Exercise 2 - Fourier versus wavelets

This exercise lets you compare the approximation capabilities of wavelets and Fourier series. You need to open `exercise2.m` in the matlab editor. There is one extra parameter to set, `thresholdfft`, which sets the threshold for the Fourier coefficients. We still use `func.m` for specifying the function to use.

If we have a discretised function f on the points $x = 0, \ldots, N-1$, matlab computes the FFT as

$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} c_k e^{2\pi i x k/N}, \tag{1}$$

$$c_k = \sum_{x=0}^{N-1} f(x) e^{-2\pi i x k/N}, \tag{2}$$

with $k = 1, \ldots, N-1$. One may easily check the following facts:

- f real valued if and only if $c_k = \overline{c_{N-k}}$
- f if and only if $c_k = \overline{c_k} \in \mathbb{R}$.

Matlab uses strictly positive indices. The Fourier coefficients however can be extended to $k < 0$ and $k > N-1$ by periodicity: $c_k := c_{\ell N+k}$. This is consistent with $e^{-2\pi i x(N-k)/N} = e^{2\pi i x k}$. We can therefore use the matlab function `fftshift` to shift the highest frequencies to the middle of the spectrum, this is more conventional.

Now try setting `Func6` in `func.m`. Set the `threshold` and `delta` to zero. If you run `exercise2` you will see what you expected, two peaks for the two cosines. (*Warning:* if you try your own functions in this exercise, make sure they are even.)

In the source you will find the commands `x = x(1:end-1)`, `domain = domain(1:end-1)` and `N=N-1`. Why is this done? Try commenting out these lines and plotting the absolute value of the imaginary part of the coefficients. Run the program again and notice the difference. Just one point 'off periodicity' makes a huge difference!

Now compare the wavelet and Fourier approximations to the functions for various wavelets, noise levels and thresholds. It seems that the wavelets and Fourier approach both yield good results. The Fourier series are superior with respect to noise filtering in this case: the frequencies are well isolated in the spectrum.

Things are quite different for a non smooth function. Do the same experiments for `Func7`. Can you explain what happens?