INTAS PROPOSAL FOR
South Caucasian Republics 2006 - Research Project

1.1 TITLE:
Polynomial Mappings: Algebra, Computation and Topology

1.1.1 Keyword 1 : Algebra
Keyword 2 : Geometry, Algebraic Geometry
Keyword 3 : Mathematical Programming

1.1.2 Free word 1 : polynomial
Free word 2 : bifurcation diagram
Free word 3 : monodromy

1.1.3 Intended Start Date: January 2007

1.1.4 Duration: 24 Months

1.2 CONSORTIUM

Universiteit Utrecht - The Netherlands
A.Razmadze Mathematical Institute - Georgia
Kiev National Taras Shevchenko University - Ukraine
University of Kaiserslautern - Germany

1.3 SUMMARY

The aim of the project is to develop new methods of algorithmic investigation of the geometry and topology of polynomial mappings. Both local and global geometric behaviour of polynomial mappings will be studied. Special attention will be given to such topics as behaviour at infinity, the structure of bifurcation diagram, monodromy around a given stratum, mixed Hodge structures of fibres, algebraic computation of topological invariants, Cohen-Macaulay modules, Lie algebras of algebraic varieties and singularities, geometry of quadratic mappings and various versions of Keller Jacobian conjecture.
The research strategy for this project is to treat each topic by using simultaneously general algebraic methods of investigating the geometry of polynomial mappings and computer algorithms for calculating various invariants and algebraic objects associated with the mapping. The teams are chosen in such way that their expertise and backgrounds will make the cooperation most effective and permit to considerably extend the knowledge in the topics under consideration.
The project brings together two research teams from the fSU: Tbilisi Mathematical Institute, 5 members, team leader G.N.Khimshiashvili; Kiev University, 3 members, team leader Yu.Drozd; - and two research teams from INTAS countries: The Netherlands, Utrecht University, 5 members, team leader, the Coordinator of the project, D.Siersma; Germany, Kaiserslautern University, 6 members, team leader G.Pfister.

2 TEAM INFORMATION

2.1 Team : Utrecht University

2.1.1 Team Description

The team is based at the Department of Mathematics of the University of Utrecht. It contains also members from the Radbout University of Nijmegen. The team has expertise in:
- algebra, geometry and topology of singular spaces
- Jacobian conjecture

More in detail:
D. Siersma : polynomials at infinity; classification of singularities
J. Steenbrink : Mixed Hodge Theory
S. Anisov : geometry of 3-manifolds
A. van der Essen : Jacobian conjecture
M. Crainic : Poisson structures

The team has direct relations with the other participating institutes and complements their tasks. The infrastructure is the equipment of the Mathematical Research Institute (MRI) in the
Netherlands, which includes international Master Classes and profits from its international connections.

2.1.2 List of publications

1 D. Siersma, M. Tibar : Deformations of polynomials, boundary singularities and monodromy, Moscow Math. J. 3 (2003), no.3, 661-679
2 J. Steenbrink, M. Schulze : Computing Hodge-theoretic invariants of singularities. In: New Developments in Singularity Theory. Nato Science Series II. Kluwer Academic Publishers, p.217-234
3 A. van den Essen : Polynomial automorphisms and the Jacobian conjecture, Progress in Mathematics, vol.190, Birkhauser, Basel, 2000
4 M.Crainic, R.L.Fernandes : Integrability of Lie brackets, Annals of Mathematics 157, 2004, No.2, 575-620.
5 D. Siersma, G. Khimshiashvili : Remarks on minimal round functions, Banach Center Publications vol.62, 2004, 159-172

2.1.3 Team Leader and address

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2.1.4 List of Senior Scientists in the team

1) First Name Sergei
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3) First Name Joseph
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   Insitute Mathematisch Instituut Radbout Universiteit Nijmegen
4) First Name Arno
   Patronic Name
   Family Name van den Essen
   Year Of Birth 1951
   Insitute Mathematisch Instituut Radbout Universiteit Nijmegen

2.1.5 Statistics

Number of Team Members involved in this project: 5
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 3

2.2 Team : Tbilisi Inst. Math.

2.2.1 Team Description

The team members are employed in A.Razmadze Mathematical Institute of Georgian Academy of Sciences. Four team members have considerable experience of international cooperation and participation in INTAS projects in algebra and singularity theory. The team has stable contacts with the dutch team and german team.

The expertise of team members may be described as follows:

G.Khimshiashvili: algebraic formulae for topological invariants of polynomial varieties and mappings, computation of the mean topological invariants of random real polynomial mappings, topology of quadratic mappings, classification of quadratic Poisson structures on affine spaces, computation of derivation Lie algebras of isolated hypersurface singularities

T.Aliashvili: effective criteria of properness and stability for real polynomial mappings, computation of topological invariants of intersections of quadrics, computation of the mean topological invariants of random real polynomial mappings

G.Chavchanidze: topological invariants of algebraic Poisson structures, effective construction of conservation laws and non-Noetherian symmetries of polynomial vector fields

A.Elashvili: derivation Lie algebras of isolated hypersurface singularities, construction of finite-dimensional Frobenius Lie algebras and associated integrable models

M.Jibladze: computer algebra, combinatorics, fast algorithms for computing topological invariants of polynomial mappings

2.2.2 List of publications

1 G.Khimshiashvili, Signature formulae for topological invariants, Proc. A.Razmadze Math. Inst. 125, 2001, 1-121
2 T.Aliashvili, G.Khimshiashvili, The Euler characteristic of intersection of quadrics, (Russian) Uspekhi matem. nauk 61, 2006, No.3.
3 T.Aliashvili, G.Khimshiashvili, Topology of the fibres of stable quadratic mappings, (Russian) Doklady Ross. Akad. nauk 408, 2006, No.1.
4 A.Elashvili, G.Khimshiashvili, Lie algebras of simple hypersurface singularities, J. Lie Theory 16, 2006, No.4, 621-649.
5 A.Elashvili, M.Jibladze, and D.Pataraia, Combinatorics of necklaces and "Hermite reciprocity", J. Alg. Combin. 10, 1999, No.2, 173-188.

2.2.3 Team Leader and address

Title	Prof.
Position	Senior Scientist
Sex	Male
Date Of Birth	28/10/1951
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Patronic Name	Nikolaevich
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2.2.4 List of Senior Scientists in the team

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2) First Name Giorgi
   Patronic Name Zurabovich
   Family Name Chavchanidze
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   Insitute A.Razmadze Mathematical Institute
3) First Name Alexander
   Patronic Name Grigorievich
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   Year Of Birth 1942
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4) First Name Mamuka
   Patronic Name Aleksandrovich
   Family Name Jibladze

Year Of Birth	1958
Insitute	A.Razmadze Mathematical Institute

2.2.5 Statistics

Number of Team Members involved in this project: 5
Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 4

2.3 Team : Kiev University

2.3.1 Team Description

The team is based on the Department of Mechanics and Mathematics, Kiev Taras Shevchenko University. The team members are experts in the following fields related to the project:

Yuriy Drozd: general theory of Cohen-Macaulay modules, matrix problems and their applications to modules and vector bundles

Vitalii Bondarenko: Cohen-Macaulay modules over surface singularities and hypersurfaces, applications of matrix problems

Olena Drozd-Koroleva: matrix problems over artinian rings and non-algebraically closed fields, explicit calculations for matrix problems and Cohen-Macaulay modules

2.3.2 List of publications

1 Y.Drozd, Derived categories of modules and coherent sheaves, Preprint MPIM 2005-27, 2005.
2 Y.Drozd, Vector bundles and Cohen-Macaulay modules, In: Representations of Finite Dimensional Algebras and Related Topics in
Lie Theory and Geometry. Field Institute Communications 40, AMS, 2004, 189-222.
3 O.Drozd-Koroleva, Representations of generalized bunches of chains, In: Algebraic Structures and their Applications, Kiev, 2003, 224-237.
4 V.Bondarenko, Tame and Wild Cauhen-Maccauley modules, Master Thesis, University of Kaiserslautern, 2003.
5 Y.Drozd, G.-M.Greuel and I.Kashuba, On Cohen-Macaulay modules on surface singularities, Moscow Math. J. 3 (2003), 397-418.

2.3.3 Team Leader and address

Title	Prof.
Position	Academician
Sex	Male
Date Of Birth	15/10/1944
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Patronic Name	
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2.3.4 List of Senior Scientists in the team

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2) First Name Olena
   Patronic Name
   Family Name Drozd-Koroleva
   Year Of Birth 1981
   Insitute Kiev Taras Shevchenko University

2.3.5 Statistics

Number of Team Members involved in this project: 3
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 0

2.4 Team : Kaiserslautern Univ

2.4.1 Team Description

The team has expertise in algebraic geometry, singularity theory, computational methods of commutative algebra and implementation of effective algorithms for local and global investigation of polynomial varieties and mappings.

More precisely, the fields of expertise of team members look as follows.

G.Pfister: algebraic geometry, quadratic forms, computational algebra

A.Fruehbis-Krueger: singularity theory, computer algebra

G.-M.Greuel: commutative algebra, algebraic geometry, singularity theory, computer algebra

B.Martin: singularity theory, deformation theory, computer algebra

H.Schoenemann: singularity theory, computer algebra

O.Wienand: commutative algebra, computer algebra

2.4.2 List of publications

1 Greuel, Gert-Martin; Pfister, Gerhard. A Singular introduction to commutative algebra. With contributions by Olaf Bachmann, Christoph Lossen and Hans Schönemann. With 1 CD-ROM (Windows, Macintosh, and UNIX). Springer-Verlag, Berlin, 2002.

2 Greuel, Gert-Martin; Pfister, Gerhard. Computer algebra and finite groups. Mathematical software (Beijing, 2002), 4--14, World Sci. Publishing, River Edge, NJ, 2002.
3 Decker, Wolfram; de Jong, Theo; Greuel, Gert-Martin; Pfister, Gerhard. The normalization: a new algorithm, implementation and comparisons. Computational methods for representations of groups and algebras (Essen, 1997), 177--185, Progr. Math., 173, Birkhäuser, Basel, 1999.
4 B.Martin, Algorithmic computation of flattenings and of modular deformations. J. Symbolic Comput. 34 (2002), no. 3, 199--212.
5 B.Martin, T.Hirsch, Modular strata of deformation functors. Computational commutative and non-commutative algebraic geometry, 156--166, NATO Sci. Ser. III Comput. Syst. Sci., 196, IOS, Amsterdam, 2005.

2.4.3 Team Leader and address

Title	Prof.
Position	Group Leader
Sex	Male
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1) First Name Anne
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2) First Name Gert-Martin
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   Family Name Greuel
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3) First Name Bernd
   Patronic Name
   Family Name Martin
   Year Of Birth 1950
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4) First Name Hans
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5) First Name Oliver
   Patronic Name
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2.4.5 Statistics

Number of Team Members involved in this project: 6

Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 3

3 OBJECTIVES

3.1 RESEARCH OBJECTIVES

The aim of the project is to use joint efforts of the involved teams for solving several topical problems concerned with polynomial mappings over the fields of complex and real numbers. In particular, it is planned to:

- Describe and classify Cohen-Macaulay tame singularities and related derived categories

- Investigate Cohen-Macaulay modules over surface singularities

- Find effective criteria of properness for inhomogeneous polynomial mappings

- Obtain exact estimates for topological invariants of intersections of real quadrics

- Develop polynomial time algorithms for computing topological invariants of quadratic mappings

- Compute Lie algebras of unimodal hypersurface singularities

- Establish Pursell-Shanks-type and Hauser-Mueller type theorems for Lie algebras of algebraic varieties and isolated hypersurface singularities

- Develop effective algorithms (in Singular) which test whether two Artinian algebras are isomorphic

- Develop algorithms for constructing bases of mutually commuting polynomial derivations in Lie subalgebras of Weil algebra

3.2 Background and Justification

Geometry of polynomial mappings is of fundamental importance in many problems of algebraic geometry, singularity theory, topology and nonlinear analysis. Some natural problems may be formulated as questions about the geometric and/or topological behaviour of polynomial mappings, a well-known example being the Keller Jacobian Problem (KJP). A number of important problems of nonlinear analysis and calculus of variations require detailed investigation of families of quadratic mappings. Many problems of singularity theory can be formulated in terms of discriminants of polynomials mappings and their behaviour at infinity. A new trend in this field is the growing importance of effective methods of commutative algebra and computer algebra which appeared extremely useful for investigating concrete examples and revealing delicate phenomena exhibited by polynomial mappings both over the field of complex and real numbers.

In line with this dominant trend, the present project focuses on a number of topical problems where considerable progress can be achieved by combining methods ot algebraic geometry, computer algebra, singularity theory and topological methods of nonlinear analysis. The problems adressed in our research proposal can be roughly distinguished as local and global ones. The local ones include description of Cohen-Macaulay modules, mixed Hodge structures and monodromy of the fibres of polynomial mappings, derivation Lie algebras of isolated hypersurface singularities and effective computation of various numerical invariants of singularities. Among the global problems addressed by the project are effective criteria of properness for polynomial mappings, various versions of KJP, structure of bifurcation diagrams and description of perestroikas of the fibres, singularities at infinity, Lie algebras of algebraic varieties, estimation and effective computation of various topological invariants such as the mapping degree and Euler characteristics of fibres. It is also planned to develop effective algebraic methods and implementations as computer programs for studying those problems in concrete cases.

The aims, tasks and research strategy for this project were worked out with regard to backgrounds of the teams participating in the project. The main tasks can be described as follows.

Cohen-Macaulay modules

Cohen-Macaulay modules over hypersurface singularities attracted considerable attention of experts in singularity theory and commutative algebra. Among the most topical problems in this field are classification of tame Cohen-Macaulay singularities and Cohen-Macaulay modules. Important results in this direction were obtained by members of the ukrainian team and german team. It is worthy of noting that some of those results were obtained jointly and the cooperation of those teams apparently has big perspectives in achieving furter progress.

A basis for further research was created, in particular, in the following papers:

Y.Drozd, Vector bundles and Cohen-Macaulay modules, In: Representations of Finite Dimensional Algebras and Related Topics in Lie Theory and Geometry. Field Institute Communications 40, AMS, 2004, 189-222.

Y.Drozd, G.-M.Greuel and I.Kashuba, On Cohen-Macaulay modules on surface singularities, Moscow Math. J. 3 (2003), 397-418.

Algebraic computation of topological invariants

An important peculiarity of polynomial mappings is that many topological invariants can be expressed by explicit algebraic formulae which give rise to effective computer algorithms for performing such computations in concrete cases. Those formulae also have interesting applications in the general study of the geometry of polynomial mappings. In particular, they yield universal estimates for topological invariants of polynomial mappings of fixed multi-degree which, in turn, have interesting applications to some well-known problems of algebraic geometry such as Hilbert sixtienth problem and effective Nullstellensatz. An important ingredient of those developments is given by the so-called signature formuale for the mapping degree and Euler characteristic developed by G.Khimshiashvili, D.Eisenbud with H.Levine, J.Bruce and Z.Szafraniec. Using the foregoing formulae, a number of general results on estimating the mapping degree and Euler characteristic were obtained by V.Arnold, A.Khovansky and G.Khimshiashvili. There is good evidence that further research in this direction will lead to new important developments and for this reason this topic is chosen as a crossroad for this project. It will be investigated from theoretical point of view and new algorithms for computing topological invariants
will be also developed by joint efforts of the georgian and german team. Special emphasis will be done on developing fast algorithms for computing holomogy groups of the fibres of quadratic mappings. Recent results of D.Grigoriev and D.Pasechnik indicate that there should exist polynomial time algorithms for solving this problem and one of our concrete tasks will be concrete implementation of such algorithms using Singular package.

The background for investigation of this topic was created, in particular, in the following publications of participants of this project:

G.Khimshiashvili, Signature formulae for topological invariants, Proc. A.Razmadze Math. Inst. 125, 2001, 1-121.

G.-M.Greuel, G.Pfister, A Singular introduction to commutative algebra. With contributions by Olaf Bachmann, Christoph Lossen and Hans Schönemann. With 1 CD-ROM (Windows, Macintosh, and UNIX). Springer-Verlag, Berlin, 2002.

T.Aliashvili, G.Khimshiashvili, Topology of the fibres of stable quadratic mappings, (Russian) Doklady Ross. Akad. Nauk 408, 2006, No.1, 1-4.

D.Grigoriev, D.Pasechnik, Polynomial-time computing over quadratic maps, Computational Complexity 14, 2005, 20-52.

Lie algebras of algebraic varieties

Lie algebras, their deformations and representations play considerable role in singularity theory. One of the cornerstones of Lie-algebraic approach to singularity theory is provided by the fact that in many cases the analytic isomorphism class of an algebraic variety is determined by the Lie algebra of derivations of its algebra of regular functions. A general theorem of such kind was proven by E.Hauser and V.Mueller. Local counterparts of this result for certain classes of singularities have been recently obtained by A.Elashvili and G.Khimshiashvili. These developments suggest a number of interesting problems concerned with the derivations of moduli algebra of an isolated hypersurface singularity.
Namely, one may hope to express various numerical invariants of a given singularity (Milnor number, Hodge numbers) purely algebraically in terms of the Lie algebra of derivations of its moduli algebra. The experience of german team in computing vector fields and Lie algebras associated with singularities fits well the approach of georgian team and gives good evidence for further progress by joint efforts. It is also planned to investigate the Lie-algebraic aspects of KJP in cooperation with the dutch team.

A basis for these studies is given, in particular, by the following publications:

A.Elashvili, G.Khimshiashvili, Lie algebras of simple hypersurface singularities, J. Lie Theory 16, 2006, No.4, 621-649.

B.Martin, Algorithmic computation of flattenings and of modular deformations. J. Symbolic Comput. 34 (2002), no. 3, 199--212.

A. van den Essen, Polynomial automorphisms and the Jacobian conjecture, Progress Math. vol. 190, Birkhauser, Basel, 2000.

4 SCIENTIFIC / TECHNICAL DESCRIPTION

4.1 Research Programme

The aim of the project is to develop new methods of algorithmic investigation of the geometry and topology of polynomial mappings. Both local and global geometric behaviour of polynomial mappings will be studied. The programme of research includes the following tasks and subtasks, which are described in more detail in Research details:

Cohen-Macaulay modules

Subtasks: Cohen-Macaulay tame isolated singularities. Cohen-Macaulay modules over non-isolated singularities. Matrix problems

Algebraic computation of topological invariants

Subtasks: Sharp estimates for topological invariants. Topological invariants of intersections of quadrics

Lie algebras of algebraic varieties

Subtasks: Lie algebras of isolated hypersurface singularities. Pursell-Shanks type theorems for algebraic varieties

4.2 Project Structure

4.2.1 Task Title : Cohen-Macaulay modules

Task coordinator : Yuriy Drozd, belonging to team: Kiev University

Objectives :

1. Description of Cohen-Macaulay tame isolated singularities (see Subtask 1).

2. Study of Cohen-Macaulay modules over non-isolated surface singularities (see Subtask 2).

3. Development of the technique of matrix problems and its application to study of modules, vector bundles and derived categories (See Subtask 3).

Methodology :

1) Kahn reduction of study of Cohen-Macaulay modules to vector bundles over projective curves and its generalizations (that have to be developed).

2) Technique of matrix problems, in particular, bimodule categories and categories of triples (already effectively used in study of modules, vector bundles and derived categories).

Task Input:

The previous results of the team members on Cohen-Macaulay modules, vector bundles and derived categories, especially on applications of the technique of matrix problems. Use of computer equipment (it is supposed to enlarge it).

Result, milestones :
1) Semi-continuity results for Cohen-Macaulay modules over isolated surface singularities. As corollary, a description of isolated Cohen-Macaulay tame surface and hypersurface singularities.

2) Description of Cohen-Macaulay modules over certain non-isolated surface singularities (espetially, degenerated cusps) and related derived categories.

3) Characterization of Cohen-Macaulay finite and tame surface singularities over fields of positive characteristic and over non-algebraically-closed fields.

4.2.1.1 Task Title : Cohen-Macaulay tame isolated singularities

Task coordinator : Yuriy Drozd, belonging to team: Kiev University

Objectives :
Characterization of Cohen-Macaulay tame surface and hypersurface singularities.

Methodology :
Kahn results on connections between Cohen-Macaulay modules over surface singularities and vector bundles over projective curves; use of deformations and semi-continuity results. Explicit calculations using Singular language.

Task Input:
Previous results of the team members (Y.Drozd, G.-M.Greuel, V.Bondarenko) on Cohen-Macaulay tame singularities, vector bundles over projective curves and semi-continuity results for Cohen-Macaulay modules and matrix problems.

Result, milestones :
Generalization of Kahn results for "partial" resolutions of singularities. Semi-continuity for Cohen-Macaulay modules over surface singularities. Classification of Cohen-Macaulay tame surface and hypersurface singularities.

4.2.1.2 Task Title : Cohen-Macaulay modules over non-isolated singularities

Task coordinator : Yuriy Drozd, belonging to team: Kiev University

Objectives :
Study of Cohen-Macaulay modules over non-isolated surface singularities and related derived categories.

Methodology :
Technique of matrix problems, in particular, bimodule categories and categories of triples (already effectively used in study of modules, vector bundles and derived categories).

Task Input:

The task is depending on : Matrix problems
The technique of matrix problems, especially generalized bunches of chains, developed by the team members (Y.Drozd, O.Drozd-Koroleva). Use of computer calculations (mainly in Singular language).

Result, milestones :
Elaboration of general methods for study of Cohen-Macaulay modules. Description

of Cohen-Macaulay modules over certain non-isolated surface singularities (especially over degenerated cusps) and related derived categories.

4.2.1.3 Task Title : Matrix problems

Task coordinator : Yuriy Drozd, belonging to team: Kiev University

Objectives :
Development of the technique of matrix problems aimed on the purposes of singularities theory and algebraic topology

Methodology :
Use of reduction algorithms, representations of bunches of chains, Auslander-Reiten theory for matrix problems, as well as computer programs for such calculations (like Singular and CRAP).

Task Input:
Previous results on matrix problems (Y.Drozd, O.Drozd-Koroleva), especially matrix problems over rings and non-algebraically-closed fields.

Result, milestones :
Generalization of bunches of chains, reduction algorithms for problems of this kind and their application to Cohen-Macaulay modules, vector bundles and algebraic topology.

4.2.2 Task Title : Algebraic computation of topological invariants

Task coordinator : Giorgi Khimshiashvili, belonging to team: Tbilisi Inst. Math.

Objectives :
1. Develop effective methods and fast algorithms for computing topological invariants of the fibres of polynomial mappings

2. Obtain exact estimates for various topological invariants of the fibres of polynomail mappings with fixed algebraic degrees of components

Methodology :
Explicit algebraic formulae for the topological invariants of real algebraic varieties and mappings will be used for developing the corresponding computer algorithms

Task Input:
Signature formulae for topological invariants. Computer programmes (in Singular) for calculating the mapping degree and Euler characteristic

Result, milestones :
Effective methods and computer algorithms for computing and estimating the Euler characteristic and Chern classes of the fibres of polynomial mappings.

4.2.2.1 Task Title : Sharp estimates of topological invariants

Task coordinator : Giorgi Khimshiashvili, belonging to team: Tbilisi Inst. Math.

Objectives :
1. To obtain sharp estimates for possible values of basic topological invariants (Euler characteristic, Chern numbers) of the fibres of polynomial mappings with fixed algebraic degrees

2. To determine, in the same setting, the whole spectrum of possible values of a topological invariant in question

Methodology :
Algebraic formulae for basic topological invariants of algebraic varieties will be used for finding the lower and upper bounds for various topological invariants.

Task Input:

The task is depending on : Algebraic computation of topological invariants Signature formulae for the mapping degree and Euler characteristic, computer algorithms for computing mapping degree of homogeneous polynomial endomorphisms.

Result, milestones :
For polynomials in n indeterminates of a fixed algebraic degree d, exact estimates of the Euler characteristic of level surfaces and gradient mapping degree will be obtained in terms of n and d. Examples of polynomials with the extremal values of those topological invariants will be also constructed for small values of n and d.

4.2.2.2 Task Title : Topological invariants of intersections of quadrics

Task coordinator : T.Aliashvili, belonging to team: Tbilisi Inst. Math.

Objectives :
1. Obtain effective criteria of properness for quadratic mappings.

2. Describe the spectra of possible values of the Euler characteristic of fibres for real quadratic mappings between affine spaces of fixed dimensions.

Methodology :
General estimates for the Euler characterisic will be obtained using Bruce formula and Petrovsky-Oleynik estimates for the mapping degree. The lower and upper bounds will be explicated using description of the possible bifurcations of fibres.

Task Input:

The task is depending on : Algebraic computation of topological invariants Signature formulae for the Euler characteristic. The spectral sequence for computing homology groups of fibres of quadratic mappings developed by Agrachev.

Result, milestones :
Exact spectra for the Euler characteristics of intersections of quadrics and explicit examples where the lower and upper bounds are attained.

4.2.3 Task Title : Lie algebras of algebraic varieties

Task coordinator : Alexander Elashvili, belonging to team: Tbilisi Inst. Math.

Objectives :
1. Characterization of solvable Lie algebras associated with isolated hypersurface singularities

2. Computation of derivation Lie algebras for unimodal singularities.

3. Computation of topological invariants and topological classification of isolated hypersurface singularities in terms of derivation Lie algebras

4. Pursell-Shanks-type and Hauser-Mueller-type theorems for Lie algebras of algebraic varieties and isolated hypersurface singularities

Methodology :
Derivation Lie algebras for unimodal singularities will be computed using the known explicit bases in moduli algebras and programmes in Singular. Pursell-Shanks-type and Hauser-Mueller-type theorems will be established using spectral invariants of arising solvable Lie algebras introduced in the paper of A.Elashvili and G.Khimshiashvili (J. Lie Theory 16, 2006, 621-649)

Task Input:

The task is depending on : Matrix problems
Algorithms for computing derivation Lie algebras in Singular. Spectral invariants of arising solvable Lie algebras introduced in the paper of A.Elashvili and G.Khimshiashvili (J. Lie Theory 16, 2006, 621-649)

Result, milestones :
Explicit formula for the Milnor number of a plane curve singularity in terms of the derivation Lie algebra of its moduli algebra. Pursell-Shanks-type theorem for Lie algebras of fewnomial singularities

4.2.3.1 Task Title : Lie algebras of isolated hypersurface singularities

Task coordinator : A.Elashvili, belonging to team: Tbilisi Inst. Math.

Objectives :
1. To compute the Lie algebras of unimodal singularities

2. To prove that unimodal singularities are topologically classified by their Lie algebras

Methodology :
Lie algebras of unimodal singularities wil be computed using existing detailed description of their moduli algebras and computer programs for calculating derivation Lie algebras.

Task Input:

The task is depending on : Lie algebras of algebraic varieties
Description of monomial bases in moduli algebras of unimodal singularities, functoriality theorems for Yau algebras and computer programs for finding generators of derivation Lie algebras

Result, milestones :
The values of dimension and explicit systems of generators for Yau algebras of unimodal singularities.

4.2.3.2 Task Title : Pursell-Shanks type theorems for algebraic varieties

Task coordinator : G.Khimshiashvili, belonging to team: Tbilisi Inst. Math.

Objectives :

Pursell-Shanks-type theorem for Lie algebras of isolated hypersurface singularities defined by fewnomials. Hauser-Mueller-type theorems for Lie algebras of algebraic varieties with simple singularities

Methodology :
Spectral invariants of solvable Lie algebras introduced in our recent paper in J. Lie Theory v.16, No.4, will be used to show that derivation Lie algebras determine the singualrities defined by fewnomials. This result will be combined with the classical Pursell-Shanks theorem for manifolds with boundary in order to obtain new versions of Hauser-Mueller theorem for varieties with simple singularities

Task Input:

The task is depending on : Lie algebras of isolated hypersurface singularities Spectral invariants of solvable Lie algebras. The Pursell-Shanks theorem for manifolds with boundary

Result, milestones :
Classification of hypersurface singularities defined by fewnomials by their derivation Lie algebras. A new version of Hauser-Mueller theorem for varieties with simple singularities

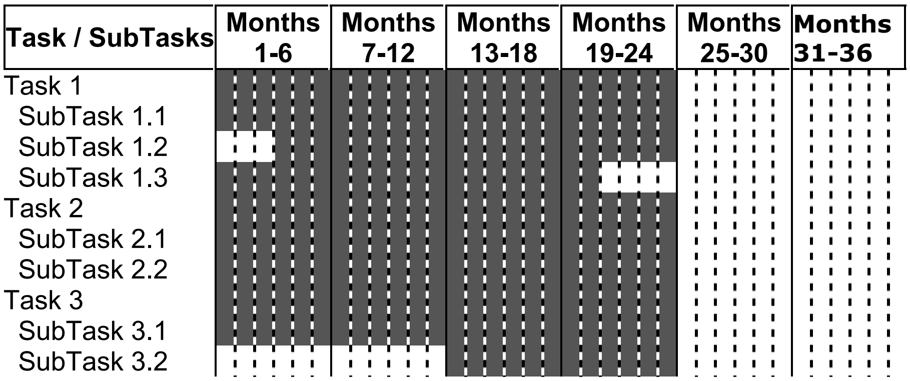
4.3 Project Management

4.3.1 Planning & Task allocation

4.3.1.1 List of Task Titles

1. Cohen-Macaulay modules
    - 1.1 Cohen-Macaulay tame isolated singularities
    - 1.2 Cohen-Macaulay modules over non-isolated singularities
    - 1.3 Matrix problems
2. Algebraic computation of topological invariants
    - 2.1 Sharp estimates of topological invariants
    - 2.2 Topological invariants of intersections of quadrics
3. Lie algebras of algebraic varieties
    - 3.1 Lie algebras of isolated hypersurface singularities
    - 3.2 Pursell-Shanks type theorems for algebraic varieties

4.3.1.2 The project will last 24 months with the activities as indicated in the diagram below



4.3.1.3 Team involvement

Teams	Task 1			Task 2		Task 3	
	Task 1.1	Task 1.2	Task 1.3	Task 2.1	Task 2.2	Task 3.1	Task 3.2
Utrecht University							
Tbilisi Inst. Math.							
Kiev University							
Kaiserslautern Univ							

4.3.2 Project Management Description

Each problem will be attacked by joint efforts of at least two teams

4.4 Project costs

4.4.1 Cost Table

The breakdown of costs of the INTAS contribution (in EURO) is given in the tables below.

INTAS MEMBER STATE TEAMS								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
1	Utrecht Unive...	0	2000	6000	0	0	0	8000
2	Kaiserslauter...	0	2000	6000	0	0	0	8000
SUBTOTAL	(EURO)	0	4000	12000	0	0	0	16000

NIS TEAMS								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
3	Tbilisi Inst....	8000	0	14000	0	3000	0	25000
4	Kiev Universi...	13000	0	8000	0	3000	0	24000
SUBTOTAL	(EURO)	21000	0	22000	0	6000	0	49000

TOTAL	(EURO)	21000	4000	34000	0	6000	0	65000

4.4.2 Justification of Costs
4.4.2.1 Labour costs (only for NIS teams)

Team name: Tbilisi Inst. Math.

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	180	10	1800
Senior Researcher	3	170	20	3400
Scientist/Engineer	1	140	20	2800
Technical or Other	0	0	0	0
TOTAL				8000

Team name: Kiev University

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	300	20	6000
Senior Researcher	1	200	20	4000
Scientist/Engineer	1	150	20	3000
Technical or Other	0	0	0	0
TOTAL				13000

4.4.2.2 Justification Labour costs

Team 1 (Utrecht University)
*

Team 2 (Kaiserslautern Univ)
*

Team 3 (Tbilisi Inst. Math.)
*

Team 4 (Kiev University)
*

4.4.2.3 Justification Operational costs

Team 1 (Utrecht University)
*

Team 2 (Kaiserslautern Univ)
*

Team 3 (Tbilisi Inst. Math.)
*

Team 4 (Kiev University)
*

4.4.2.4 Justification Overheads

Team 1 (Utrecht University)
*

Team 2 (Kaiserslautern Univ)

*

Team 3 (Tbilisi Inst. Math.)
*

Team 4 (Kiev University)
*

4.4.2.5 Comments

Team 1 (Utrecht University)
*

Team 2 (Kaiserslautern Univ)
*

Team 3 (Tbilisi Inst. Math.)
*

Team 4 (Kiev University)
*

4.5 Project innovation potential and dissemination of results

The methods and algorithms which will be developed in the framework of the present project will have wide range of applications - from investigation of the global and local geometric behaviour of concrete polynomial mappings to topological classification of configuration spaces of mechanical linkages and molecular conformations. The results will be published in recognized journals and presented at international conferences and workshops on algebraic geometry and computer algebra. The algorithms will become publicly available through the Internet and Singular package.