the variables can be plotted and the distance to the cluster can be computed.

(Continued on fourth page)

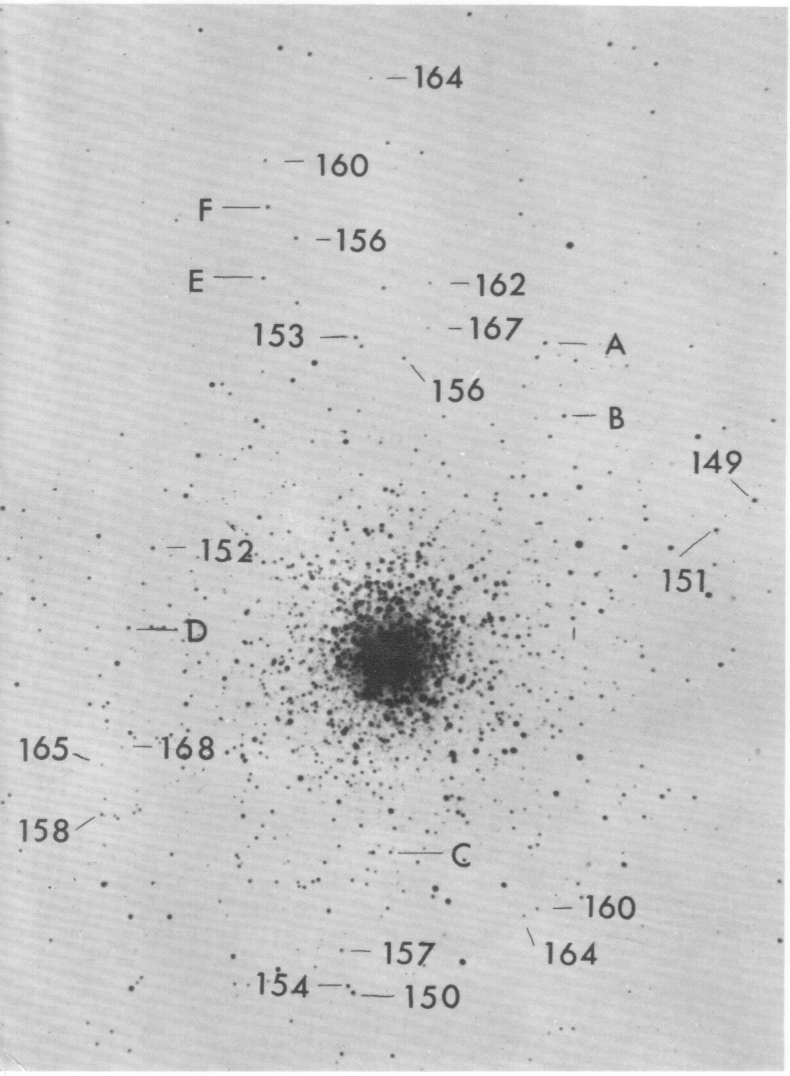


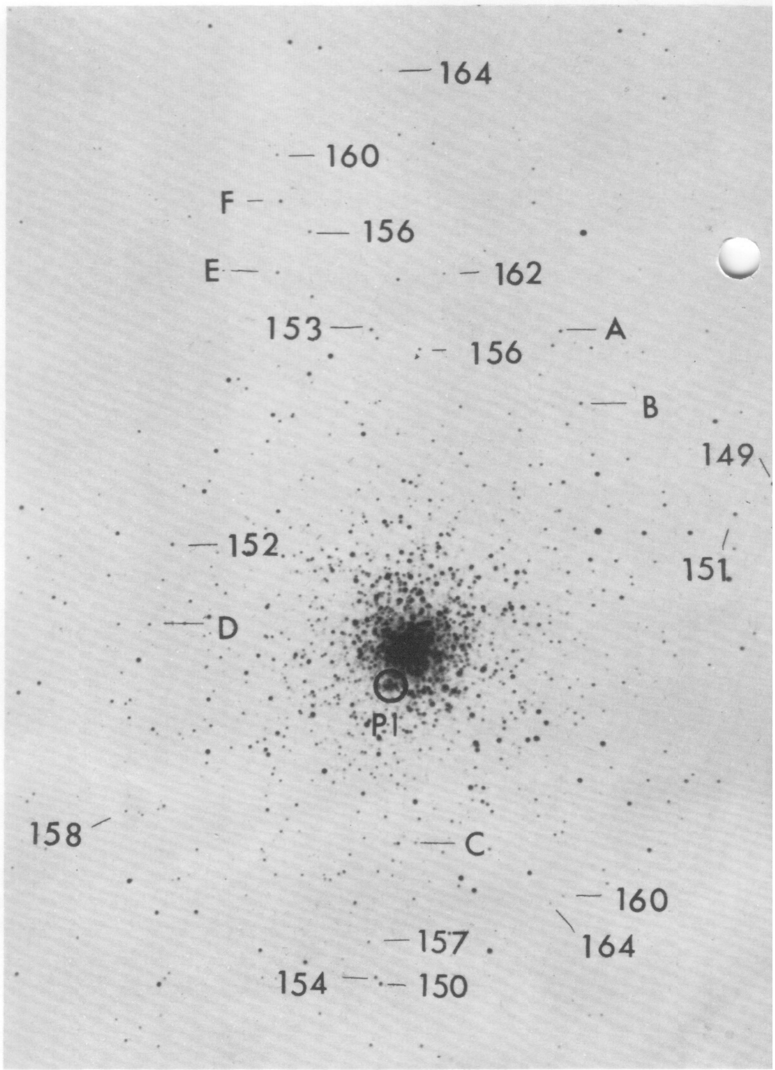
This photograph of M15 and the eight on the following pages were taken with the 100-inch Hooker reflector at Mount Wilson Observatory. The original plates were made available to the author by Whitin Observatory, Wellesley College. South is at the top, east to the left.

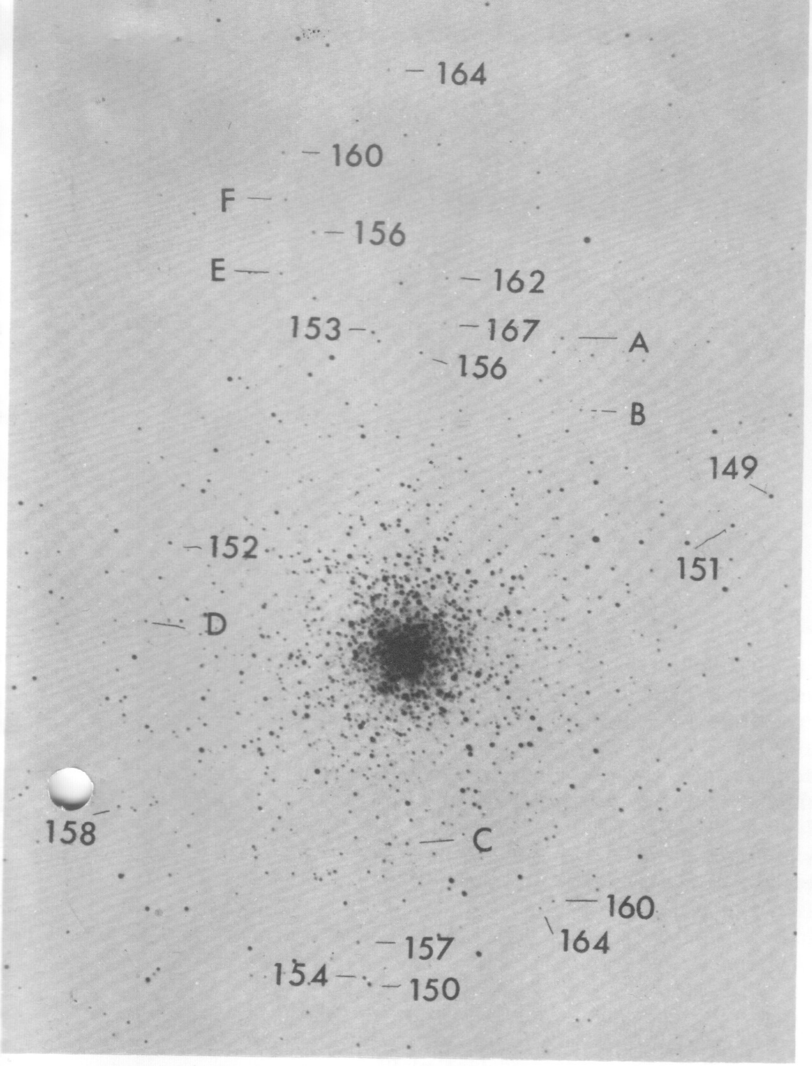


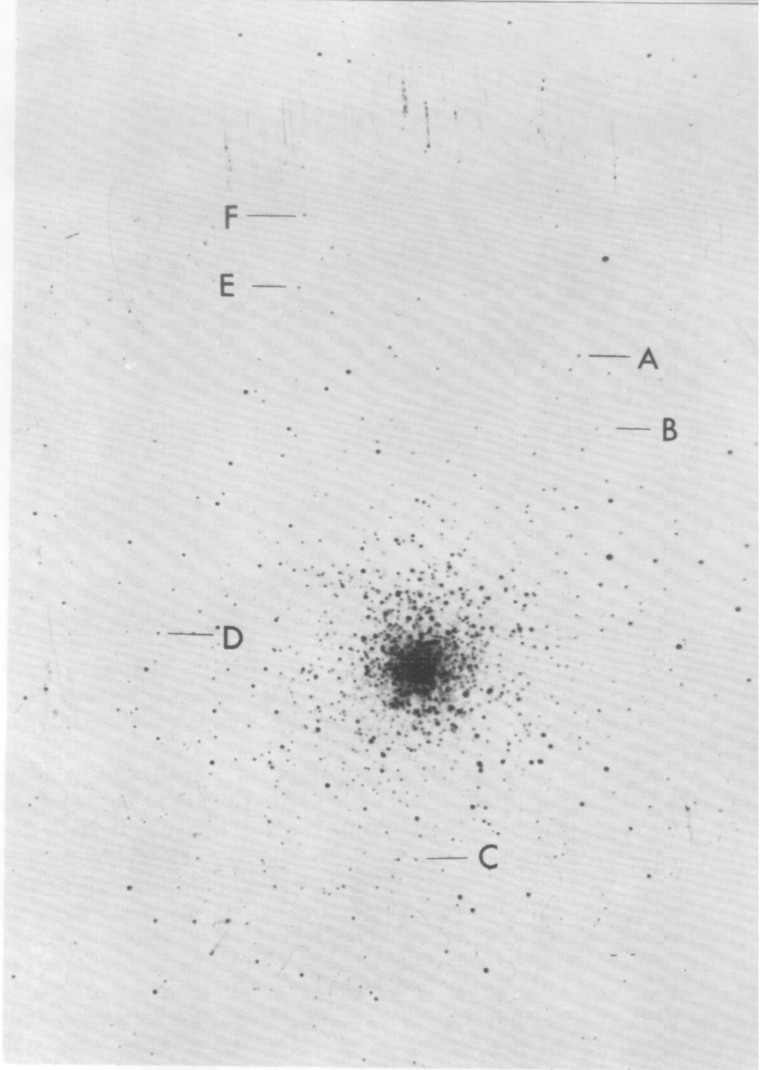


Reading across these two pages, the photographs are in order of time. Above: Duncan 465,

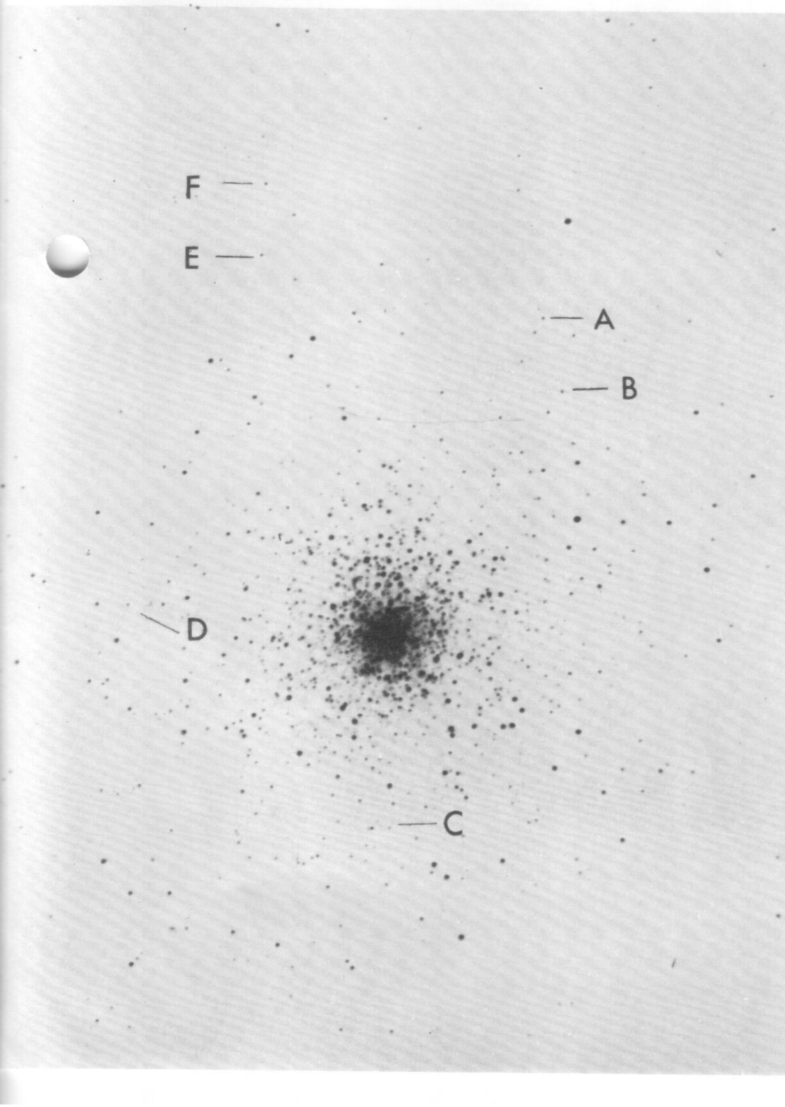


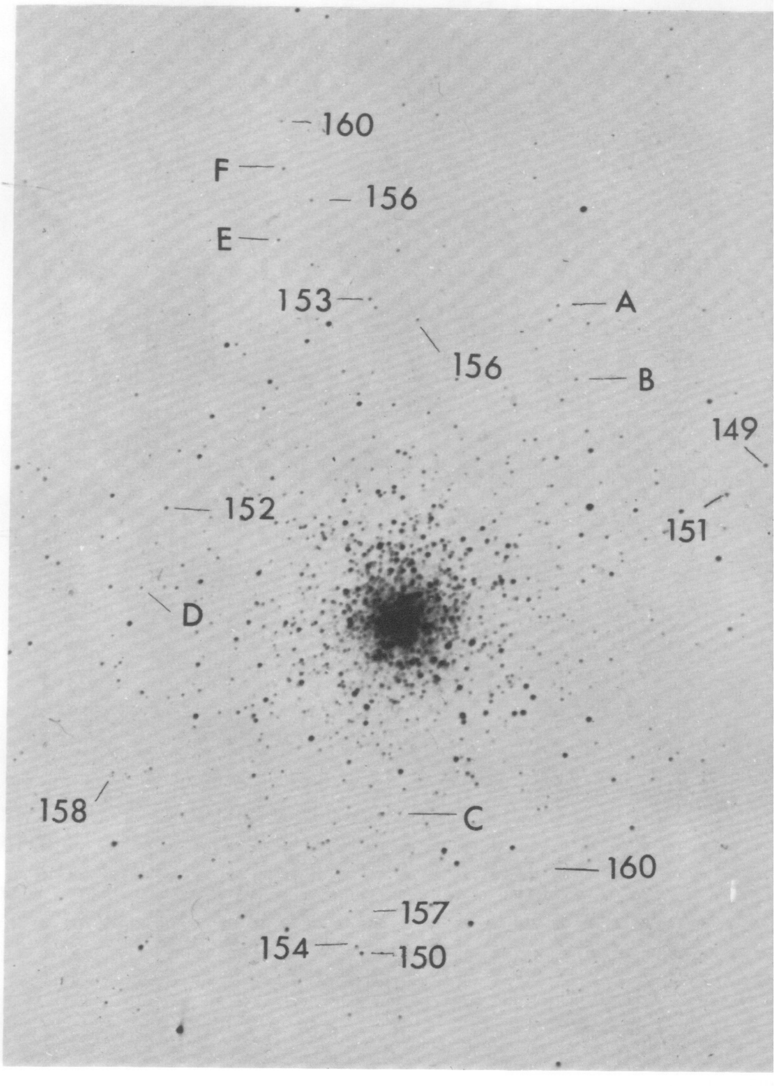


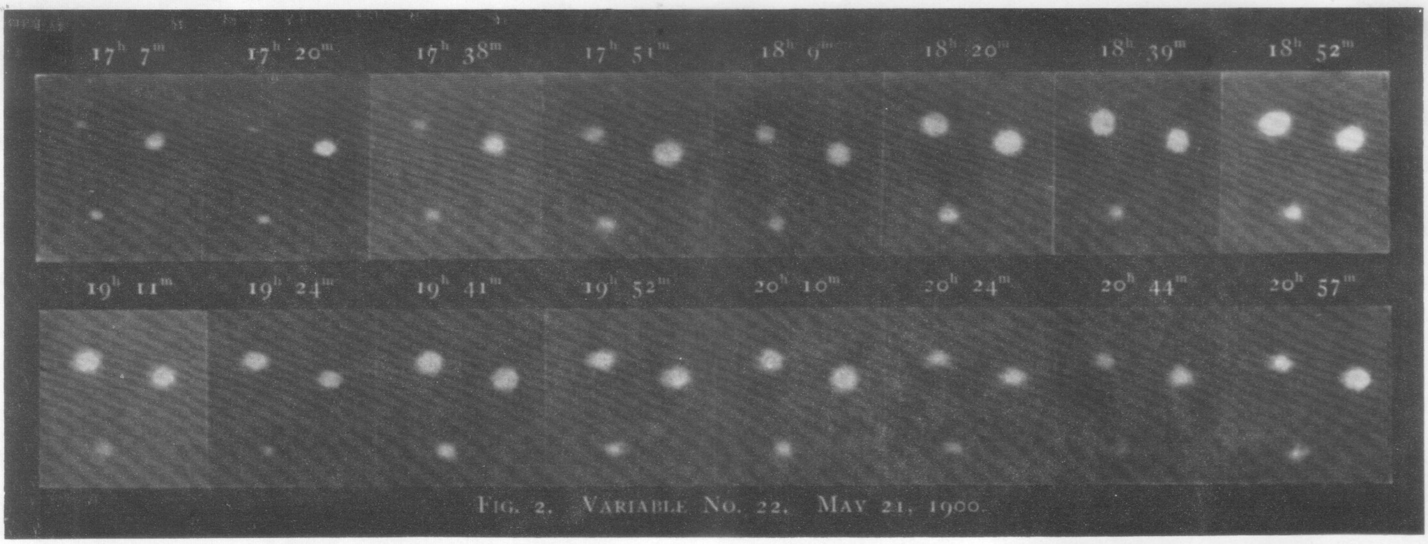




470, and 472; below: Duncan 475, 477, 479, and 481. The scale of all pictures is the same.







At upper left in each frame is a 0.48-day RR Lyrae star in the globular cluster M3. Note how it brightened rapidly and then faded in this four-hour series of 10-minute exposures made at Lick Observatory. The rapidity of its changes is typical of stars in M3, wrote Solon I. Bailey in Harvard *Annals, 78,* Part 1, from which this illustration is taken.

Following are the Universal times of the plates' mid-exposures, in fractions of a day after 0^h on July 30, 1932.

Plate No.	*Time*	*Plate No.*	*Time*
Δ465	0.174	Δ475	0.381
Δ468	0.228	Δ477	0.409
Δ470	0.279	Δ479	0.449
Δ472	0.332	Δ481	0.478

To understand how to estimate the brightness of one of the variables, look at star **C** on plate Δ468, directly below the nucleus of the cluster. Nearby are several comparison stars labeled with their magnitudes. Because decimal points are easily confused with star images, it is customary to omit them when marking plates. Thus, the magnitude of the star labeled 157 is 15.7. Note that the limiting magnitude of plate Δ468 is about 15.8; that is, stars 160 and 164 are invisible on this reproduction.

The variable **C** is slightly more conspicuous than star 154 and slightly less so than 150. Therefore, by interpolating **C** between these standards, its magnitude can be estimated as 15.3 or 15.2. Although a comparison star marked 153 is near the top of the field, it is poor practice to compare the variable with a star so far away. Although these reproductions have been enlarged from the negatives, a magnifier aids in making estimates.

Note that some of the variables may at times be near the limit of visibility or not seen at all. In such cases, use the standard stars to estimate the limiting magnitude of the plate and record the observation as fainter than that value. Do not take averages for these values.

Before starting to estimate the variables, draw up a table having eight lines and eight columns. The first column is for plate numbers, and the second for mid-exposure times. Head each of the remaining six columns with the letter of one variable; each line of the table will contain the magnitude estimates for the appropriate plate.

Now, determine each variable's magnitude by interpolating it between two standard stars (preferably near the variable), one slightly brighter and one slightly fainter. Enter the estimated magnitude in the table.

After all the variables on all the plates have been observed, prepare on graph paper the light curve of each star. Plot the fraction of the day along the x-axis and the magnitudes along the y-axis. Because the star is brightest when its magnitude is numerically least, plot the smaller magnitudes above the larger ones.

Next, draw a smooth curve to satisfy as well as possible the general run of your estimates. Because of observational scatter, the curve may not pass through every point. Most of the curves will be fragmentary, not spanning an entire cycle of light changes, since the plate material covers only a third of a day. This is less than the periods of most of the variables.

For any of the variables whose light curve is sufficiently complete, average the magnitudes at maximum and minimum to obtain an approximate value of m, the mean magnitude.

Even if the light curve is fragmentary, an approximate value can be obtained from each variable that is visible on all eight photographs, by taking the arithmetic mean of the eight magnitude estimates. (Can you explain why this procedure may lead to a biased result? Estimate how the incompleteness of the data affects the average magnitude.) Finally, form the average of all values of m.

Assuming that the mean absolute magnitude M of RR Lyrae stars is +0.5, compute the distance to M15, using the formula above. This gives the distance d in parsecs; multiply by 3.26 to convert to light-years.

We have assumed that the light from the cluster is not dimmed by interstellar material. Suppose that intervening cosmic dust does obscure the cluster. How would this affect your determination of the distance? Also, suppose that the magnitudes of the comparison stars are systematically too bright by half a magnitude. Calculate how this would affect the distance.

NOTE FOR TEACHERS. Messier 15 is the only globular cluster known to contain a planetary nebula, first reported by F. G. Pease in 1928. The position of this planetary has been circled on Δ477 and labeled **Pl.**

Since the Hooker 100-inch reflector has a focal length of 508 inches, and the M15 photographs have been enlarged 3.2 times, it is possible to determine the scale of the reproductions and the angular size of the cluster. Thus, from the distance determined in this exercise, the linear diameter of M15 can be computed.

Because the quality of the exposures improved through the night, students may have more success if they reserve the first three plates to be measured last. To shorten the exercise, some of the stars can be omitted, but **B** should be retained for it goes through a complete cycle.

The following table shows the correspondence between the letter designations in this exercise and Bailey's original numbers. The periods and amplitudes have been taken from his work.

Variable	*Bailey*	*Period*	*Amplitude*
A	5	$0^d.385$	$0^m.67$
B	4	0.314	0.77
C	8	0.646	0.92
D	11	0.344	0.79
E	14	0.382	0.56
F	15	0.584	0.94

Variables quite far from the center of the cluster have been chosen to avoid interference from nearby stars. A better idea of the distribution of variables in M15 can be obtained by examining the frontispiece of Bailey's original memoir, which is Harvard Observatory *Annals,* Vol. 78, Part 3, 1919.