23 februari 2004

Seminar: Iterative Methods, spring 2004

The problems that are to be numerically computed in science and technology are often high dimensional, millions of unknowns is not exceptional. To facilitate efficient computations, these problems are projected onto low dimensional spaces. The low dimensional projected problems should provide accurate approximations to the solution of the original high-dimensional problems. The low dimensional space, the so-called search subspace, is expanded until the extracted approximation is sufficiently accurate. Fundamental question are how to find effective expansion vectors and how to extract to best information from the projected problem. Stability and efficiency issues play an important role.

Solution methods of this type are GMRES and Bi-CGSTAB for linear systems of equations and Arnoldi and Jacobi-Davidson for eigenvalue problems. In this seminar, we will also study more complicated problems that can be solved with similar approaches.

Internet page for this seminar:

`http://www.math.uu.nl/people/sleijpen/Opgaven/Seminaria/2004`

Teachers

Prof. H.A. van der Vorst and Dr. G.L.G. Sleijpen

Teaching form: seminar

Each of the participants will give a presentation on one or more research papers and will actively participate with the presentation of the other participants. If you sign in for the seminar then it is obligatory to attend all talks. (Be prepared to get an additional assigment if you mis a talk.)

There are two rounds of talks. Each of the participants will give a presentation of 45 minutes in each of the rounds.

Guidelines for preparing a talk

Start with the preparation at least three weeks before you have to give the talk.
Use the first week to understand the mathematics and to get a clear view on what the highlights are and what side issues.
Use the second week to find clear but concise explanations and illuminating examples: you should avoid (proofs with) technical details in your talk, unless the details are essential for (understand) the theory. Often an example is more instructive then a proof. Nevertheless you should also understand the details: the audience might be interested!
The third week is for making your talk. You have to organize your talk (what should go in the introduction, what in the kernel, ...), to decide on what you put on transparencies (highlights in order to more clearly bring out the overview, technical details to avoid errors and boring copying from paper to blackboard, ...) and what you write on the blackboard (some aspects require slow digestion). And, of course, you have to make the transparencies and prepare clear but concise formulations: your talk should fit in a 45 minute timeslot. You should realize that the audience will welcome an early stop, but may stop paying attention if you take more time.

Do not forget to address in your talk the important questions: “Why would you like to have a new theory or a new method? What is lacking in the old ones?”, “Does the new theory (method) solve all problems?”

Do not hesitate to contact the teachers, not only for mathematical details, but also for the overview, illuminating examples or structuring the talk: the quality of your talk determines your grade, not the way you prepared your talk. Preparing a talk is part of the educational process.

Program

For references to [31], you may also have to consult the references therein. An early version of [31] is [30], which is available from Prof. van der Vorst website. Note, however, that there is a shift in the numbering of the chapter in the earlier version,

Day 1, February, 18

TD LECTURE 1: Krylov subspaces and basis for Krylov subspaces [31, Ch.3], effects of loss of orthogonality (see also the paper by Daniel, Gragg, Kaufmann and Stewart [9]. More information in [26, 28, 19, 7, 8, 16, 13, 12, 14]).

MW LECTURE 2: Conjugate Gradients (CG), CG for least square systems, Graig’s method [31, Ch.5]

Day 2, February, 25

EN LECTURE 1: Convergence CG, super linear convergence, consequences for CG for least square systems [31], [29]

TL LECTURE 2: Bi-CG [31, Ch.7] ([10, 11, 2]).

Day 3, March, 3

No seminar.

Day 4, March, 10

No seminar

Day 5, March, 17

TD LECTURE 1: FOM and GMRES [27], comparison GMRES, CG least squares, Bi-CG [25].

MW LECTURE 2: Convergence GMRES [15].

Day 6, March, 24

EN LECTURE 1: Preconditioning I: Incomplete LU (ILU), modified ILU (MILU) [24], [18] (see also [31, Ch.13]).

TL LECTURE 2: Preconditioning II: Sparse approximate inverse (SPAI) [17] ([20]),[4], [22] ([21]).

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