September 13, 2017

Matlab Assignments – Lecture 12, Fall 2016

Assignment 12.1. **Pseudo-spectrum.** Let $\mathbf{A}$ be a (complex) $n \times n$ matrix. For a discussion of the notion 'pseudo-spectrum', see Theorem 12.17 and preceeding paragraph and subsequent two paragraphs in the set of exercises.

(a) Consider the matrix $\mathbf{A} = (A_{ij})$ with ones on the $+1$ and $-k$ diagonal for $k = 2$, (i.e., $A_{i,j} = 1$ if $j - i \in \{1, -k\}$) and zeros elsewhere.
Plot the spectrum of $\mathbf{A}$ for $n \in \{100, 200, 300\}$. Discuss the results.
Is there a difference between the (computed) spectrum of $\mathbf{A}$ and $\mathbf{A}^*$? Is there a difference between the results of MATLAB's `eig(A)` and `eig(A,'nobalance')`?[1]
Plot also the eigenvalues of $\mathbf{A} + \Delta$, where Δ is random $n \times n$ matrix with $\|\Delta\|_2 \leq \varepsilon$ ($\varepsilon = 10^j$ for $j = -2, -4, -6, -8$).

(b) Download the MATLAB tool `EigTool` from Oxford university: follow the appropriate link on the course homepage. This tool allows you to compute the pseudo-spectrum of matrices (of modest dimension).
Run the command `eigtool(gallery('grcar',32))` in MATLAB to plot the pseudo eigenvalues of the `grcar` matrix of dimension 32.
Plot the pseudo-spectrum (with `eigtool`) of the matrix $\mathbf{A}$ from from part (a).
Compare the results with the plot of the eigenvalues $\mathbf{A} + \Delta$ as computed in part (a).

(c) Is the sensitivity of the eigenvalues to perturbations reflected in the angle between right and left eigenvector: plot the condition number of the eigenvalues of the matrix $\mathbf{A}$ from from part (a) (eigenvalue λ for $\lambda \in (0, \infty)$ versus $\mathcal{C}_\lambda(\mathbf{A})$)? Recall that

$$\mathcal{C}_\lambda(\mathbf{A}) = \frac{\|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2}{\mathbf{y}^* \mathbf{x}}$$

if λ is a simple eigenvalue with right eigenvector $\mathbf{x}$ and left eigenvector $\mathbf{y}$, i.e., $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$, $\mathbf{y}^*\mathbf{A} = \lambda\mathbf{y}^*$ and both $\mathbf{x}$ and $\mathbf{y}$ are non-zero n-vectors; see also MATLAB's routine `condeig`.

(d) Compute the conditioning of the (right) eigenvectors as well.

(e) Derive an analytic expression for the eigenvalues of the matrix $\mathbf{A}$ as defined in part (a).

Assignment 12.2. Let $\mathbf{A}$ be a Hermitian $n \times n$ matrix (real symmetric if you wish) with a few negative eigenvalues.

(a) Extend the Lanczos code for computing Ritz values to compute harmonic Ritz values. Make sure your code is efficient. (The results in Exercise 6.4(f) can be used. Note that in this case H_k is Hermitian. Unfortunately, the matrix in 4) nor the one in 5) is Hermitian. There is also an Hermitian version that allows more stability).

(b) Plot Ritz values and harmonic Ritz values of $\mathbf{A}$ (around 0).

(c) Do harmonic Ritz values lead to faster convergence?

[1] What is purpose of 'balancing' the matrix A, i.e., of `eig(A,'balance')`?