September 13, 2017

Matlab Assignments – Lecture 7, Fall 2016

In this assignment we will investigate the superlinear convergence of CG.

Assignment 7.1.

(a) Implement the CG algorithm. Start with $\mathbf{x}_0 = \mathbf{0}$. Your algorithm should be called as follows:

```
[x,res] = cg(A,b,m_iter,eps);
```

.

The input parameters are:

`A`: the system matrix,

`b`: the right-hand side,

`m_iter`: maximum number of iterations,

`eps`: residual tolerance.

The output parameters are:

`x`: iterative solution (as produced by CG at termination),

`res`: *convergence history*, i.e., the row $(\|\mathbf{r}_0\|_2, \|\mathbf{r}_1\|_2, \ldots)$ of residual norms $\|\mathbf{r}_k\|_2$ in every iteration step,

(b) Define a sparse diagonal matrix $\mathbf{A}$ and right-hand-side vector $\mathbf{b}$ of dimension 1000

$$\mathbf{A} = \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & \ddots & \\ & & & 1000 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.$$

Use the system $\mathbf{Ax} = \mathbf{b}$ to check if your CG code is correct. Plot the residual norm as function of the iteration number. Determine the "rate of convergence" (reduction of the residual norm per iteration) around the 30th iteration. How does this compare to the theoretical rate of convergence? What is the condition number of $\mathbf{A}$?

(c) With $\mathbf{b}$ again all 1s, now define the diagonal matrix $\mathbf{A}$, also of dimension 1000,

$$\mathbf{A} = \begin{bmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & 11 & & \\ & & & & \ddots & \\ & & & & & 1000 \end{bmatrix}.$$

Solve the new system $\mathbf{Ax} = \mathbf{b}$. Plot the residual norm as function of the iteration number, and determine the rate of convergence around the 30th iteration. How does this compare to the theoretical rate of convergence? What is the condition number of $\mathbf{A}$? Explain the difference with the previous assignment.

(d) Extend your code with the possibility to compute the Ritz values. Determine for the above examples the convergence to the eigenvalue 1. Use this information to explain the superlinear CG convergence of the above examples.