September 13, 2017

Matlab Assignments – Lecture 5, Fall 2016

Assignment 5.1. This assignment uses the codes `main.m`, `problem1.m`, `polynomialsolver.m` and `gcr.m` (and modifications)

(a) The file `problem1.m` defines the matrix vector multiplication (MV). The matrix is a diagonal" $\texttt{A} = \text{diag}(\texttt{MyLAMBDA})$, where `MyLAMBDA` is a column vector and $\text{diag}(\texttt{MyLAMBDA})$ is the diagonal matrix with (i, i) entry equal to $\texttt{MyLAMBDA}(\texttt{i}, \texttt{1})$. The MV can be performed as $\texttt{c} = \texttt{A} * \texttt{u}$, but also as $\texttt{c} = \texttt{MyLAMBDA}. * \texttt{u}$. Can you observe a difference? Explanation? The command `spdiags` can also be used to define the matrix. Discuss the effectiveness of defining the matrix with this command.

The matrix in the above experiments is diagonal. Does it effect the number of iterations steps if we replace the diagonal matrix $\Lambda = \text{diag}(\texttt{MyLAMBDA})$ by the matrix $\mathbf{A} \equiv \mathbf{Q}^*\Lambda\mathbf{Q}$, with $\mathbf{Q}$ unitary (and we replace $\mathbf{b}$ by $\mathbf{Q}^*\mathbf{b}$)? The file `problem2.m` is prepared to do this transformation by means of a Householder reflection. It is not advisable to form the matrix $\mathbf{A}$ explicitly. Why not? Fill in the details in this file `problem2.m` and answer the question on the number of iteration steps? Explain the result?

Do you come to the same conclusion if you replace Λ and $\mathbf{b}$ by $\mathbf{T}^{-1}\Lambda\mathbf{T}$ and $\mathbf{T}^{-1}\mathbf{b}$, respectively? Take for $\mathbf{T}$ the bi-diagonal matrix $\mathbf{I} - \alpha\mathbf{S}$ with $\alpha \in (0, 1)$ en $\mathbf{S}$ the matrix with all zeros except on the first co-diagonal, where it has the values 1 ($S_{i+1,i} = 1$). How do you code the application of $\mathbf{T}^{-1}$ to a vector?

Conclusion: the convergence history for symmetric problems depends on the eigenvalues and not on the eigenvectors.

(b) What method do you prefer, Richardson with optimal α, or Minimal Residuals in case $\mathbf{A}$ is a diagonal with eigenvalues in $[\lambda_-, \lambda_+] \subset (0, \infty)$. Does it make a difference whether the n eigenvalues are uniformly distributed between $[1, 400]$ or $[1, 2]$?

(c) Does the ordering of the parameters in Chebyshev iteration with a fixed polynomial (of degree ℓ) affect the convergence?

(d) Let $\mathbf{A}$ be the $n \times n$ tridiagonal matrix with 2 on all the diagonal entries and -1 on all the co-diagonal entries. Use `eig` (for a number of (small) n) to find an interval $[\lambda_-, \lambda_+] \subset (0, \infty)$ that contains the eigenvalue. How sensitive does Chebyshev iteration depend on the estimates of λ_-? (Use $\gamma\lambda_-$ in the estimates instead of λ_- for γ between 0.5 and 2.

Investigate how effective Chebyshev iteration is on a problem $\mathbf{Ax} = \mathbf{b}$ with spectrum in the complex plane close to an interval $[\lambda_-, \lambda_+] \subset (0, \infty)$. (Replace Λ and $\mathbf{b}$ by $\mathbf{T}\Lambda\mathbf{T}$ and $\mathbf{T}^{-1}\mathbf{b}$ with $\mathbf{T}$ as in (a)).

(e) Write a code for three terms Chebyshev iteration, i.e., the degree of the Chebyshev polynomial equals the iteration step.

(f) Compare Chebyshev iteration and GCR (check the strong and weak points of these methods as listed on the transparencies).