1 Hoofdstuk 7

Opgave 7.3.3

a) The matrix of this game is:

	rood	geel	blauw
groot	6	1	3
klein	2	5	4

b) We will search for the best strategy for Yvonne. Her strategy will be given by $\{x_1, x_2, x_3\}$. These variables must satisfy the condition:

$$x_1 + x_2 + x_3 = 1, \quad x_1, x_2, x_3 \geq 0$$

Furthermore, since Yvonne would like to find a strategy such that she has to pay the smallest possible amount to Xander, then our goal is to minimize z (or maximize $-z$) under the condition that,

$$6x_1 + x_2 + 3x_3 \leq z$$
$$2x_1 + 5x_2 + 4x_3 \leq z$$

When we will be done, then z will be the expectation value of the game.

We introduce two "slack" variables x_4, x_5 satisyfing $x_4, x_5 \geq 0$ and we set $x_6 = -z$. We obtain the following linear programming problem,

$$\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 6 & 1 & 3 & 1 & 0 & 1 & 0 \\ 2 & 5 & 4 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & \text{-z} \end{array}$$

x_6 is not bounded by any further condition (it does not have to be positive), we thus solve for it by applying row operations (vegen),

$$\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 6 & 1 & 3 & 1 & 0 & 1 & 0 \\ -4 & 4 & 1 & -1 & 1 & 0 & 0 \\ \hline -6 & -1 & -3 & -1 & 0 & 0 & \text{-z} \end{array}$$

The second row tells us that $z = 6x_1 + x_2 + 3x_3 + x_4$. We now remove the second row and sixth column, and are left with the problem of finding $\{x_1, x_2, x_3, x_4, x_5\}$ that maximize $-z = -6x_1 - x_2 - 3x_3 - x_4$, in other words, with the following matrix,

$$\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 1 \\ -4 & 4 & 1 & -1 & 1 & 0 \\ \hline -6 & -1 & -3 & -1 & 0 & \text{-z} \end{array}$$

We first need to find an admissible basis, we do so by applying row operations to obtain a basis vector in the first column,

1	1	1	0	0	1
0	8	5	-1	1	4
0	5	3	-1	0	-z+6

We see that the bottom row contains positive numbers, so this basis does not achieve the maximum yet. We choose to increase x_2, and since $\min\{4/8, 1\} = 4/8$, then we see that we must isolate the element in the second row and second column,

1	0	3/8	1/8	-1/8	1/2
0	1	5/8	-1/8	1/8	1/2
0	0	-1/8	-3/8	-5/8	-z+ 7/2

And we are done. We see that the best strategy for Yvonne is $\{x_1, x_2, x_3\} = \{1/2, 1/2, 0\}$ and that the expectation value for the game is then $z = 7/2$.

c) We can now find an optimal strategy for Xander without much computation. By the Minimax theorem, we know already that he has to find a strategy so that the value of the game will be 7/2. We want to find a strategy $\{y_1, y_2\}$ so that,

$$
\begin{aligned}
6y_1 + 2y_2 &\geq 7/2 \\
y_1 + 5y_2 &\geq 7/2 \\
3y_1 + 4y_2 &\geq 7/2
\end{aligned}
$$

Since the value of the game is 7/2, the by the Minimax theorem we know that we cannot find a strategy where all inequalities are strict. At least one of the inequalities must be an equality. Let us guess that the solution is reached when the first inequality becomes an equality, i.e. that $6y_1 + 2y_2 = 7/2$ under the condition $y_1 + y_2 = 1$. We solve this:

$$6y_1 + 2(1 - y_1) = 4y_1 + 2 = 7/2 \quad \Rightarrow y_1 = 3/8, \quad y_2 = 5/8$$

Indeed, when we insert this in the remaining two inequalities, we see that they hold, and therefore $\{y_1, y_2\} = \{3/8, 5/8\}$ is an optimal strategy for Xander.