NUMERICAL LINEAR ALGEBRA
ACADEMIC YEAR 2007-2008

Review quiz day 3

1. Let $\mathcal{R}(A)$ denote the range of A and $\mathcal{N}(A)$ denote the nullspace of A. Prove that
   - (i) $\mathcal{N}(AB)$ contains $\mathcal{N}(B)$.
   - (ii) $\mathcal{R}(AB)$ is contained in $\mathcal{R}(A)$.
   - (iii) The left null space of AB, which is $\mathcal{N}((AB)^H)$, contains $\mathcal{N}(A^H)$.
   - (iv) The row space of AB, which is $\mathcal{R}((AB)^H)$, is contained in $\mathcal{R}(B)$.
2. Let the permutation matrix P be given by
$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} .$$
   - What is the inverse of this matrix?
   - What is its 2-norm condition number?
3. Let Q be the matrix
$$Q = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{pmatrix} .$$
   - What is the rank of Q?
   - What is the rank of Q^T?
   - Determine the orthogonal projection matrix onto $\mathcal{N}(Q^T)$.
4. Let R be the matrix
$$R = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} .$$
   - Determine the eigenvalues and eigenvectors of R.
   - Determine $\|R\|_1$, $\|R^{-1}\|_1$ and $c_1(R)$.

- Determine $\|R\|_2$, $\|R^{-1}\|_2$ and $c_2(R)$.
- Determine $\|R\|_\infty$, $\|R^{-1}\|_\infty$ and $c_\infty(R)$.

5. Let A be the matrix
$$A = \frac{1}{2}\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}.$$
   - Prove that A is positive definite.
   - Compute $\sqrt{A}$.