Quantum trajectory pictures of laser cooling

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We have applied the method of single atom trajectories to study the mechanism behind some cooling schemes in laser cooling. In several cases we recognize the cooling mechanism as being due to a "Sisyphus" process, where the atoms move in a spatially varying light shift potential and are optically pumped towards the most light shifted states. In other cases we identify a "Sisyphus" process in time, where the light shift is constant and the force on the atom alternates between positive and negative. This process is interrupted by quantum jumps at random instants and in each case we depict the mechanism leading to a cooling force on the atom. In the special case of sub-Doppler laser cooling in a strong magnetic field we obtain 12 jump operators and identify the jump operators responsible for the cooling. The versatility of the single atom trajectory method allows it to be applied to any cooling process and is therefore a very valuable tool in unraveling the physical mechanisms behind cooling processes.

I. INTRODUCTION

Simulation methods of quantum mechanical open systems are based upon the representation of the system's density matrix $\sigma(t)$ as an ensemble $\{|\ \psi_i(t)>\}$ of pure state vectors. Each of these time-dependent state vectors may be thought of as a possible history of the evolution of the system. The requirement is, of course, that ensemble averages of physical quantities evaluated on the basis of such a history, reproduce the results obtained by using the density matrix.

A prototype case of an open system in quantum optics is a two-state atom coupled to the continuum of empty modes of the radiation field, which gives rise to spontaneous decay. Then the density matrix of the total system consisting of the atom and the field modes can be written as a sum of terms, each corresponding to a specific number of photons. Each of these terms represents a pure state of the atom. This expansion of the state in photon numbers has been applied by Mollow [1] to the study of the spectrum of resonance fluorescence, and by Cook [2] to the number statistics of fluorescent photons.

The recent renewed interest in the representation of the solution of a quantum master equation as an ensemble of pure states arises in part from the advantage of numerically simulating an n-dimensional state vector over the evaluation of an $n \times n$ density matrix in cases where the number of states n is large. This has led to the introduction of Monte Carlo simulations of the ensemble of atomic wave functions, where spontaneous emission is described in terms of quantum jumps [3–5]. The method has been applied also to the evaluation of the spectrum of resonance fluorescence of atoms in an optical molasses [6]. The same technique is shown to be applicable to a general class of master equations, describing a small system coupled to a large reservoir [7]. Apart from the computational advantage of the method of quantum trajectories of open systems, it also provides a new insight in the physics of the processes involved, in particular of the statistical nature of the evolution of open systems that is added to the normal probabilistic character of the quantum mechanical description. For complex processes in quantum optics, such as sub-Doppler laser cooling, this can be important, in particular in situations where a clear physical picture of the cooling mechanism is lacking. Also it provides a means to test whether the adopted qualitative picture really applies. For example, the laser cooling process of velocity-selective coherent population trapping has been simulated by various ensembles of trajectories with quantum jumps [8].

In the present paper, we evaluate the quantum trajectories based on the picture of quantum jumps for several situations of sub-Doppler cooling. We consider an atom moving with a given classical velocity v through the light field. The quantum trajectories only refer to the internal state of the atom. Both the case of polarization gradients [9] and magnetically induced cooling [10] will be considered.

II. STRUCTURE AND SIMULATIONS OF THE MASTER EQUATION

A. Separation of the density matrix in pure states

A system undergoing dissipative relaxation due to its coupling to a reservoir is commonly described by a density matrix σ that obeys a master equation of the form

$$\frac{d}{dt}\sigma = -\frac{i}{\hbar}[H_0, \sigma] + \sum_\lambda \left(C_\lambda \sigma C_\lambda^\dagger - \frac{1}{2}C_\lambda^\dagger C_\lambda \sigma - \frac{1}{2}\sigma C_\lambda^\dagger C_\lambda \right), \tag{2.1}$$

with H_0 the Hamiltonian of the uncoupled system, and C_λ are jump operators describing the coupling to the reservoir. This form of the Master equation is rather generally valid when the evolution of an open system has a Markovian nature [11]. In the standard case of a two-state atom undergoing spontaneous decay, only a single type of operator C occurs, which is equal to $S_- \sqrt{\Gamma}$, with Γ the spontaneous decay rate, and $S_- = | g ><e |$ the atomic lowering operator, coupling the excited state $| e >$ to the ground state $| g >$ [12]. Another simple case is the decay of a cavity mode, where C must be taken proportional to the annihilation operator. The Hamiltonian H_0 may contain external fields driving the system, and it can be time-dependent. In general, master equations of the type of (2.1) arise when the coupling to the reservoir can be described by fast disruptive events, for which the operators C_λ are the transition operators [13].

The Monte Carlo simulation of the solution σ of Eq. (2.1) in terms of an ensemble of pure states executing quantum jumps has been discussed in refs. [3–5]. Equation (2.1) can be expressed in the alternative form

$$\frac{d}{dt}\sigma = -\frac{i}{\hbar}H\sigma + \frac{i}{\hbar}\sigma H^\dagger + G\sigma, \tag{2.2}$$

where

$$H = H_0 - iH_1 \tag{2.3}$$

is an effective non-Hermitian Hamiltonian, with

$$H_1 = \sum_\lambda \frac{1}{2}\hbar C_\lambda^\dagger C_\lambda. \tag{2.4}$$

The (super)operator G, which acts on density matrices, is defined by

$$G = \sum_\lambda G_\lambda, \tag{2.5}$$

with

$$G_\lambda \sigma = C_\lambda \sigma C_\lambda^\dagger. \tag{2.6}$$

When σ represents the state of the system before a jump, the state after the jump is proportional to $G\sigma$. Hence, the operator G represents the gain term in the master equation. A simple way to represent the solution of the master equation as a statistical mixture of pure states is to write the density matrix $\sigma(t)$ as a Dyson expansion in powers of G. This gives

$$\sigma(t) = U(t,0)\sigma(0) + \int_0^t dt_1 U(t,t_1)GU(t_1,0)\sigma(0)$$
$$+ \int_0^t dt_2 \int_0^{t_2} dt_1 U(t,t_2)GU(t_2,t_1)GU(t_1,0)\sigma(0) + \ldots, \tag{2.7}$$

The evolution operator U is defined by its action on a density matrix σ as

$$U(t,t_0)\sigma = u(t,t_0)\sigma u^\dagger(t,t_0), \tag{2.8}$$

with u the Schrödinger evolution operator corresponding to the Hamiltonian H. Hence, u obeys the equation of motion

$$\frac{d}{dt}u(t,t_0) = -\frac{i}{\hbar}H(t)u(t,t_0), \tag{2.9}$$

with the initial condition $u(t_0,t_0) = 1$. As H is not Hermitian, the operator u is not unitary. Equation (2.7) depicts the evolution of the system as a continuous evolution determined by the operator u, interrupted at discrete instants of time by a jump, described by G. The first term in Eq. (2.7) gives the contribution to $\sigma(t)$ corresponding to the case that no jumps occur in the time interval $[0,t]$, the second term describes the case that a single jump occurs, etc. This illustrates the nature of the jumps as random events. In the case of an atom driven by an external radiation field, Eq. (2.7) is a convenient starting point to study the statistics of fluorescent photons [14]. When the initial state is a pure state, we may write

$$\sigma(0) = \mid \psi(0) > < \psi(0) \mid . \tag{2.10}$$

Then if we replace the gain operator G by the summation (2.6), one easily checks that each integrand in the expansion in (2.7) is a summation over pure states. For instance, the first integrand is equal to

$$U(t,t_1) \ G \ U(t_1,0)\sigma(0)$$
$$\sum_\lambda u(t,t_1)C_\lambda u(t_1,0) \mid \psi(0) > < \psi(0) \mid u^\dagger(t_1,0)C_\lambda^\dagger u^\dagger(t,t_1). \tag{2.11}$$

Hence, Eq. (2.11) defines the pure states in which the state $\sigma(t)$ can be separated.

B. Simulations of pure-state histories

The simulation of single histories $\mid \psi(t) >$ of normalized pure states as introduced in refs. [4] and [5] is now obvious. The probability $P_0(t,0)$ that no jump occurs in the time interval $[0,t]$ is given by the trace of the first term on the right-hand side of (2.7). For the initial state (2.10), this is equal to

$$P_0(t,0) = < \psi(0) \mid u^\dagger(t,0)u(t,0) \mid \psi(0) > . \tag{2.12}$$

This probability can be evaluated as a (monotonously decreasing) function of t. The decay of P_0 is due to the anti-Hermitian part H_1 of the effective Hamiltonian. Since the complement $W(t,0) = 1 - P_0(t,0)$ is the probability that the first jump occurs before the time t, the probability distribution for the time intervals that one has to wait for the first jump is

$$w(t \mid 0) = \frac{d}{dt}W(t,0) = -\frac{d}{dt}P_0(t,0). \tag{2.13}$$

This waiting-time distribution can also be expressed as

$$w(t \mid 0) = \sum_\lambda w_\lambda(t \mid 0), \tag{2.14}$$

with

$$w_\lambda(t \mid 0) = < \psi(0) \mid u^\dagger(t,0)C_\lambda^\dagger C_\lambda u(t,0) \mid \psi(0) > . \tag{2.15}$$

The physical significance of the partial waiting-time distribution w_λ is that $w_\lambda(t_1 \mid 0)dt_1$ is the probability that the first jump after time zero occurs between the times t_1 and $t_1 + dt_1$, and that it is of the type λ. In general, the probability $P_0(t,0)$ of zero jumps, and the waiting-time distribution function $w(t \mid 0)$ depend on the initial state $\mid \psi(0) >$.

The time instant t_1 of the first jump is now simulated by drawing a random number ξ, homogeneously distributed between 0 and 1, and determining t_1 by $\xi = W(t_1,0)$. This is verified by noticing that for a homogeneous distribution of ξ, the density of points t_1 is equal to $w(t_1 \mid 0)$. Up to the jump instant t_1, the system is described by the normalized pure state

$$\mid \psi(t) >= \frac{1}{\sqrt{P_0(t,0)}}u(t,0) \mid \psi(0) >, \tag{2.16}$$

where the normalization factor follows from (2.12).

After determining the instant t_1 of the first jump, we must decide the type of this jump. The probability that it is of type λ_1 is

$$p_{\lambda_1} = w_{\lambda_1}(t_1 \mid 0) / \left[\sum_\lambda w_\lambda(t_1 \mid 0) \right], \qquad (2.17)$$

and a second random number is needed to determine λ_1. The normalized state vector after this first jump at time t_1 is

$$\mid \psi(t_1) > = \frac{1}{\sqrt{w_{\lambda_1}(t_1 \mid 0)}} C_{\lambda_1} u(t_1, 0) \mid \psi(0) > . \qquad (2.18)$$

The normalization factor in (2.18) follows from (2.15). This final state after the first jump serves as the initial state for a jumpless evolution until the instant t_2 of the second jump, that is to be determined in the same fashion. In this way ensembles of pure-state histories can be created, which correspond to the result of a *Gedanken* measurement, in which the time instants and the types of the jumps are continuously recorded. The only stochastic input that is needed is just the outcome of this measurement, which consists of the instants and the types of the jumps. With this information, the pure state $\mid \psi(t) >$ is fully determined. Apart from normalization, the evolution in between jumps is governed by the operator u, and the effect of a jump of type λ is given by the action of the jump operator C_λ.

By repeating this procedure, one can create an ensemble of N such histories $\mid \psi_1(t) >, \mid \psi_2(t) >, \ldots$. This ensemble $\{\mid \psi(t) >\}$ can be used to evaluate expectation values $< Q(t) >$ of any physical quantity, according to the prescription

$$< Q(t) > = \frac{1}{N} \sum_{i=1}^{N} < \psi_i(t) \mid Q \mid \psi_i(t) > . \qquad (2.19)$$

For large values of N, the results are indistinguishable from the result $\text{Tr}\sigma(t)Q$, determined with the solution $\sigma(t)$ of the master equation. This is explicitly demonstrated in ref. [7]. We point out that this is obvious already from the Dyson expansion (2.7).

In summary, a single pure-state history is obtained by simulating successively the time instants and the types of the jumps. The series $(t_1, \lambda_1), (t_2, \lambda_2), \ldots$ then determines the pure state $\mid \psi(t) >$, which for $t_k < t < t_{k+1}$ can be expressed as

$$\mid \psi(t) > = [\mathcal{N}(t)]^{-1/2} u(t, t_k) C_{\lambda_k} u(t_k, t_{k-1}) \ldots C_{\lambda_1} u(t_1, 0) \mid \psi(0) > \qquad (2.20)$$

with $\mathcal{N}(t)$ a time-dependent normalization constant. The simulation of (t_i, λ_i) can be drawn as soon as the normalized state $\mid \psi(t_{i-1}) >$ is known.

C. Evolution between jumps

Equation (2.20) demonstrates that when we ignore the normalization, the time dependence of a single history $\mid \psi(t) >$ is simply determined by the action of the evolution operator u, interrupted by the action of C_λ at the jump instants. Without normalization, the evolution would be linear. However, the normalization is essential in the final step, where the ensemble is used to evaluate physical averages, in order to give each history its proper weight. For a specific outcome of the continuous measurement, it is the normalized state vector $\mid \psi(t) >$ that describes the actual history of the system.

The continuous evolution in between two jumps can be described as in Eq. (2.16). The differential form of this equation yields the differential equation [7]

$$\frac{d}{dt} \mid \psi > = -\frac{i}{\hbar} H_0 \mid \psi > -\frac{1}{\hbar} H_1 \mid \psi > +\frac{1}{\hbar} \mid \psi > < \psi \mid H_1 \mid \psi >, \qquad (2.21)$$

where the last term is needed to conserve the normalization. It is remarkable that this evolution equation, which describes the actual state during a single run, is nonlinear. The expectation value

$$< Q(t) > = < \psi(t) \mid Q \mid \psi(t) > \qquad (2.22)$$

of a physical quantity Q obeys the corresponding differential equation

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$$\frac{d}{dt} < Q > = \frac{i}{\hbar} < [H_0, Q] > - \frac{1}{\hbar} < (H_1 Q + Q H_1 > + \frac{2}{\hbar} < H_1 > < Q > . \tag{2.23}$$

The last two terms in (2.23) express the correlation between the quantity Q and the non-Hermitian part H_1 of the effective Hamiltonian. Only in the presence of such correlation does the evolution equation deviate from the result for a closed system. This may be understood as resulting from the information that is obtained from the null result of the *Gedanken* measurement of the jumps. It is noteworthy that the evolution equation (2.23) implies that energy or momentum is not always conserved during a single history.

III. TWO-STATE ATOM IN TRAVELLING WAVE

In this section we briefly discuss the quantum-trajectory picture in the simplest possible case of a two-state atom in a single travelling wave with wave vector $\vec{K}$. It is well-known that in this case the net force on the atom in the steady state is simply $\hbar \vec{K} \Gamma \sigma_{ee}$ [2], with σ_{ee} the constant population of the excited state. This corresponds to the picture of a constant rate of photon scattering by the atom, with a momentum change of $\hbar \vec{K}$ per scattered photon. Here we wish to point out that the picture arising from single quantum trajectories is slightly different. We make the dipole and rotating-wave approximation, and describe the atomic density matrix $\sigma(t)$ in a frame rotating with the light frequency ω. In the atomic rest frame, the Hamiltonian H_0 of the driven atom is

$$H_0 = -\hbar \Delta S_z - \hbar \Omega S_x, \tag{3.1}$$

with Ω the Rabi frequency, and $\Delta = \omega - \omega_0$ the detuning of the light frequency from resonance. The quasi spin operators have their usual significance $S_z = [P_e - P_g]/2$ and $S_x = [S_+ + S_-]/2$, with

$$P_e = |e><e|, \quad P_g = |g><g|, \quad S_+ = |e><g|, \quad S_- = |g><e|. \tag{3.2}$$

The master equation (2.1) reproduces the well-known optical Bloch equations, when we substitute Eq. (3.1) for H_0, and the single jump operator C_λ is $S_- \sqrt{\Gamma}$. In the present case of a single travelling wave, the force exerted by the radiation field on the atom is given by [15]

$$\vec{F} = \hbar \vec{K} A, \tag{3.3}$$

where

$$A = \frac{1}{2} i \Omega < S_+ - S_- > . \tag{3.4}$$

The quantity A is the power transferred from the beam to the atomic dipole, measured in photon energies, which is equal to the rate of photon absorption.

Now we turn to the picture of a single pure-state trajectory. For the internal evolution of a two-state atom, this picture has been discussed in ref. [4]. We represent the pure state in terms of two amplitudes as

$$|\psi(t)> = a_e(t) |e> + a_g(t) |g> . \tag{3.5}$$

The force on the atom during this single history is given by (3.3), where the photon absorption rate (3.4) is

$$A(t) = \frac{1}{2} i \Omega [a_e^* a_g - a_g^* a_e]. \tag{3.6}$$

The time dependence of the state vector (3.5) is given by the continuous evolution (2.21), disrupted at discrete stochastic time instants by a quantum jump. In the present case, the decay Hamiltonian H_1 is

$$H_1 = \frac{1}{2} \hbar \Gamma P_e. \tag{3.7}$$

The jumps occur at a rate $\Gamma |a_e|^2$, and simply reduce the atom to its ground state. Equation (2.21) for the continuous evolution between jumps gives the nonlinear evolution equations

$$\frac{d}{dt} a_e = \frac{1}{2} \left[i\Delta a_e - i\Omega a_g - \Gamma a_e |a_g|^2 \right],$$
$$\frac{d}{dt} a_g = \frac{1}{2} \left[-i\Delta a_g - i\Omega a_e + \Gamma a_g |a_e|^2 \right]. \tag{3.8}$$

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The terms proportional to Γ result from the coupling to the vacuum field, which leads to decay of the excitation even in the period between two spontaneous emissions. For the solution $\sigma(t)$ of the optical Bloch equations, the time derivative of the excited-state population is equal to the absorption rate. This is an expression of energy conservation. However, in the present case of the evolution in between two jumps we find from (3.8) that

$$\frac{d}{dt} \mid a_e \mid^2 = A(t) - \Gamma \mid a_e \mid^2 \mid a_g \mid^2, \tag{3.9}$$

so that the energy gain of the atom is smaller than the energy loss of the field during this evolution. This temporary loss of energy is regained on average during the subsequent jump, which can take place also when $\mid a_e \mid^2$ is less than 1.

In Fig. 1 we plot for a single trajectory the value of the excited-state population $\mid a_e \mid^2$ and the value of the force F, which is proportional to the absorption rate $< A(t) >$ according to Eq. (3.3). At $t = 0$ the atom is in the ground state and starts to absorb radiation, which causes the force to become negative. The population then oscillates back and forth between the ground and excited state and the force alternates between negative and positive. In the absence of quantum jumps the force becomes on the average zero. However, after a certain time a quantum jump occurs (indicated in the lower part of Fig. 1) and the atom jumps back to the ground state. Then the cycle starts over again. So after a quantum jump the atom absorbs radiation and the force will always be negative, whereas the quantum jumps occur at random instants, in which case the force before the jump can be either positive or negative. The average force will then always be negative and oppose the atomic motion.

Note that the distribution of positions where a quantum jump occurs is completely uniform, since the intensity and the polarization of the light field are constant. Therefore there is no "Sisyphus" mechanism to explain the force, as is done for instance in the case of sub-Doppler laser cooling in the lin $\perp$ lin configuration (see ref. [9] and below). Consequently we cannot define in this case a spatially dependent light shift for different states and identify the positions at which the atom predominantly jumps. In fact the jumps occur at random instants in time and we would therefore call this mechanism a "Sisyphus" mechanism in time. Although this mechanism seems rather trivial in this case, we will see that this mechanism plays a role in other cooling schemes and that the same features also occur in these cases.

IV. RADIATIVE FORCES ON ATOMS WITH TWO DEGENERATE LEVELS

A. Force and absorption rates

We consider an atom with two degenerate levels in a monochromatic radiation field that is expressed as a superposition of travelling waves. The atom is assumed to follow a classical path $\vec{R}(t)$, so that it experiences a time-dependent field

$$\vec{E}(t) = \sum_n \vec{E}_n^+ \exp[i\vec{K}_n.\vec{R}(t) - i\omega t] + \text{c.c.}. \tag{4.1}$$

The Hamiltonian of the atom coupled to the field is

$$H_0 = H_{\text{at}} - \hbar[\mathcal{R}(t) + \mathcal{R}^\dagger(t)], \tag{4.2}$$

with

$$\mathcal{R}(t) = \sum_n \frac{1}{\hbar} \vec{\mu}_{eg} \cdot \vec{E}_n^+ \exp[i\vec{K}_n.\vec{R}(t)] = \sum_n \mathcal{R}_n(t) \tag{4.3}$$

a Rabi operator that is the sum of partial Rabi operators for each of the beams. Here $\vec{\mu}_{eg}$ is the raising part of the atomic dipole operator, which couples a lower level g to an excited level e, with angular momenta J_g and J_e. Both levels can have magnetic degeneracy. The free-atom Hamiltonian in the rotating frame is

$$H_{\text{at}} = -\frac{1}{2}\hbar\Delta[P_e - P_g], \tag{4.4}$$

with P_e and P_g projection operators on the substates of the two levels. The time dependence of the Hamiltonian H_0 results from the motion of the atom.

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The density matrix σ of the atom obeys the master equation (2.1), with H_0 given by (4.2). Since the ground state can have magnetic degeneracy, the effect of spontaneous decay depends on the polarization of the emitted photon. Therefore for the jump operators C_λ in (2.1) one can take the three spherical components of the lowering part of the dipole. As before [16], we introduce the spherical components Q_β of the dimensionless transition vector, coupling the ground-level substates to the excited states. These operators, which generalize the raising operator S_+ of a two-state atom, are defined by the statement that their matrix elements are Clebsch-Gordan coefficients, so that [16]

$$< J_e M_e \mid Q_\beta \mid J_g M_g > = < J_e M_e \mid J_g M_g; 1\beta > . \tag{4.5}$$

The jump operators C_λ, which transfer excited states to ground states, are then proportional to the Hermitian conjugates of the operators Q_β. Equation (2.1) gives the correct evolution equation for σ if we take

$$C_\beta = Q_\beta^\dagger \sqrt{\Gamma}. \tag{4.6}$$

for $\beta = -1, 0, 1$, with Γ the rate of spontaneous decay. With the substitution (4.6), the anti-Hermitian part H_1 of the Hamiltonian is still given by (3.7), which reflects the isotropy of the decay of the excited state.

In the field (3.1), the radiative force on the atom is

$$\vec{F} = \sum_n \hbar \vec{K}_n A_n, \tag{4.7}$$

where

$$A_n = < i[\mathcal{R}_n - \mathcal{R}_n^\dagger] > \tag{4.8}$$

is the rate of photon absorption from beam n [15].

B. Low-intensity limit

If we substitute Eq. (4.2) and (4.6) in the master equation (2.1), we obtain the equation of motion for the atomic density matrix $\sigma(t)$. This equation may be viewed as a set of coupled linear equations for the four submatrices $\sigma_{ee}, \sigma_{gg}, \sigma_{eg},$ and σ_{ge}. When the atomic velocity $\vec{v}$ is sufficiently low, so that the Doppler shift $\vec{K} \cdot \vec{v}$ is smaller than the decay rate Γ, the imposed time dependence is slow, and the optical coherences σ_{eg} and σ_{ge} can be shown to follow the excited-state and ground-state submatrices σ_{ee} and σ_{gg} adiabatically. When moreover the intensity is so low that the absorption rates are much smaller than the decay rate Γ, the submatrix σ_{ee} for the excited states follows the coherences. This situation of an atom slowly moving through a weak light field is typical for situations of sub-Doppler cooling [9].

In this case, one can derive a closed equation of motion for the ground-state submatrix σ_{gg}, in the form [17]

$$\frac{d}{dt}\sigma_{gg} = -\left(\mathcal{P} + i\mathcal{S}\right)\sigma_{gg} - \sigma_{gg}\left(\mathcal{P} - i\mathcal{S}\right) + \frac{\Gamma}{\Gamma^2/4 + \Delta^2}\sum_\beta Q_\beta^\dagger \mathcal{R}\sigma_{gg}\mathcal{R}^\dagger Q_\beta, \tag{4.9}$$

with the Hermitian operators $\mathcal{P}$ and $\mathcal{S}$ defined by

$$\mathcal{P} + i\mathcal{S} = \frac{1}{\Gamma/2 - i\Delta}\mathcal{R}^\dagger \mathcal{R}. \tag{4.10}$$

The light-shift operator $\mathcal{S}$ has a dispersive dependence on the detuning Δ, whereas the operator $\mathcal{P}$ has a Lorentzian line shape. It is obvious that (4.10) is a master equation of the type (2.1), or, equivalently, (2.2). One recognizes in (4.10) three jump operators

$$C_\beta = \sqrt{\frac{\Gamma}{\Gamma^2/4 + \Delta^2}}Q_\beta^\dagger \mathcal{R}, \tag{4.11}$$

for $\beta = -1, 0, 1$. These operators reduce a substate of the ground level into a different substate. This transfer represents an optical pumping cycle, consisting of absorptive excitation by the light field, and a subsequent spontaneous emission of a photon with polarization specified by β. The effective Hamiltonian H is now given by

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$$H = \hbar(\mathcal{S} - i\mathcal{P}). \tag{4.12}$$

The pure-state histories $| \psi(t) >$ corresponding to this Master equation are now state vectors for the atom in the lower state only. The force on the atom can be expressed as an average

$$\vec{F} = -\frac{i/\hbar}{\Gamma/2 + i\Delta} < \mathcal{R}^\dagger \vec{\nabla}\mathcal{R} > + \text{c.c.}. \tag{4.13}$$

Since both the effective Hamiltonian (4.12) and the jump rates are proportional to the intensity, the ensemble of trajectories $\{| \psi(t) >\}$ obeys a simple scaling law with intensity. When we multiply both the velocity and the intensity by a factor ζ, the resulting single histories $| \psi_\zeta(t) >$ can be found from the original ensemble by the simple substitution

$$| \psi_\zeta(t) > = | \psi(\zeta t) > . \tag{4.14}$$

Hence at a lower intensity, an atom passes through the same histories, but at a slower pace. This result is analogous to the scaling law derived before for the density matrix [17].

C. lin $\perp$ lin

In this section we discuss the single-history simulation for the configuration of two counterpropagating plane waves with orthogonal linear polarization and the same intensity. An equivalent picture is provided by two standing waves with opposite circular polarization σ_+ and σ_-, and which are spatially shifted by a quarter wavelength. Then the Rabi operator $\mathcal{R}$ is given by

$$\mathcal{R} = \frac{1}{\sqrt{2}}\Omega[Q_1 \cos(KZ) + Q_{-1} \sin(KZ)], \tag{4.15}$$

with Ω an effective Rabi frequency. For simplicity we will discuss the case of a $J_g = 1/2$ to $J_e = 3/2$ transition. This is the standard configuration where the Sisyphus mechanism for sub-Doppler cooling has been discussed [9]. We shall demonstrate how the picture changes when one considers single histories. These results also hold for other angular momentum values.

The two lower states $| \pm >$ with $M_g = \pm 1/2$ are eigenstates of the light-shift operator $\mathcal{S}$, with eigenvalues

$$S_\pm = \frac{1}{3}\Delta s_0[2 \pm \cos(2KZ)], \tag{4.16}$$

where

$$s_0 = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} \tag{4.17}$$

is the saturation parameter. The jump operators C_0 transfers an atom in the state $| - >$ to the state $| + >$ at a rate

$$|< + | C_0 | - >|^2 = \frac{2}{9}\Gamma s_0 \cos^2(KZ), \tag{4.18}$$

and the jump rate for the opposite transition is

$$|< - | C_0 | + >|^2 = \frac{2}{9}\Gamma s_0 \sin^2(KZ). \tag{4.19}$$

The other jump operators $C_{\pm 1}$ leave the states $| \pm >$ unchanged, and they play no part in the evolution of a single history. The force in the state $| \pm >$ is given by

$$F_\pm = \pm\frac{2}{3}\hbar\Delta K s_0 \sin(2KZ). \tag{4.20}$$

Hence an atom moving with a given velocity through the light field simply jumps back and forth between the two eigenstates $| \pm >$ of the light-shift operator, so that it has a potential energy $\hbar S_\pm$. In Fig. 2 we have plotted a single quantum trajectory for this situation. The force at any instant during the history can be expressed as a positive

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constant times $\sin\phi$, whereas the light shifted energy of its state is proportional to $\cos\phi$. The effect of a jump of the atomic state is a phase jump of π, which according to (4.16) implies a change in the optical potential equal to

$$\Delta S = -\frac{2\hbar}{3} \mid \Delta \mid s_0 \cos\phi. \tag{4.21}$$

During the motion of the atom it has to climb again an additional potential step, which leads to the net momentum change $\Delta P = \Delta S/v$,

$$\Delta P = -\frac{2}{3v}\hbar \mid \Delta \mid s_0 \cos\phi. \tag{4.22}$$

The histories are fully characterized by the normalized phase distribution $R(v,\phi)$ of the rate of these jumps as a function of the velocity v, which gives rise to the average force

$$F = -\frac{2}{3v}\hbar \mid \Delta \mid s_0 \int d\phi\, R(v,\phi)\cos\phi. \tag{4.23}$$

If the jumps occur at random phases, the average force will be zero. In Fig. 3 we have depicted the situation for the potential energy of the atom for four different values of ϕ.

In Fig. 4 we have plotted for various velocities the distribution over the phase ϕ just before the jumps. For an atom at a very low velocities $v \ll s_0\Gamma/K$, this distribution is proportional to the product of the jump rates (4.18) and (4.19), so that the phase distribution is proportional to $\sin^2\phi$. In Fig. 4a we have plotted the results for an atom with a reduced velocity $w = Kv/s_0\Gamma$ of 0.01. For this low velocity, the distribution of jumps is almost identical to the prediction for an atom at rest (indicated by the dashed line). The atom adiabatically adjusts its populations to the local field. If we increase the velocity, the population lags behind the local field [9]. According to the Sisyphus picture, the probability to be optically pumped from the state $\mid + >$ to the state $\mid - >$ near the antinode of the σ_- standing wave will increase, which leads to a damping force for negative detuning Δ. However, the plots of Fig. 4b and 4c show that the main change is that the distribution $R(\phi)$ attains an asymmetry, which favors jumps at instants that the force is negative over instants with positive values of the force. This demonstrates that the Sisyphus picture does not really apply for velocities below the capture range, which is the region where the cooling is most prominent, and where the limiting temperature is determined. For high velocity, where $v \gg s_0\Gamma/K$ the atomic state has no time to adapt its state during the passage of a wavelength. Then the distribution $R(\phi)$ is simply determined by the rates (4.18) and (4.19), so that it proportional to $-\cos^2(\phi/2)$ for positive Δ, and to $\cos^2(\phi/2)$ for negative Δ. Since the distribution R becomes independent of the velocity, one recognizes with (4.23) that the force scales as $1/v$. In Fig. 5 we compare this high-velocity result with numerical calculations [15] and our MCWF results. The agreement between the high-velocity result and the numerical calculations is good for velocities larger than the capture velocity. The conclusion is that the standard Sisyphus picture applies best in the high-velocity limit.

The simplicity of this picture is due to the fact that the states $\mid \pm >$ are position-independent eigenstates of the Hermitian part H_0 of the Hamiltonian (the light-shift operator), and of the contributions $C_\beta^\dagger C_\beta$ to the anti-Hermitian part. Therefore, in a single history the atom is always in one of the eigenstates.

D. σ_+-σ_-

Our second example is the other standard configuration of sub-Doppler cooling in a polarization gradient, consisting of two counter-propagating waves with opposite circular polarization. We will discuss the case of a $J_g = 1$ to $J_e = 2$ transition, which case has been treated in the literature before [9]. The main contribution to the force arises from the difference in absorption rate of the two counterpropagating travelling waves, which is due to a motionally induced population imbalance between the ground states with $M_g =\mid \pm 1 >$. The Rabi operator for the σ_+-σ_- configuration is

$$\mathcal{R} = \frac{1}{2}\Omega\left[Q_1\exp(iKZ) + Q_{-1}\exp(-iKZ)\right]. \tag{4.24}$$

Since the field has a linear polarization for all values of Z, each position is fully equivalent for the cooling mechanism. The direction of the polarization is $\hat{y}\cos KZ + \hat{x}\sin KZ$. It is convenient to introduce the Z-dependent rotating states

$$\mid M_g >' =\mid M_g > \exp(iM_g KZ) \tag{4.25}$$

for $M_g = 0, \pm 1$. Then the Z-dependence of the linear states

$$| x >' = -\frac{1}{\sqrt{2}} [| 1 >' - | -1 >'],$$

$$| y >' = +\frac{i}{\sqrt{2}} [| 1 >' + | -1 >'],$$

$$| z >' = | 0 >' \tag{4.26}$$

is simply given by the rotation

$$| x >' = | x > \cos KZ - | y > \sin KZ,$$

$$| y >' = | x > \sin KZ + | y > \cos KZ \tag{4.27}$$

These states $| x >'$ and $| y >'$ are eigenstates of the operator $\mathcal{R}^{\dagger}\mathcal{R}$ with eigenvalues $\Omega^2/4$ and $\Omega^2/3$ respectively. The third eigenstate $| z >' = | z >$ has the same eigenvalue $\Omega^2/4$ as $| x >'$.

The effect of the jump operators C_β on the states $| M >'$ is simply expressed in terms of the rate of optical pumping $\Gamma_p \equiv \Gamma s_0/2$. The operator C_0 couples the state $| 0 >'$ to the states $| \pm 1 >'$ and *vice versa*, according to the rules

$$C_0 | 0 >' = \frac{\sqrt{\Gamma_p}}{2} [| 1 >' + | -1 >'],$$

$$C_0 | 1 >' = C_0 | -1 >' = \frac{\sqrt{\Gamma_p}}{3} | 0 > . \tag{4.28}$$

On the other hand, the operators $C_{\pm 1}$ transfer the state $| 0 >'$ to itself, and a linear combination of $| \pm 1 >'$ to a different linear combination, according to

$$C_{\pm 1} | 0 >' = \frac{\sqrt{\Gamma_p}}{2} | 0 >',$$

$$C_1 [\alpha_+ | 1 >' + \alpha_- | -1 >'] = \sqrt{\Gamma_p} \left[\alpha_+ | 1 >' + \frac{1}{6}(\alpha_+ + \alpha_-) | -1 >' \right] \tag{4.29}$$

The jump rate for $\beta = 1$ in the state $\alpha_+ | 1 >' + \alpha_- | -1 >'$ is equal to $\Gamma_p \left[| \alpha_+ |^2 + \frac{1}{36} | \alpha_+ + \alpha_- |^2 \right]$. Hence, during a single trajectory the atom jumps back and forth between the state $| 0 >'$ (which is independent of Z), and a linear combination of the states $| \pm 1 >'$. While the atom is in $| 0 >'$, the force is zero, and the atom can only leave this state by a jump with $\beta = 0$. When the atom is in the linear combination $\alpha_+ | 1 >' + \alpha_- | -1 >'$, the force is equal to

$$F = \frac{1}{2}\hbar K s_0 \left\{ \frac{5}{6}\Gamma \left[| \alpha_+ |^2 - | \alpha_- |^2 \right] + 2i\Delta \left[\alpha_-^* \alpha_+ - \alpha_+^* \alpha_- \right] \right\} . \tag{4.30}$$

From eq. (4.28) it follows that immediately after the atom has left the state $| 0 >'$ by a jump with $\beta = 0$, the force is still zero, since the amplitudes $\alpha_{\pm}$ are equal. Moreover, the change of the basis states $| \pm 1 >'$ during the motion of the atom in the Z-direction does not modify the population of these states. Equation (4.29) shows that when the atom is in a linear combination of $| \pm 1 >'$ with (almost) balanced populations, a jump with $\beta = 1$ strongly enhances the population of $| 1 >'$. However, since the probabilities for jumps with $\beta = \pm 1$ are (almost) equal, these mechanisms do not explain the average population imbalance that is responsible for the net force. In fact, the only source of an average population imbalance is the evolution in between jumps. This evolution is described by the effective Hamiltonian (4.12). The eigenstate $| y >'$ suffers a stronger damping and dephasing than the eigenstate $| x >'$. For the pure state of the atom the net result is that the population of $| -1 >'$ is favored for negative detuning Δ. This is illustrated in Fig. 6. Notice that the evolution of the state and of the resulting force following a jump with $\beta = 0$ is always identical, up to the instant of the first next jump. Single histories do not always correspond to an eigenstate of the light-shift operator. The mixing of eigenstates arises from non-adiabatic coupling, due to the fact that these eigenstates vary with position.

Hence the single histories of the force on an atom display finite periods of zero force, when the atom is in the state $| M_g = 0 >$. After the atom has left this state by a jump with $\beta = 0$, it starts its evolution until the next jump with an equal population of the states $| \pm 1 >$, and zero force. It is the evolution between jumps that is the seed of a net population imbalance, and subsequent jumps with $\beta = \pm 1$ enhance this imbalance. The next jump with $\beta = 0$ brings the atom back in the state $| 0 >$, and makes the force disappear again. The probability distribution of the positions at which these jumps can occur is completely homogeneous, and all positions are fully equivalent.

E. Laser cooling in weak magnetic field

Sub-Doppler cooling at low intensities is possible when the anisotropy of the atomic ground state varies appreciably during the passage of a wavelength. In the examples discussed above, this variation was due to the polarization gradient of the field. An alternative possibility is the use of a magnetic field, so that the combined action of the Zeeman precession and the level shifts leads to a position-dependent anisotropy [10,19]. We discuss now the characteristics of single histories of the atomic state when a $J_g = 1/2$ to $J_e = 3/2$ transition is driven by a standing wave with σ_+ circular polarization, in the presence of a transverse magnetic field. The Zeeman precession frequency ω_Z has a similar magnitude as the rate of optical pumping, which is low compared to the natural width of the transition. The Rabi operator is

$$\mathcal{R} = \Omega Q_1 \cos(KZ), \tag{4.31}$$

and the effective Hamiltonian (4.12) must be supplemented by the Zeeman term $H_Z = \hbar \omega_Z S_x$, in terms of a Pauli matrix. When the atom is in the state $\alpha_+ \mid + > + \alpha_- \mid - >$, the force on the atom is given by

$$F = 2\hbar K s_0 \Delta(\mid \alpha_+ \mid^2 + \frac{1}{3} \mid \alpha_- \mid^2) \sin(2KZ). \tag{4.32}$$

Jumps with $\beta = 0$ occur at a rate $\frac{4}{9} \Gamma s_0 \cos^2(KZ) \mid \alpha_- \mid^2$, and put the atom in the state $\mid + >$. Jumps with $\beta = 1$ occur at a rate $2\Gamma s_o \cos^2(KZ)(\mid \alpha_+ \mid^2 + \frac{1}{9} \mid \alpha_- \mid^2)$. Their effect is to enhance the population of the state $\mid + >$. Therefore the jumps drive the atom towards the state $\mid + >$, whereas the Zeeman precession mixes the states. The evolution in between jumps is described by the total effective Hamiltonian. The Hermitian part (comprising the light-shift operator and the Zeeman term) can be viewed as the action of a magnetic field, with a fictitious longitudinal component proportional to the local field intensity. The anti-Hermitan part tends to enhance the population of the state $\mid - >$, since its absorption rate is lower than that of the state $\mid + >$.

The intermittent process of Zeeman mixing and optical pumping leads to a Sisyphus-type cooling when the detuning Δ is negative [10,19]. The naive picture of this process is that the atom is optically pumped near the antinodes to $\mid + >$ by jumps with $\beta = 0$, where its light-shifted energy is minimal. We have calculated an ensemble of single histories, and results are plotted in Fig. 7. Inspection shows that in reality the optical pumping towards $\mid + >$ takes place mostly by jumps with $\beta = 1$, whereas jumps with $\beta = 0$ are relatively rare. When the atom passes a node just after making a jump, the population of $\mid + >$ is typically decreasing. This decrease arises both due to the Zeeman precession and to the stronger damping of this state by the operator $\mathcal{P}$. When accidentally no jumps occur between two successive nodes, the populaton of $\mid + >$ tends to be minimal at the antinode, and it is increasing at the second node, in contrast to the average behavior. The net negative force arises since in the first half of the distance between two nodes (where the atom is sliding down a slope), the population of $\mid + >$ tends to be lower than in the second half (where the atom is climbing a hill). But this is by no means true for all passages from node to node.

Since the Hamiltonian contains a Zeeman term in addition to the light-shift operator, the eigenstates $\mid \pm >$ of the operators $C_\beta^\dagger C_\beta$ do not coincide with the eigenstates of H_0, so that a single history is generally a linear combination of these states. This makes the picture arising from single histories more complex than in configurations without a magnetic field.

V. LASER COOLING IN A STRONG MAGNETIC FIELD

The picture of jumps at random instants is modified in the presence of a strong magnetic field that gives a Zeeman splitting that is much larger than the pump rate Γ_p. On the other hand, the Zeeman splitting is still small compared with the natural width, so that the atomic evolution can be described completely within the ground level. In two counterpropagating plane waves, a moving atom sees running waves with different Doppler-shifted frequencies, and when this effective frequency difference coincides with the Raman resonance between two Zeeman substates, a resonant enhancement of the atom-field coupling occurs. This leads to cooling of atoms with non-zero velocities [20]. This situation can be conveniently described in a frame rotating about the magnetic field, by neglecting rapidly oscillating terms [18]. In the effective rotating-wave approximation, the evolution operator is independent of time.

As an illustrative case, we take the situation of a standing wave with σ_+ circularly polarized light driving a $J_g = 1/2$ to $J_e = 3/2$ transition, in the presence of a strong transverse magnetic field. Then the force displays a strong resonant velocity dependence at velocities $v \cong \pm \omega_Z/2K$ [18], so that the atom traverses about half a wavelength during a precession period. Here we investigate the velocity region around $\omega_Z/2K$. It is natural to take the quantization axis (the Z-direction) along the magnetic field, and the X-direction is taken as the propagation direction. The circular

components of the polarization vector are then $\epsilon_0 = 1/\sqrt{2}, \epsilon_{\pm 1} = 1/2$. The effect of the rotating-wave approximation is that the jump operators (4.11) are separated in various terms, each with a different oscillation frequency in the rotation frame. Mixing of terms with different frequencies is then neglected. The evolution equation in the rotating frame then takes the general form (2.2), where the jump operators are labeled by two indices α and β, indicating spherical components of $\vec{Q}$. Their explicit expression is

$$C_{\beta\alpha} = \sqrt{\Gamma_p} Q_\beta^\dagger \left[\epsilon_\alpha Q_\alpha + \epsilon_{\alpha+1} Q_{\alpha+1} \right]. \tag{5.1}$$

The term with label α describes the coupling to the travelling wave in the positive X-direction, whereas the term $\alpha+1$ arises from the counterpropagating part of the standing wave (see Fig. 8). The index α can take the values -2, -1, 0 and 1, where for the lowest value only the term $\alpha+1$ contributes. In the present case of a $J_g = 1/2$ to $J_e = 3/2$ transition the index β cannot take the value 1 for $\alpha = -2$, whereas for $\alpha = 1$ the value $\beta = -1$ does not contribute. Hence there are effectively 10 different jump operators, and the picture gets rather complex. The jump operators with $(\alpha, \beta) = $ (-2,0) and (-1,1) simply transfer the state $| + >$ to the state $| - >$, and conversely the jumps with $(\alpha, \beta) = $ (0,-1) and (1,0) transfer $| - >$ to $| + >$. The other six jump operators mix the two counterrunning travelling waves, and create coherence between the states $| \pm >$.

The effective Hamiltonian in the present case is given by

$$H = (\omega_Z - 2Kv)J_Z + \hbar(\mathcal{S} - i\mathcal{P}), \tag{5.2}$$

where the light shift operator and the pump operator are specified by

$$\begin{aligned}
\mathcal{P} + i\mathcal{S} &= (\frac{1}{2} + i\frac{\Delta}{\Gamma}) \sum_{\alpha,\beta} C_{\beta\alpha}^\dagger C_{\beta\alpha} \\
&= \frac{1}{4} s_0 (\frac{\Gamma}{2} + i\Delta) \left[Q_{-1}^\dagger Q_{-1} + 2Q_0^\dagger Q_0 + Q_1^\dagger Q_1 \right. \\
&\quad \left. + \frac{1}{\sqrt{2}} (Q_{-1}^\dagger Q_0 + Q_0^\dagger Q_1 + Q_0^\dagger Q_{-1} + Q_1^\dagger Q_0) \right]
\end{aligned} \tag{5.3}$$

When the atom is in the state $\alpha_+ | + > + \alpha_- | - >$, the force in the propagation direction is given by [18]

$$F = \frac{1}{3} \hbar K s_0 \Delta i (\alpha_- \alpha_+^* - \alpha_+ \alpha_-^*), \tag{5.4}$$

which is proportional to the orientation of the atomic ground state in the Y-direction.

In Fig. 9 we illustrate the evolution of a single history for this case. One should notice that each position is fully equivalent, since the position dependence of the evolution operator has disappeared after the rotating-wave approximation. This is due to the fact that the only coherence between the two counterpropagating travelling waves that matters is that between the polarization component ϵ_α of the beam in the positive X-direction, and the component $\epsilon_{\alpha+1}$ in the negative X-direction.

Jumps of the type 2, 5, 6 and 9 always end up with zero force, since they reduce the atom to one of the states $| \pm >$. All other jumps tend to diminish the force. The evolution between jumps is mainly due to the net Zeeman precession in the rotating frame, which arises from the term proportional to J_Z in (5.2). The force attains its maximal value when the orientation of the state is in the Y-direction. Then the two populations are equal.

Obviously, the picture of single histories remains quite complicated. This is mainly due to the large number of jump types. Also, each jump operator $C_{\beta\alpha}^\dagger C_{\beta\alpha}$ has different eigenstates. The non-zero average value of the force is due to the effect of the first term in (5.2), which breaks the left-right symmetry of the configuration.

VI. CONCLUSIONS

We have analyzed the internal state dynamics of an atom during single histories in several situations where radiative forces arise. Some standard situations of laser cooling are considered, both with and without a transverse magnetic field. Even in situations where each position is physically equivalent, so that the atomic density matrix is constant, the single histories in which the density matrix can be unraveled display wildly varying characteristics. In the simple case of a two-state atom in a single travelling wave (Sec. III), the net force may be understood as arising from a Sisyphus-type mechanism in time (not in position), since the spontaneous emissions have a preference to occur

when the atom has picked up a photon momentum. In the situation of sub-Doppler laser cooling in weak counter-propagating beams with opposite circular polarizations (Sec. IV D), the atom displays periods where the force leads to cooling, interrupted by periods when the force is exactly zero. The population imbalance that is responsible for the cooling arises exclusively from the coherent evolution between quantum jumps. The Sisyphus picture explaining the cooling effect in two counterrunning plane waves with linear polarization turns out to be most convincing at higher velocities, where the force is proportional to $1/v$ (Sec. IV C).

The picture of single histories becomes more complicated in the presence of a transverse magnetic field. This is partly due to the fact that the eigenstates of the jump operators do not coincide with the eigenstates of the Hamiltonian H_0. When the magnetic field is high (Sec. V), the effective number of types of quantum jumps becomes large, and the picture of cooling to non-zero velocities in terms of single histories gets surprisingly complicated. These examples demonstrate that the history of a single atom in situations where laser cooling occurs leads to pictures that are quite different from the standard descriptions in terms of steady-state density matrices. Moreover, the ensemble of these single histories contains not only the average behavior of an atom, but also all higher moments of the fluctuating force. In particular, when an ensemble has been calculated, the heating effect of the momentum diffusion can be directly extracted.

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FIG. 1. Excited state population $|a_e|^2$ and force F on the atom for a single atom-trajectory in the case of a two-level atom at rest in a travelling wave. The detuning $\delta = 0$ and the saturation parameter $s_0 = 50$. The lower part indicates the instants at which a quantum jump occurs (denoted by the diamonds).

FIG. 2. Populations $|a_{-1/2}|^2$ and $|a_{+1/2}|^2$ and the force F on the atom as a function of the position for a single atom trajectory in the case of the lin $\perp$ lin configuration. The velocity of the atom is $w = Kv/s_0\Gamma = 1$ and the detuning of the laser $\delta = -2\Gamma$. Bottom part of the figure indicates when the quantum jumps with $\beta = 0, \pm 1$ occur.

FIG. 3. Schematic diagram of the potential energy of an atom travelling in a polarization gradient of lin $\perp$ lin for different values of the phase ϕ when a jump occurs. The thin lines indicate the potential energy, when the atom is in either state $|-1/2>$ or state $|+1/2>$, the thick line indicates the path of the atom. Note, that the maximum energy loss for the atom occurs for $\phi = 0$.

FIG. 4. The distribution $R(\phi)$ of the phase ϕ when the atom makes a jump from one state to another state with a quantum jump $\beta = 0$ for several reduced velocities $w = Kv/s_0\Gamma$.

FIG. 5. Comparison for the reduced force $\phi = F/\hbar k\Gamma s_0$ on the atom as a function of reduced velocity $w = Kv/s_0\Gamma$ for three different situations: 1) Numerical calculation using optical Bloch Equations [15], 2) Monte-Carlo Wavefunction Method (this paper) and 3) Estimate using the high-velocity limit (see text).

FIG. 6. Populations $\mid a_{-1}\mid^2$ (——), $\mid a_0 \mid^2$ (...) and $\mid a_{+1} \mid^2$ (– – – –) and the force F on the atom as a function of position for a single atom trajectory in the case of the σ_+-σ_- configuration. Bottom part of the figure indicate when the quantum jumps with $\beta = 0, \pm1$ occur. Note, that from $KZ/\pi = 10.5$ to 18 the atom runs through a cycle as described in the text.

FIG. 7. Populations $\mid a_{-1/2}\mid^2$ (——) and $\mid a_{+1/2} \mid^2$ (...) and the force F on the atom as a function of position for a single atom trajectory in the case of the MILC configuration. Bottom part of the figure indicate when the quantum jumps with $\beta = 0, \pm1$ occur. Note, that since the light is σ_+ polarized that no jumps with $\beta = -1$ are possible.

FIG. 8. Schematic diagram for the quantum jumps in the case of cooling in a strong magnetic field. The coupling of one laser beam is indicated by an open arrow, the coupling of the other beam by a solid arrow. Spontaneous emission is indicated by wavelets.

FIG. 9. Populations $\mid a_{-1/2}\mid^2$ (——) and $\mid a_{+1/2} \mid^2$ (...) and the force F on the atom as a function of position for a single atom trajectory in the case of the laser cooling in a strong magnetic field. Bottom part of the figure indicate when the different quantum jumps occur (see Fig. 8 for the numbering of the quantum jumps).