Rekenkunst met strepen
Digitalized Geometry

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extended for UU GAGA seminar 28/04/2015

Trefwoorden:

- frequentie
- richting
- digitaal
- rekenen:

$$\begin{aligned} 0+0&=0 & 0\times 0&=0\\ 0+1&=1 & 0\times 1&=0\\ 1+0&=1 & 1\times 0&=0\\ 1+1&=0 & 1\times 1&=1 \end{aligned}$$

Zhegalkin (1927):
Polynomial functions over $\mathbb{F}_2$ give a powerful alternative for the Boolean algebra used in logic and set theory.

Zebra:

- frequentie : $\frac{1}{2\times\text{breedte zwarte strepen}}$
- richting : $\perp$ zwarte strepen
- digitaal : wit/zwart

Standaardisatie:

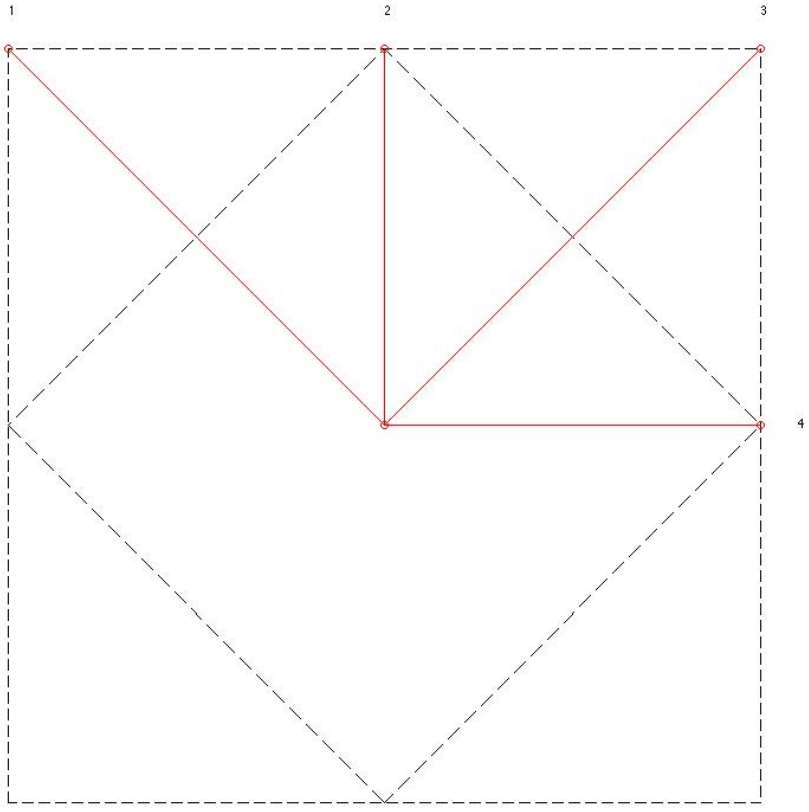
- Kies vast referentiepunt O in het vlak.
- Bekijk alleen zebra's met een zwart/wit overgang in O.

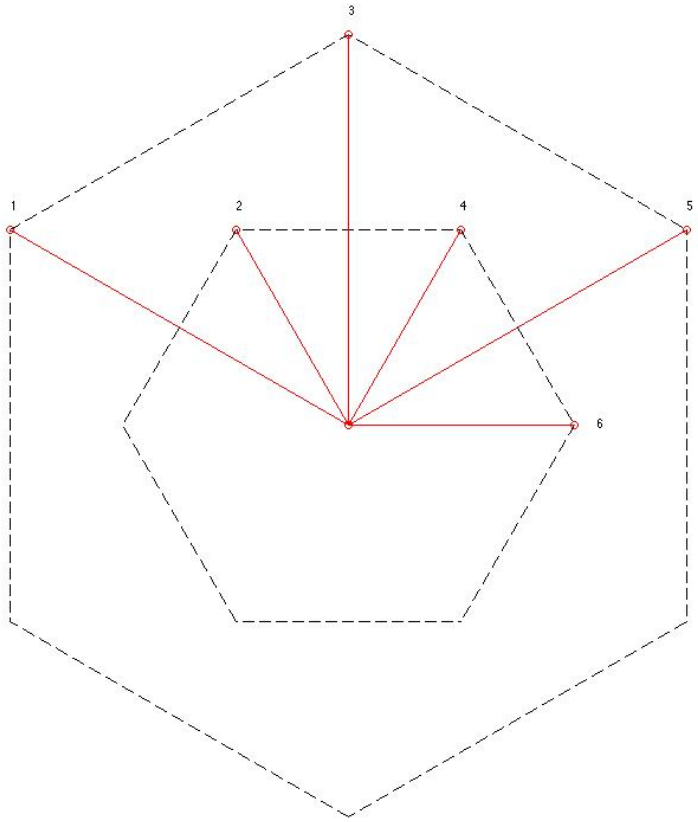
Codeer beschrijving van **zebra** als **pijl** **(= frequency vector v)**

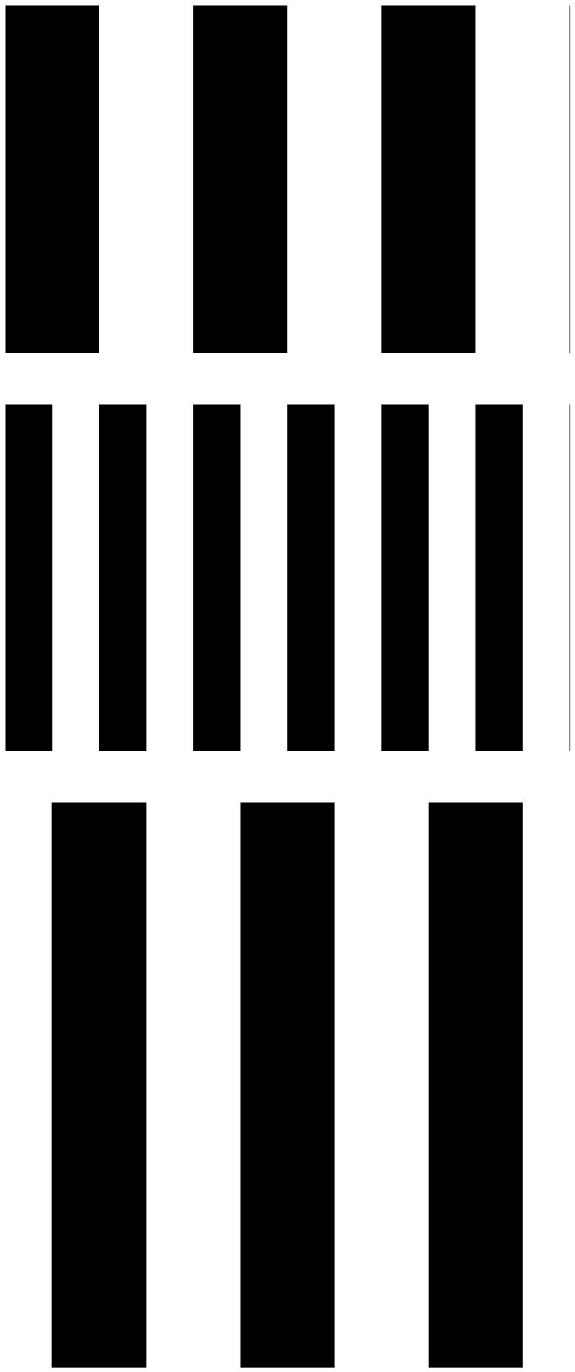
- vanuit O,
- richting: wit$\rightarrow$zwart , $\perp$ scheidingslijn,
- lengte $= \frac{1}{2\times\text{breedte zwarte strepen}}$.

- Defining formula:

$$Z_{\mathbf{v}}(\mathbf{x}) \;=\; \lfloor \mathbf{2x} \cdot \mathbf{v} \rfloor \bmod 2$$


1
2
3
4

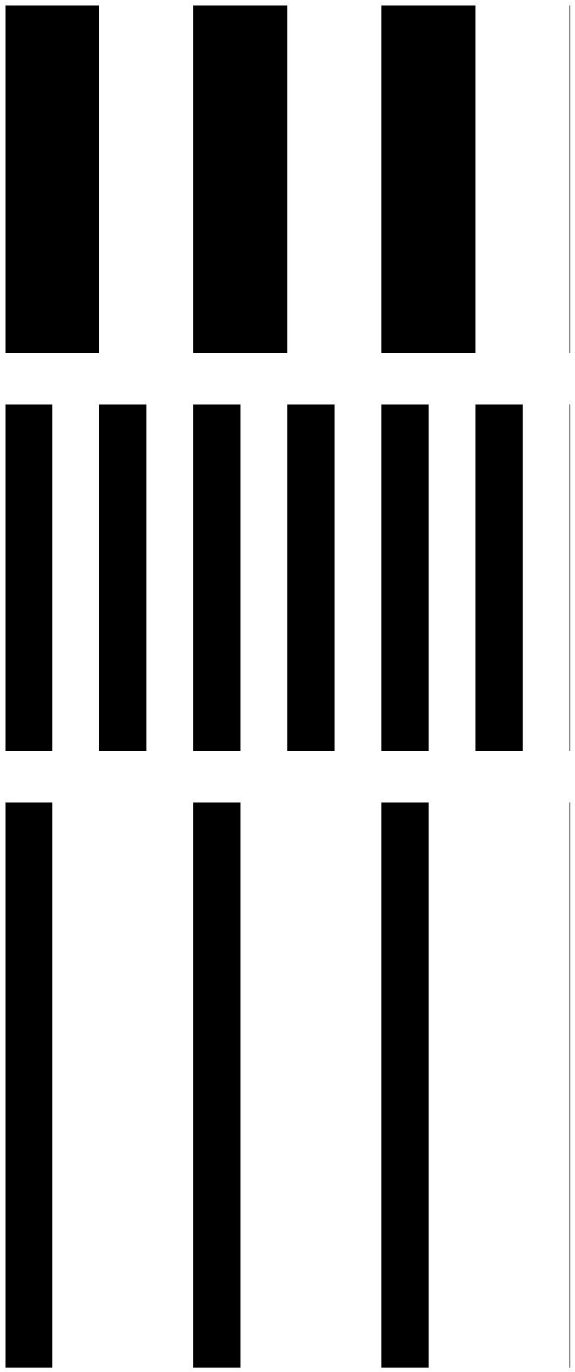

3
1
2
4
5
6



Z^{141}

Z^{142}

$Z^{141} + Z^{142}$

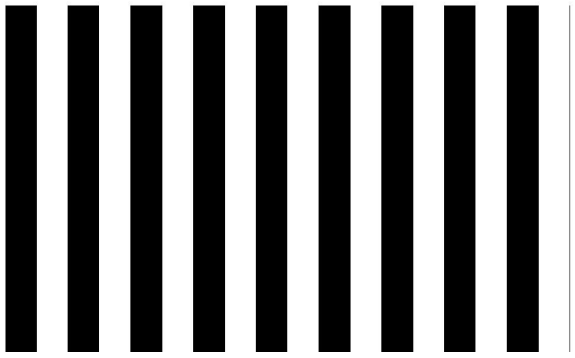


Z^{141}

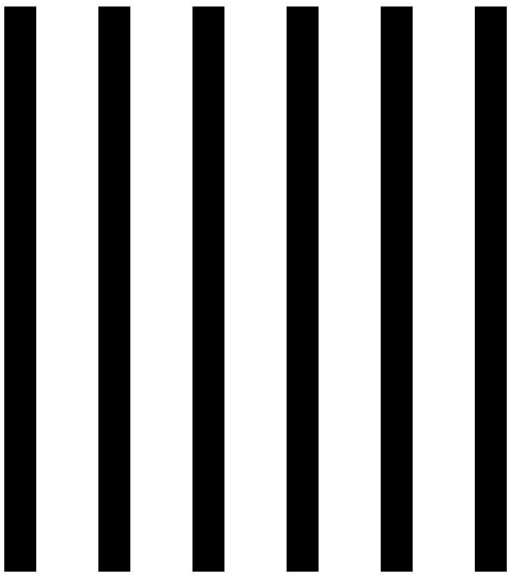
Z^{142}

$Z^{141} \times Z^{142}$

Z^{141}

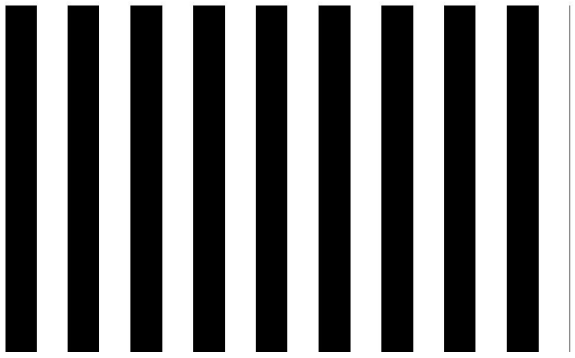


Z^{143}

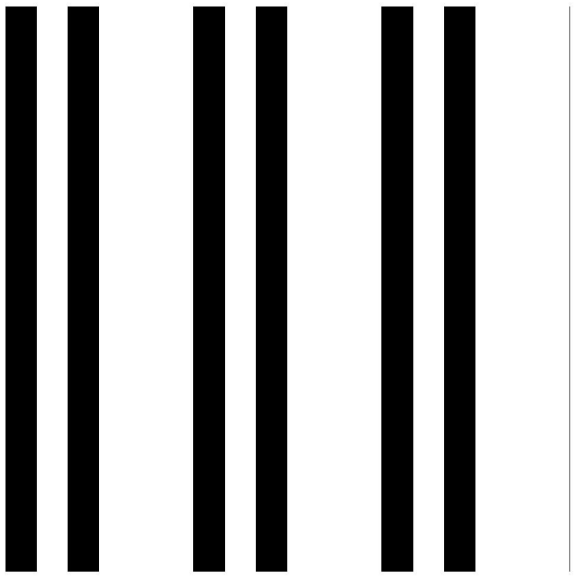


$Z^{141} + Z^{143}$

Z^{141}



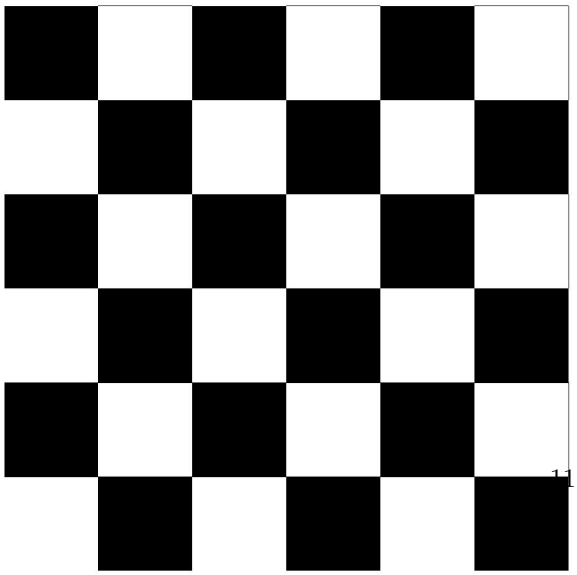
Z^{143}



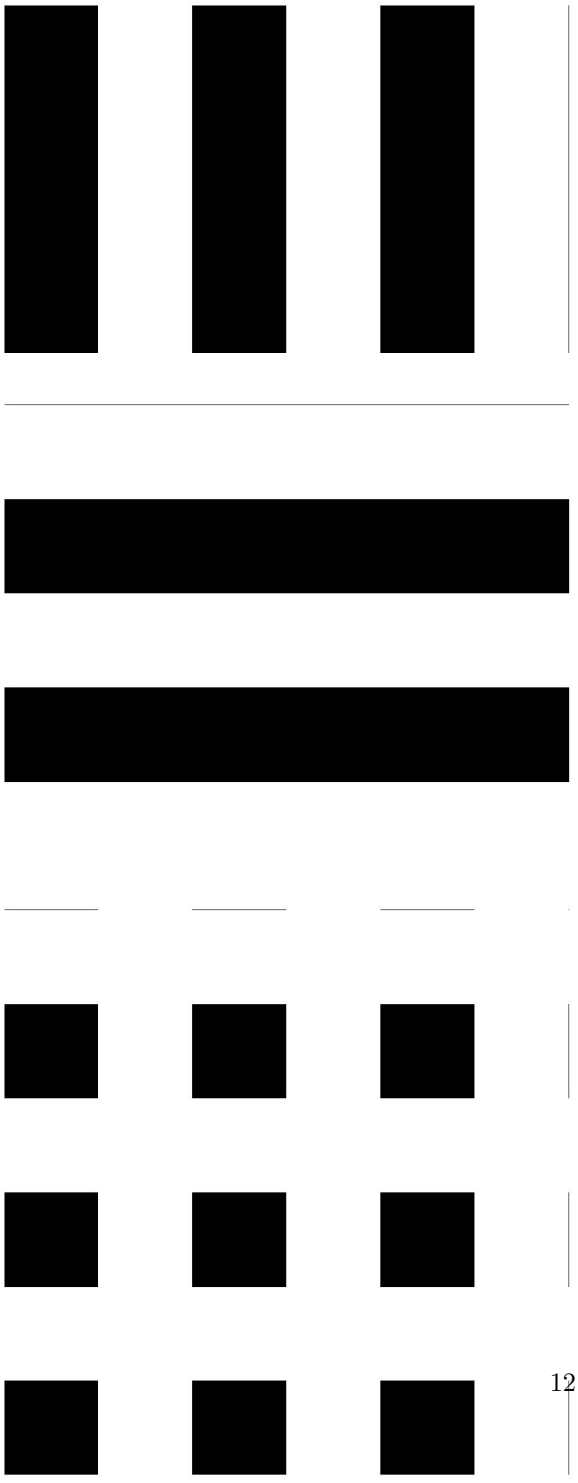
$Z^{141} \times Z^{143}$

Z^{141}

Z^{121}



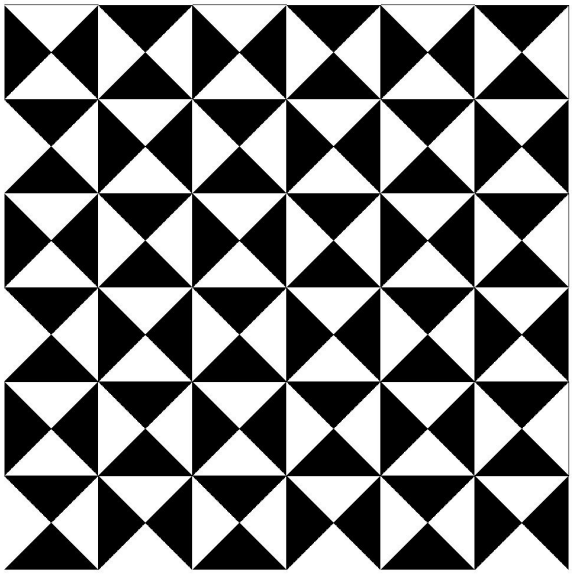
$Z^{121} + Z^{141}$



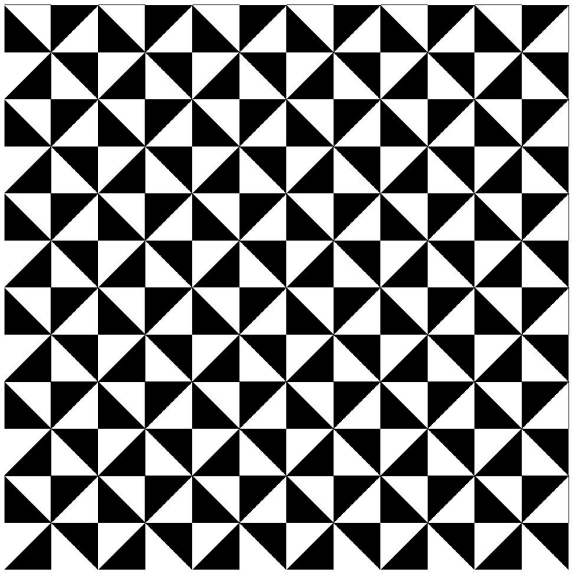
Z^{141}

Z^{121}

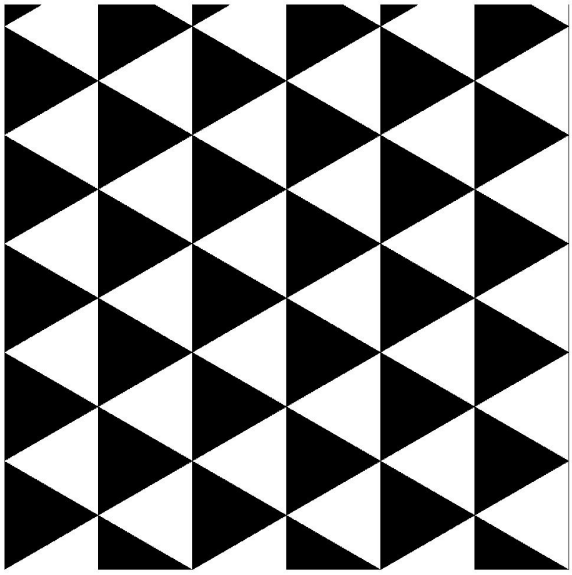
$Z^{121} \times Z^{141}$



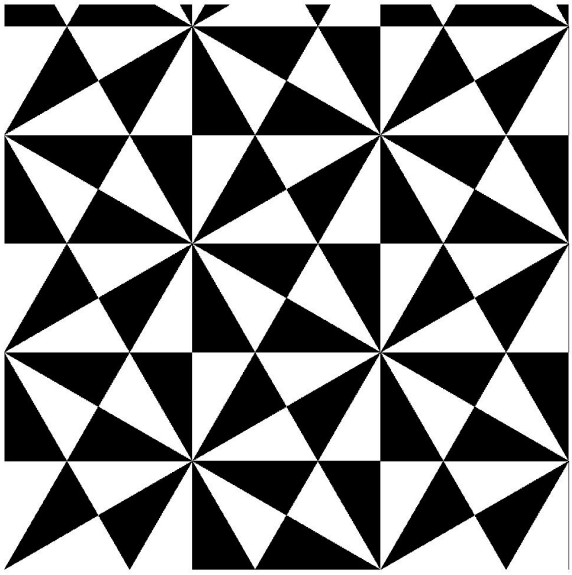
$$Z^{111} + Z^{121} + Z^{131} + Z^{141}$$



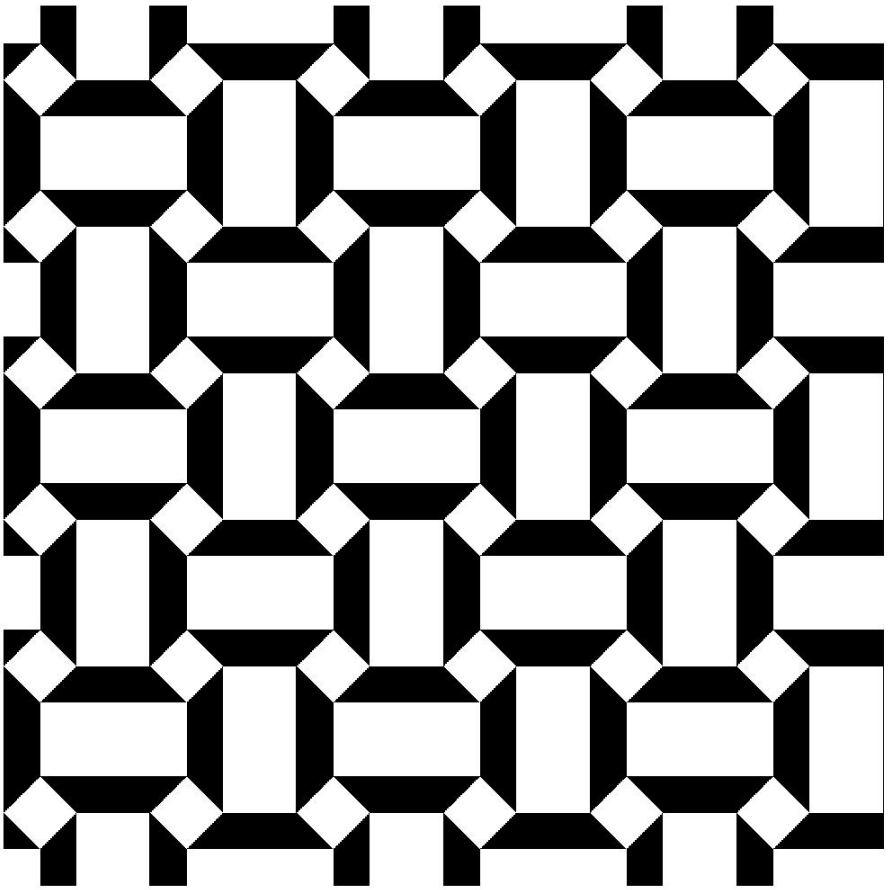
$$Z^{111} + Z^{122} + Z^{131} + Z^{142}$$



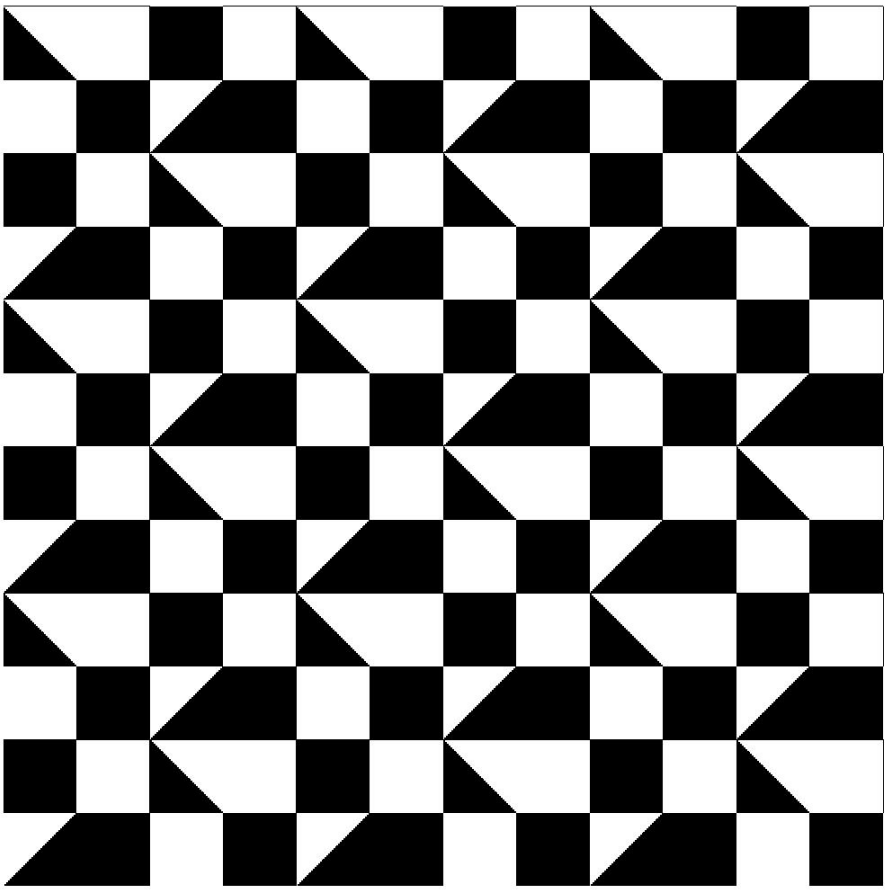
$$Z^{221} + Z^{241} + Z^{261}$$



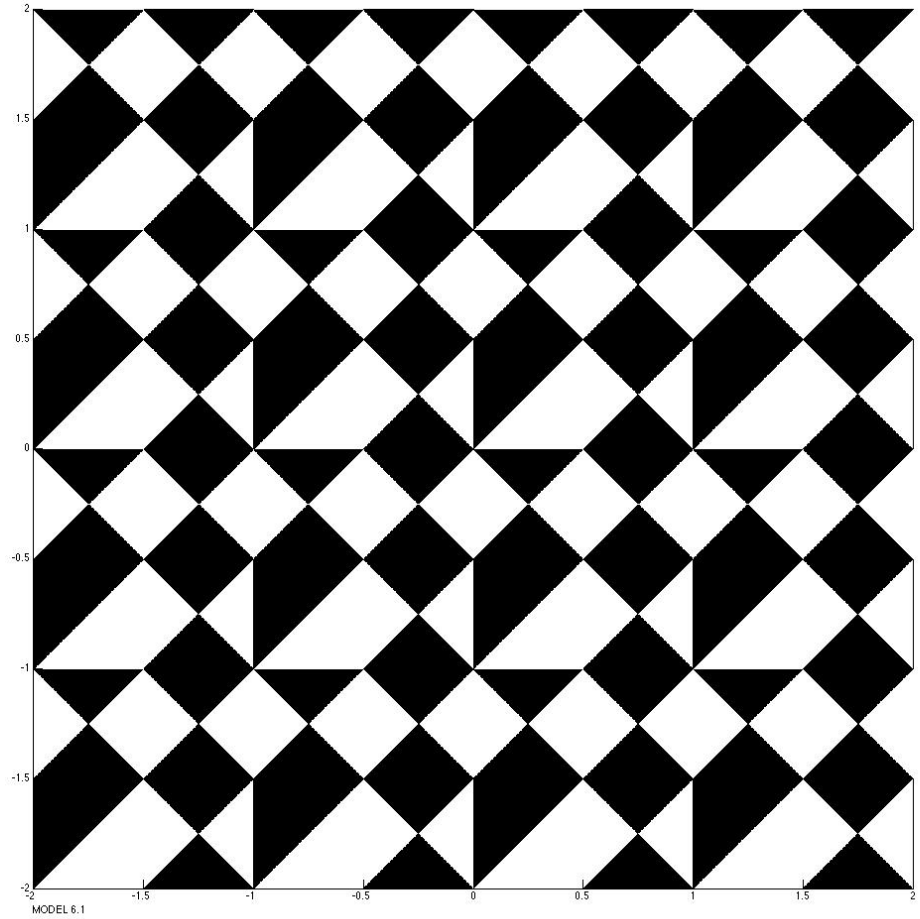
$$Z^{211} + Z^{221} + Z^{231} + Z^{241} + Z^{251} + Z^{261}$$



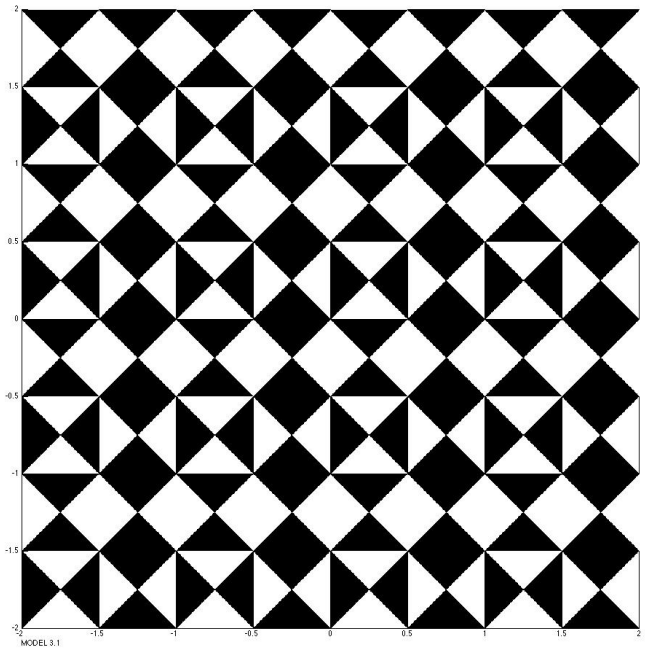
$$
\begin{aligned}
Z^{124} &+ Z^{121}Z^{142} + Z^{141}Z^{142} + Z^{122}Z^{142} \\
&+ Z^{122}Z^{141} + Z^{121}Z^{122} + Z^{124}Z^{141} \\
&+ Z^{121}Z^{124} + Z^{122}Z^{124} + Z^{142}Z^{144} \\
&+ Z^{112}Z^{122}Z^{142} + Z^{122}Z^{132}Z^{142} \\
&+ Z^{112}Z^{124}Z^{142} + Z^{124}Z^{132}Z^{142} \\
&+ Z^{122}Z^{142}Z^{144} + Z^{124}Z^{142}Z^{144}
\end{aligned}
$$



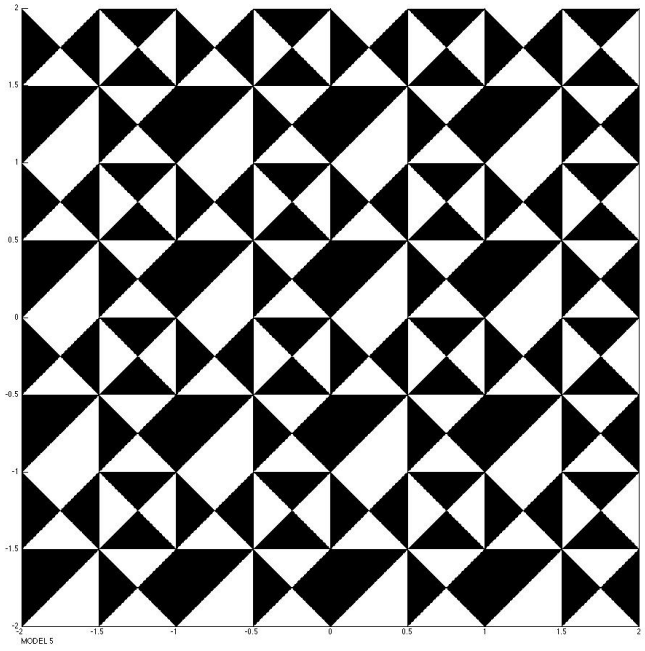
$$
\begin{aligned}
& Z^{122} + Z^{142} \\
& + Z^{131} Z^{141} Z^{142} + Z^{111} Z^{141} Z^{142} \\
& + Z^{121} Z^{131} Z^{142} + Z^{111} Z^{121} Z^{142} \\
& + Z^{122} Z^{131} Z^{142} + Z^{111} Z^{122} Z^{142} \\
= \\
& Z^{122} + Z^{142} \left(1 + (Z^{111} + Z^{131})(Z^{121} + Z^{122} + Z^{141})\right)
\end{aligned}
$$



$$Z^{111} + Z^{131} + Z^{121}Z^{131}Z^{141}$$



$$Z^{111} + Z^{131} + Z^{121}Z^{141}$$



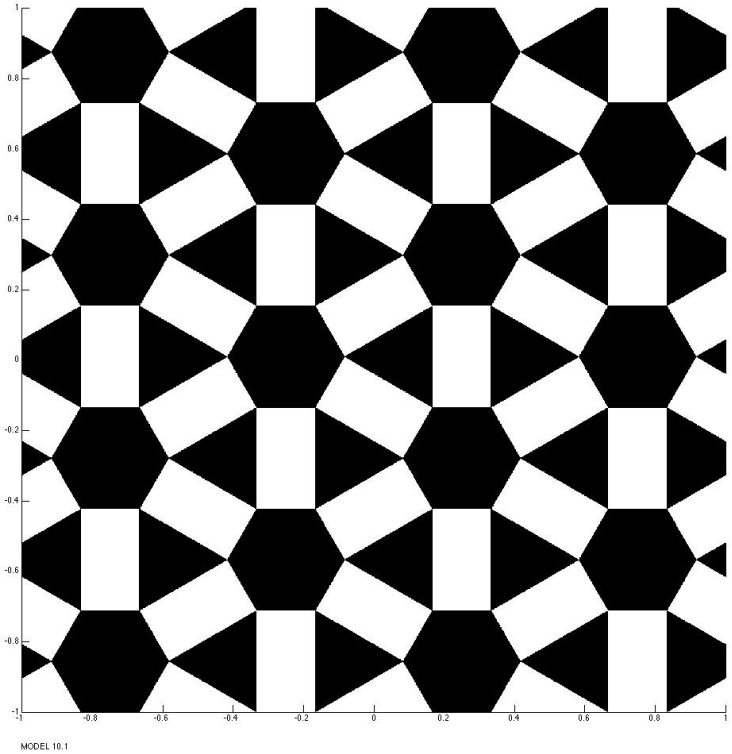
$$Z^{111} + Z^{121} + Z^{131} + Z^{141} + Z^{121}Z^{131}Z^{141}$$

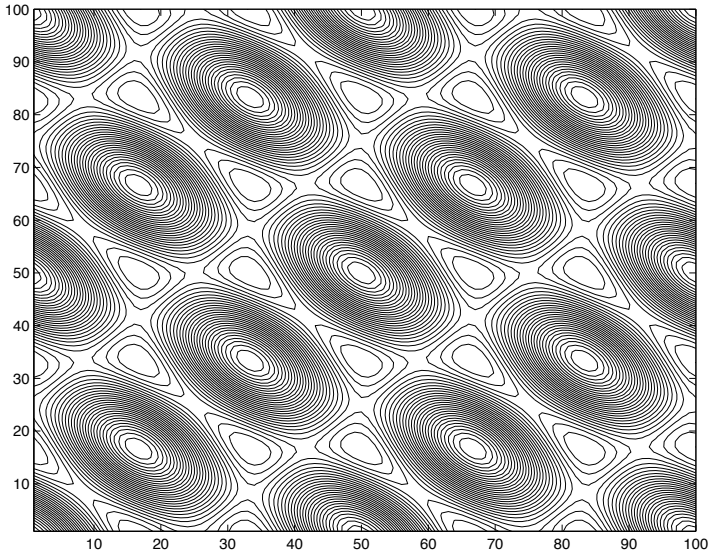
$$Z^{111} + Z^{121} + Z^{131} + Z^{141} + Z^{121}Z^{131}Z^{141}$$

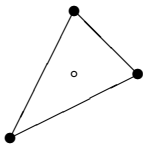
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
σ_0	11	3	17	7	10	13	18	5	15	8	14	9	19	16	12	1	2	4	6
σ_1	3	12	7	17	4	10	1	15	18	14	8	19	9	6	11	5	16	13	2

ZPKAST =

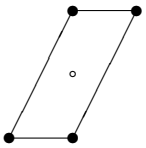
sign	edge	black	white	source	target
−	1	3	5	6	1
−	2	4	4	4	1
+	3	4	5	1	5
−	4	5	3	2	5
+	5	1	3	6	2
−	6	6	6	4	2
+	7	5	5	5	6
−	8	1	1	3	6
−	9	2	2	7	6
+	10	1	6	2	3
+	11	3	1	1	3
+	12	2	4	1	7
+	13	6	2	2	7
+	14	3	6	3	4
+	15	2	1	6	1
+	16	3	3	4	6
+	17	4	3	5	4
+	18	5	2	6	2
+	19	6	4	7	4


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
MODEL 10.1

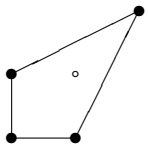

100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100



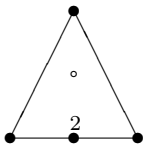
$B_1 : 3$
HS16



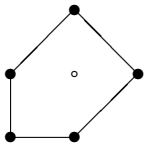
$B_4 : 4, 5$
HS15: a, b



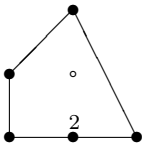
$B_2 : 4$
HS14



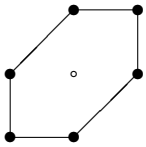
$B_3 : 4$
HS13



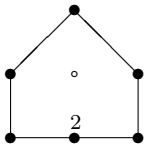
$B_6 : 5, 6$
HS12: a, b



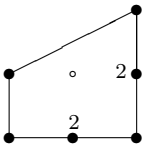
$B_5 : 5$
HS11



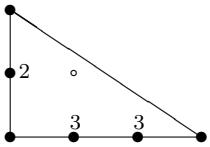
$B_7 : 6, 7, 8, 11$
HS10: a, b, c, d



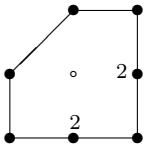
$B_8 : 6, 7, 8$
HS9: a, b, c



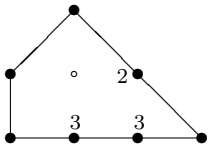
$B_9 : 6, 7$
HS8: a, b



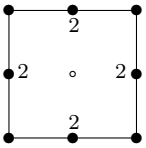
$B_{10} : 6$
HS7



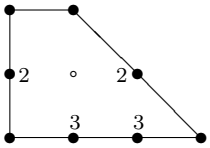
$B_{11} : 9, 9, 12$
HS6: a, b, c



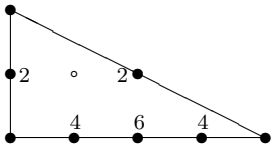
$B_{12} : 9$
HS5



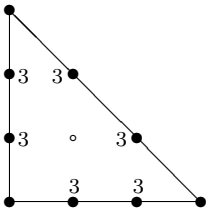
$B_{14} : 12, 12, 14, 21$
HS4: a, b, c, d



$B_{15} : 12, 14$
HS3: a, b



$B_{13} : 12$
HS2



$B_{16} : 21$
HS1