Strongly-interacting Bose gases

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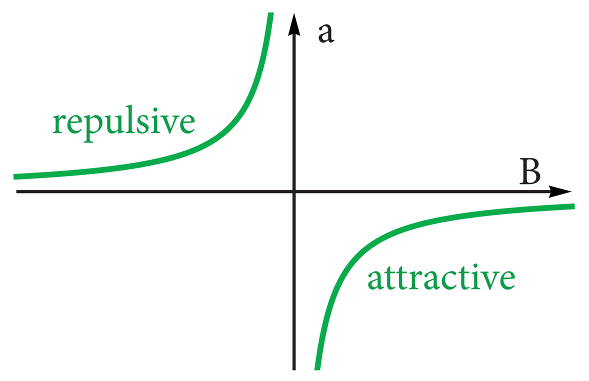
Synopsis

We present an analytical approach to describe the crossover of an atomic Bose gas from weak to strong two-body interactions, which can be systematically improved with renormalization-group methods and reduces to the Bogoliubov theory in the weak-coupling limit.

Strong interactions

In these systems also strong two-body interactions can be achieved using a so-called Feshbach resonance, which allows the interaction strength "a" to be tuned by an externally applied magnetic field.



The interaction strength "a" as a function of the external magnetic field "B".

Many-body corrections

To describe strong interactions many-body corrections need to be calculated, however, these corrections give rise to troublesome logarithmic divergencies due to the linear (sound) mode in the excitation spectrum.

Renormalized bosonization

To describe a strongly-interacting Bose gas we get rid of the troublesome logarithmic corrections by isolating the phase fluctuations of the theory by writing the field as

$$\phi(\mathbf{x},\tau) = \sqrt{n_0(\mathbf{x},\tau)}\exp\left[i\theta(\mathbf{x},\tau)\right] + \phi'(\mathbf{x},\tau),$$

where ϕ' explicitely does not contain phase fluctuations. This extraction of the phase is similar to *bosonization* for fermions.

The propagator of the non-phase fluctuations ϕ' is given by that of Bogoliubov theory without phase fluctuation contributions and is used to systematically improve the action of a Bose gas using *renormalization* group.

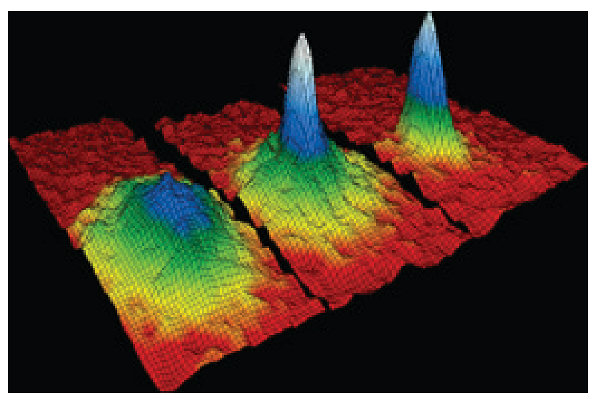
In the end, the contribution of the phase fluctuations is re-introduced using the exact form of the phase fluctuation propagator.

Important properties of the theory

- It avoids troublesome logarithmic divergencies due to phase fluctuations
- The theory can be systematically improved using renormalization group
- It reduces to the Bogoliubov theory in the weak-coupling limit
- The exact single-particle propagator in the long-wavelength limit as derived by [3] can be reproduced
- All quantities, such as the chemical potential, speed of sound, contact, bound-state energy, and the many-body recombination rate, are found as a function of two-body interaction strength
- Generalization to non-zero temperature is straightforward
- It might be applicable to other systems with broken continuous symmetries

Bose-Einstein condensation

At low temperatures a gas of bosons will form a Bose-Einstein condensate, i.e., the particles macroscopically occupy the lowest energy state. This was first achieved in weakly-interacting dilute cold atomic gases confined in a harmonic magneto-optical trap [1].



The momentum-space distribution as the temperature is lowered.

Universality

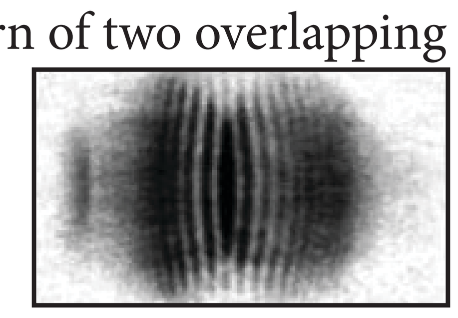
When the two-body interaction strength diverges, no length scale besides the interparticle distance is available and the gas is expected to display universal behavior, i.e., it is independent of the particle species and the details of the scattering processes. For example, the chemical potential is then given by

$$\mu = (1+\beta)\epsilon_F,$$

with β a universal constant and the Fermi energy ϵ_F is solely related to the density of the gas and the mass of the particles, as in its fermionic definition.

Phase fluctuations

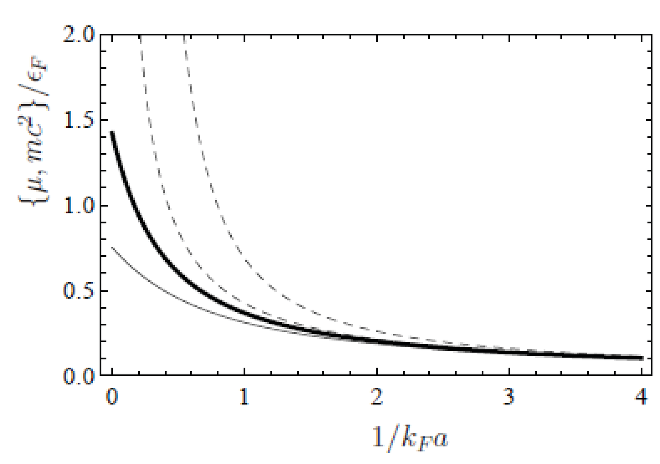
Similar to magnets, where a magnetization direction is spontaneously chosen below a certain temperature, a Bose-Einstein condensate has a fixed phase, as is verified by the interference pattern of two overlapping BEC's [2]:



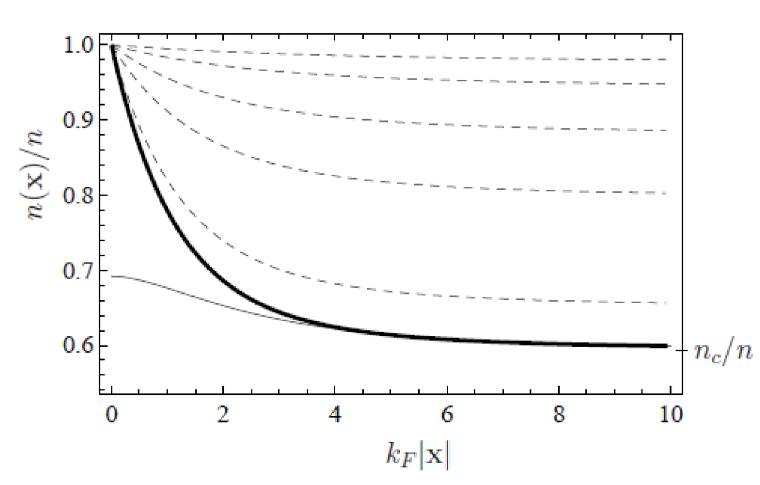
The low-energy excitations of these systems correspond to fluctuations in the spins or phase, respectively.
The linear mode in the excitation spectrum can be identified with the fluctuations in the phase of the BEC.

Application of the theory

As a first application of the general framework, which we call renormalized bosonization, we have determined several quantities of the Bose gas as a function of the two-body interaction strength.



The chemical potential (thick), speed of sound (thin) and their weak-coupling results (dashed).



The universal one-particle density matrix (thick), its contributions without non-phase fluctuations (thin), and for several finite interaction strengths (dashed).

Here the suitable approximation of a momentum and frequency independent effective interaction was used, which is determined from the renormalization group equations.

[1]: E.A. Cornell & C.E. Wieman, NIST/JILA/CU-Boulder
[2]: M.R. Andrews, et. al., Science 275, 637-641 (1997)
[3]: Y. Nepomnyashchii and A. A. Nepomnyashchii, JETP 48, 493 (1978)