As a first application of the general framework, which we call renormalized bosonization, we have determined several quantities of the Bose gas as a function of the two-body interaction strength. Here the suitable approximation of a momentum and frequency independent effective interaction was used, which is determined from the renormalization group equations.

- It avoids troublesome logarithmic divergencies due to phase fluctuations.
- The theory can be systematically improved using renormalization group methods.
- It reduces to the Bogoliubov theory in the weak-coupling limit.
- The exact single-particle propagator in the long-wavelength limit as derived by [3] can be reproduced.
- All quantities, such as the chemical potential, speed of sound, bound-state energy, and the many-body recombination rate, are found as a function of two-body interaction strength.
- Generalization to non-zero temperature is straightforward.
- It might be applicable to other systems with broken continuous symmetries.

At low temperatures a gas of bosons will form a Bose-Einstein condensate, i.e., the particles macroscopically occupy the lowest energy state. This was first achieved in weakly-interacting dilute cold atomic gases confined in a harmonic magneto-optical trap [1].

When the two-body interaction strength diverges, no length scale besides the interparticle distance is available and the gas is expected to display universal behavior, i.e., it is independent of the particle species and the details of the scattering processes. For example, the chemical potential is then given by

$$\mu = (1 + \beta)\epsilon_F$$

where $\beta$ is a universal constant and the Fermi energy $\epsilon_F$ is solely related to the density of the gas and the mass of the particles, as in its fermionic definition.

Similar to magnets, where a magnetization direction is spontaneously chosen below a certain temperature, a Bose-Einstein condensate has a fixed phase, as is verified by the interference pattern of two overlapping BEC's [2]:

The low-energy excitations of these systems correspond to fluctuations in the spins or phase, respectively. The linear mode in the excitation spectrum can be identified with the fluctuations in the phase of the BEC.

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