BOOK FOR SPRINGER-VERLAG

COOLING AND TRAPPING OF NEUTRAL ATOMS

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Introduction

Introduction

Since the discovery of the kinetic theory of gases, the concept of thermal energy and its associated atomic motion has permeated experimental physics and has had a profound influence on a wide variety of laboratory measurements. The entire fields of atomic and molecular beams, collisions, and gas phase spectroscopy are dominated by considerations of atomic motion.

The sensitivity of atomic beams and collision experiments is always limited by the conflicting requirements of collimation and flux: smaller slits that allow a narrow angular spread also limit the flow of atoms into the target region. Before the advent of the optical techniques described below, the data rate in such experiments was limited by the brightness of the source, since no collimation method could increase the beam brightness. Many extremely sensitive measurements were reduced count rates of a few per second.

The thermal motion of atoms also causes two distinct limits on ultrahigh resolution spectroscopy. One of these arises from the linewidth produced by the finite interaction time between the measuring equipment and a rapidly moving atom. Since there are only a few ms to interact with a free atom in an apparatus of few meter size, the linewidth imposed by the classical relation $\Delta\nu\Delta t \geq 1$, places restriction on the accuracy of such measurements. The other limit arises from the relativistic time differences between reference frames in relative motion (sometimes called the second order Doppler effect). The velocity distribution can be calculated, but the details of the distribution at the low velocity end depend very sensitively on the details of the source, and sometimes cannot be adequately known. A sample of slowly moving or monovelocity atoms could provide a substantial improvement in spectroscopic resolution.

In Einstein's famous paper on blackbody radiation [1] he predicted that there is a transfer of momentum in spontaneous emission processes. He further stated that "the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practise", but Frisch [2] demonstrated the deflection of an atomic beam by 0.01 mm with a sodium lamp. Even though the momentum of photons is much smaller than the momentum of thermal atoms, the process of absorption and spontaneous emission can be cycled many times because of the short lifetime of atomic excited states. It

was not until the discovery of the laser, and its succesful use in atomic experiments, that real progress could be made towards manipulating atomic motion. The breakthrough was initiated by Wineland and Dehmelt [3] and Hansch and Schawlow [4] who discussed new ideas for using laser light to cool atoms. They realised that in principle atoms can be cooled to very low temperatures using laser light pressure.

This article describes recent experiments that are directed at reduction and control of thermal motion. There are many applications for a collection of slowly moving and/or monovelocity atoms. We can expect new and exquisitely more sensitive atomic beam experiments to probe the electrical neutrality of matter, the effects of parity violating interactions, gravitation, quantum statistics in optical absorption and emission, and other fundamental interactions. Such atoms could serve as an ideal target or beam for a variety of low energy and/or high energy-resolution scattering experiments, chemical reaction studies, or surface studies.

Chapter 1

Deceleration of an Atomic Beam

1.1 Introduction

The use of electromagnetic forces to influence the motion of neutral atoms has been a subject of interest for some years (two topical issues on this subject have been published by the J. Opt. Soc. Am. B; one in November 1985[5] and the other in November, 1989[6]). We consider the force caused by radiation, particularly with light at or near the resonance frequencies of atomic transitions. The origin of this force derives from the momentum associated with light. In addition to energy $E = h\nu$, a quantum of light carries momentum $h\nu/c \equiv \hbar k$ and angular momentum $\hbar$. When an atom absorbs light, it stores the energy by going into an excited state; it stores the momentum by recoiling from the light source with a momentum $\hbar k$; and it stores the angular momentum in the form of internal motion of its electrons. It is the velocity change of the atom, $\hbar k/M \simeq$ few cm/s, that is of special interest here, and although it is very small compared with thermal velocity, multiple absorption can be used to produce a large total velocity change. Proper control of this velocity change constitutes a radiative force that can be used to decelerate and/or to cool free atoms.

Although there are many ways to decelerate and cool atoms, the one that has received the most attention by far is the scattering force that uses this momentum transfer between the atoms and a radiation field resonant with an atomic transition. By making a careful choice of geometry and of the light frequency one can exploit the Doppler shift to make the momentum exchange (hence the force) velocity dependent. Because the force is velocity dependent, it can not only be used for deceleration, but also for cooling.

1.2 Techniques of Beam Deceleration.

The idea that the radiation scattering force on free atoms could be velocity dependent and therefore be used for cooling a gas was suggested by Wineland and Dehmelt[3], Hansch and Schawlow[4], and Wineland and Itano[7], although Kastler, Landau, and others had made allusions to it in earlier years. The possibility for cooling stems from the fact that atomic absorption of light near a resonance is strongly frequency dependent, and is therefore velocity dependent because of the Doppler shift of the laser frequency seen by the moving atoms relative to the laboratory fixed laser. Of course, a velocity dependent dissipative force is needed for cooling.

The simplest form of this force to study, that from a low intensity single plane wave of light, has been exploited for cooling of an atomic beam. Early experiments in several laboratories[5, 6, 8, 9] have used this force, along with a variety of methods to overcome technical problems[10, 11, 12], to decelerate and cool thermal atomic beams to only a few hundredths of a Kelvin.

One very obvious implementation of radiative deceleration and cooling is to direct a laser beam opposite to an atomic beam as shown in Fig. 1.3.2 [8, 9]. In this case each atom could absorb light very many times along its path through the apparatus. Of course, excited state atoms can not absorb light efficiently from the laser that excited them, so between absorptions they must return to the ground state by spontaneous decay, accompanied by emission of fluorescent light. The emitted fluorescent light will also change the momentum of the atoms, but its spatial symmetry results in an average of zero net momentum transfer after many such scattering events. So the net deceleration of the atoms is in the direction of the laser beam, and the maximum deceleration is limited by the spontaneous fluorescence rate.

The maximum attainable deceleration is obtained for very high light intensities, and is limited because the atom must then divide its time equally between ground and excited states. High intensity light can produce faster absorption, but it also causes equally fast stimulated emission; the combination produces neither deceleration nor cooling because the momentum transfer to the atom in emission is then in the opposite direction to what it was in absorption. The deceleration therefore saturates at a value $\vec{a}_{max} = (1/M)(\Delta\vec{p}/\Delta t) = \hbar\vec{k}\gamma/2M$ where $1/\gamma \equiv \tau$ is the excited state lifetime, and the factor of 2 arises because the atom spends half its time in each state.

The Doppler shifted laser frequency in the moving atom's reference frame should match that of the atomic transition to maximize the light absorption and scattering rate. This rate Γ_{sc} is given by the Lorentzian

$$\Gamma_{sc} \equiv \frac{\gamma s/2}{1 + s + \Delta^2} \tag{1.1}$$

where $s \equiv I/I_s$ is the ratio of the light intensity I to the saturation intensity $I_s \equiv \pi hc/3\lambda^3\tau$, which is a few mW/cm^2 for typical atomic transitions. Also

atom	H	He*	He*	Li	Na	K	Rb	Cs
wavelength (nm)	12.2	1083	1083	670	589	766	780	852
lifetime (ns)	1.6	100	100	32	17	27	27	31
γ/k (m/s)	128	1.7	1.7	4.0	5.9	4.7	4.6	4.2
$\hbar k/M$ (cm/s)	325	9.2	9.2	8.5	2.9	1.3	0.59	0.35
$\gamma_R/2\pi$(kHz)	13328	42	42	74	25	8.7	3.7	2.0
Oven Temp. (K)	1000	4	650	1017	712	617	568	544
$\sqrt{3kT/}$ (m/s)	5000	158	2013	2051	876	626	402	319
stop length (m)	0.012	0.03	4.4	1.15	0.42	0.77	0.75	0.93
stop time (ms)	0.005	0.34	4.4	1.12	0.96	2.45	3.72	5.82

Table 1.1: Parameters of interest for slowing various atoms. The stopping length and time are minimum values. The oven temperature that determines the peak velocity is chosen to give a vapor pressure of 1 Torr. Special cases are H at 1000 K and He in the metastable triplet state, for which two columns are shown: one for a 4 K source and another for the typical discharge temperature.

$\Delta \equiv 2(\delta + \omega_D)/\gamma$ where the laser detuning from resonance is $\delta \equiv (\omega_L - \omega_A)$, ω_L is the laser frequency and ω_A is the atomic resonance frequency. The Doppler shift seen by the moving atoms is $\omega_D \equiv -\vec{k}\cdot\vec{v}$ (note that $\vec{k}$ opposite to $\vec{v}$ produces a positive Doppler shift). Maximum deceleration requires $\Delta \ll 1$ so that the laser light is nearly resonant with the atoms in their rest frame. The net force $\vec{F}$ on the atoms is

$$\vec{F} = \hbar\vec{k}\Gamma_{sc} \tag{1.2}$$

which saturates at large s to $M\vec{a}_{max} = \vec{f}_{sc} \equiv \hbar\vec{k}\gamma/2$.

The choice of an atom to use for the easiest deceleration and cooling experiments depends on the availability of suitable lasers and the ease of making an atomic beam. Of course, there should also be some physical interest for the particular choice as well. In the table below are some of the parameters for slowing a few atoms of interest from the peak of the velocity distribution.

1.2.1 Doppler Compensation.

If the light source is spectrally narrow, then as the atoms in the beam slow down, their changing Doppler shift will take them out of resonance. They will eventually cease deceleration after their Doppler shift has been decreased by a few times $\gamma\sqrt{1+s}$ corresponding to Δv few times γ/k for low intensity. Although this few m/s is considerably larger than the atomic recoil velocity $\hbar k/M$ few cm/s, it is still only a small fraction of the atoms' average thermal velocity, so that significant further cooling or deceleration cannot be accomplished. The two most common methods for overcoming this problem are sweeping the laser frequency to keep it in resonance with the decelerating atoms[13, 14, 15],

and spatially varying the atomic resonance frequency with an inhomogeneous dc field to keep the decelerating atoms in resonance with the fixed frequency laser[8, 16]. Both methods have now succeeded in several laboratories.

With the first method, the laser frequency is swept upward at rate $\dot{\omega}_L$ to compensate the decreasing Doppler shift as the atoms slow down. Of course, Δ must be kept $\ll 1$ in order to sustain atomic resonance. This requires $\dot{\delta}_L + \dot{\omega}_D = 0$, from which we find that $a < a_{max}$ requires $\dot{\omega}_L = \vec{k} \cdot \vec{a} < \hbar k^2 \gamma / 2M \equiv \gamma_R \gamma$.

The second method uses a spatially varying magnetic field to tune the atomic levels along the beam path[8, 16]. It works as long as the g-factors and thus the Zeeman shifts of the ground and excited states are different so that the resonant frequency is shifted. The field can be tailored to provide the appropriate Doppler shift along the moving atom's path. For uniform deceleration $a \equiv \eta a_{max}$ from initial velocity v_0, the appropriate field profile is $B(z) = B_0 \sqrt{1 - z_0/z}$, where $z_0 \equiv m v_0 / \eta \hbar k \gamma$ is the length of the magnet, $B_0 = \hbar k v_0 / \mu'$, $\mu' \equiv (g_e m_e - g_g m_g)\mu_B$, subscripts g and e refer to ground and excited states, g is the Landé g-factor, μ_B is the Bohr magneton, and m is the magnetic quantum number. The design parameter $\eta < 1$ determines the length of the magnet z_0. A solenoid that can produce such a spatially varying field has layers of decreasing lengths as shown in Fig. 1.3.2. The technical problem of extracting the beam of slow atoms from the end of the solenoid can be simplified by reversing the field gradient[10].

The equation of motion of an atom in the magnet can not be easily solved in general because of the velocity-dependent force, but we can transform to a decelerating frame $\mathcal{R}$[17]. For the special case of uniform deceleration the velocity of this frame in the lab is $v_{\mathcal{R}} = v_0 \sqrt{1 - z_0/z}$, and the Doppler shift associated with this velocity is compensated by the position-dependent Zeeman shift in the magnet. The resulting equation of motion for the velocity of atoms $v' \equiv v - v_{\mathcal{R}}$ relative to this frame is given by

$$m \frac{dv'}{dt} = -\vec{f}_{sc} \left(\frac{s}{1 + s + (\Delta')^2} - \eta \right) \tag{1.3}$$

where $\Delta' \equiv 2(\delta - \vec{k} \cdot \vec{v}'/\gamma)$. For $dv'/dt = 0$ we find the steady state velocity v'_{ss} is given by

$$k v'_{ss} = -\delta \pm \frac{\gamma}{2} \sqrt{s \frac{1 - \eta}{\eta} - 1}. \tag{1.4}$$

There are two values of v_{ss} but the one with the (+) sign is unstable. The magnitude of v'_{ss} is typically of order δ/k. This velocity is constant as an atom decelerates along its path through the magnet so the decreasing Doppler shift is compensated by the decreasing Zeeman shifts.

The changing Doppler shift can also be compensated by a Stark shift using an inhomogeneous dc electric field. In this case the choice of the atomic transition is more important because Stark shifts are generally small and a highly polarizable excited state is desirable. Some new experiments are in progress [18, 19].

More recently, compensation of the changing Doppler shift using spectrally narrow light to slow atoms in a beam has been achieved by exploiting the angular dependence of the Doppler shift embodied in $\omega_D = -\vec{k}\cdot\vec{v}$ [20, 21]. Atoms moving through diffuse light see a range of frequencies that vary with the angle between the velocity and the light direction. If the light is tuned below resonance in the lab frame by an amount δ, then the Doppler shift will be towards the blue, closer to resonance, if a component of the light's propagation direction is antiparallel to the atomic velocity. (Light propagating nearly parallel to the atomic velocity is shifted further to the red, further out of resonance.) Atoms can be efficiently slowed by scattering the counter-propagating blue-shifted light if the incident angle required for the Doppler effect to shift it close to atomic resonance is not too close to $\pi/2$ so that there is a considerable component of the momentum vector $\hbar\vec{k}$ antiparallel to $\vec{v}$. Of course, as the atoms slow down, the Doppler shift of the fixed frequency, red-tuned light needed to achieve resonance doesn't change, but the smaller $\vec{v}$ requires a larger contribution from the angular part of $\vec{k}\cdot\vec{v}$. Thus atoms will interact with counterpropagating light from a cone of different angle, closer to the velocity direction, until they have decelerated to nearly $v = \delta/k$. Below this velocity, there is no angle for which the Doppler effect can shift the light into resonance, and deceleration becomes very inefficient.

Still another method of deceleration uses light that is not spectrally narrow, but is white over a spectral region from ω_A to $(\omega_A - \vec{k}\cdot\vec{v}_0)$. Then Doppler compensation is not necessary because atoms of any velocity below v_0 will find resonant light in a counterpropagating beam. Such white light slowing has been considered long ago [22, 23] and has recently been demonstrated[24, 25]. One important disadvantage is that saturation of the atomic transition for all velocities below v_0 requires much more light power because the spectral density must be I_s/γ, and since $\vec{k}\cdot\vec{v}_0$ is typically 100γ, the overall power must be 100 times larger than in the Doppler compensation techniques.

A quite different slowing scheme using the large Stark shifts of Rydberg states was proposed in 1981[26]. In this method, the force on the atoms does not come from the momentum of the light but from the energy associated with their Stark shifts in a dc electric field. Atoms are optically excited to a Rydberg state whose Stark shifted energy is downward-going in a region of strong dc electric field. If the field is very inhomogeneous, produced for example by a pair of small electrodes of few mm size and separation, then atoms gain potential energy and thus must lose kinetic energy as they leave the region between the electrodes and travel to a region of zero field. When they entered such a region of strong field in the ground state they did not gain as much energy as they lose when they leave it in an excited state.

The lifetime of the selected Rydberg state is chosen so that the atoms will decay to the ground state outside the field region. Thus the scale of the experiment is determined by the Rydberg state lifetime and the atomic speed. Travel through regions of alternately small and large fields, coupled with proper

excitation by well-focused laser beams, causes repeated kinetic energy loss. The atoms always climb up bigger hills than they fall down, and radiate higher frequency light from the tops of those hills than they absorb at the bottoms. Thus their kinetic energy is converted into potential energy and then radiated away. This method has the advantage that the slowing distance depends on the atom's kinetic energy, not its speed. Therefore the distance can be much less for light but fast atoms such as He* at discharge temperatures (see Table1.1).

1.2.2 Apparatus and Results.

In this section we present a detailed description of some deceleration experiments that used the Zeeman tuning technique to compensate the changing Doppler shift. In the apparatus of Fig. 1.3.2,[8] a 450^o oven produces a slightly supersonic atomic beam through a 0.12 mm aperture that is collimated to about 0.01 rad. The beam then enters a 1.1 m long solenoid that has windings to produce the inhomogeneous or tapered field to compensate for the changing Doppler shift where the deceleration occurs. The atomic velocity distribution is analyzed by observing the fluorescence induced by a very weak probe laser that crosses the beam at a small angle about 40 cm beyond the end of the solenoid. This probe laser is sufficiently weak that it doesn't perturb the atomic velocity distribution, and crosses the atomic beam sufficiently far away from the end of the solenoid that there is no residual magnetic field from it. The slowing laser beam is chopped to allow convenient observation of fluorescence induced by the probe while the main cooling laser beam is blocked. Because of the Doppler shift, the absorption and hence the intensity of this fluorescence will depend on the atomic velocity, and a slow scan of this laser's frequency will result in a fluorescence signal that reflects the velocity distribution as shown in Fig. 1.3.2. The velocity resolution of this measurement scheme is dictated by the natural width of the atomic transition and corresponds to about $\gamma/k \cong 6$ m/s. There has also been a detailed study of the deceleration process by monitoring the fluorescence along the atoms' flight path[12].

The production of such very slow, cold atomic beams is no longer an interesting demonstration, but has been transformed into a practical tool that can now be exploited for other experiments. For example, such an atomic sample has an energy width comparable to the capture range of optical molasses or the well depth of a neutral atom trap (see below).

1.3 Further Considerations.

It is important to recognize that deceleration of atoms is not the same as cooling. That is, the "temperature" of a sample of atoms is not determined by their average speed in the lab frame, but by the width of their velocity distribution. Of course, there is strictly speaking no temperature involved here because the

atoms are not in equilibrium with any reservoir, their velocity distribution is not Maxwellian, and there is a flow of energy from the laser beam to the fluorescent bath. Still, we use the word to mean average kinetic energy in the center of mass frame divided by k_B.

1.3.1 Cooling During Deceleration.

In order to see how this notion applies to laser deceleration of an atomic beam, we will consider the example of Zeeman compensation of the Doppler shift [27]. Some atoms emerging from the oven are moving too fast to be decelerated at all because, for them, the laser frequency is Doppler shifted too far out of resonance to absorb light, even where the magnetic field is strongest at the solenoid entrance. Others have velocities whose Doppler shift causes the laser frequency to match the Zeeman shift and begin slowing down as soon as they enter the solenoid. Still others are moving so slowly that they do not absorb light until they have travelled to a point where the static but spatially varying magnetic field has decreased to the appropriate value to match their smaller Doppler shift and produce resonance. Thus *all atoms* with velocity lower than v_0 can be decelerated by the laser beam to some smaller velocity at the end of the solenoid. This final velocity is determined by the atomic resonance condition at the chosen laser frequency in the field at the end of the solenoid. Thus *all atoms* with velocities below v_0 are swept into a narrow velocity group around this final velocity. The result is that the originally wide thermal velocity distribution is compressed and shifted to lower velocities as shown in Fig. 1.3.2: this process is called laser cooling of an atomic beam.

This can be viewed in more detail by expanding the velocity in the reference frame $\mathcal{R}$ around v'_{ss}. Then Eq. (1.3) can be rewritten as $dv'/dt = -(\beta/M)(v' - v'_{ss})$ where $\beta \equiv (4M\gamma_R\eta^2/s)\sqrt{(1-\eta)s/\eta - 1}$ when only the lowest order term in $(v' - v'_{ss})$ is retained. The damping rate β/M is maximum at $\beta = \hbar k^2/4$ for $\eta = 0.5$ and $s = 2$. The damping time M/β is typically 10 μs which is much smaller than the time atoms spend in the magnet. Thus atoms are not only decelerated in the lab frame, but cooled toward this velocity.

1.3.2 Transverse Motion During Deceleration.

As the longitudinal motion of atoms in a beam is slowed by counterpropagating laser light, their transverse motion becomes more important if it, too, is not compensated. For example, if an atomic beam of mean transverse velocity of 1000 m/s and angular spread 0.01 radian is decelerated to 50 m/s, its angular divergence expands to 0.2 radian at the end of the slowing region. It is hardly recognizable as a beam! In order to compensate this effect, the laser beam is not parallel but is converging toward the atomic beam oven. Thus there is a small transverse component of the optical force that opposes the transverse velocity just enough to maintain the angular spread of the original atomic beam

[8, 9]. Note that for the case of diffuse light slowing described above, such considerations are not necessary because the optical force is always directly opposite to the atomic velocity[20, 21].

There is another source of transverse motion that is not as easily controlled. Each time an atom spontaneously decays to the ground state, it receives and impulse $\hbar\vec{k}$ in a random direction. Although the average value of these impulses is zero, the rms value is not. Thus the atoms diffuse transversely as they move along their paths. The resulting distance from the axis Δx as a result of this diffusion is $\Delta x = \int v(t)dt$ where $v(t) = (\hbar k/M)\sqrt{n}$ and $n = \Gamma_{sc}t$ is the number of photons scattered. Integrating from $t = 0$ to $v_0/\eta a_{max}$ gives $\Delta x = (2\hbar k/3M)\sqrt{\Gamma_{sc}(v_0/\eta a_{max})^3}$ which is several mm for typical experiments. Thus the focused laser beam must be about 1 cm in diameter at the end of the deceleration region if too many atoms are not to escape out the sides as a result of this transverse diffusion.

As atoms move down the magnet, their velocity is determined by s, δ, and B as discussed above. However the light intensity distribution across the counter-propagating laser beam is not uniform, but is given by $I(x, z) = I_0 e^{-2x^2/w(z)^2}$ where $w(z)$ is the beam waist. If the beam is focused, $w(z)$ is given by the usual expression for Gaussian beam propagation. Thus atoms in a plane perpendicular to the laser beam have different velocities, and the shape of the surface containing atoms of the same velocity is determined by the Gaussian intensity profile. Using $v_{\mathcal{R}} = \sqrt{v_0 - 2az}$ and Eq. (1.2) with the Gaussian spatial profile in Γ_{sc}, we find this surface to be of the form $z(x) = z_1 e^{2x^2/w(z)^2}$ for small x, and to be undefined where x is large enough for the intensity to fall below ηI_0, the minimum required to maintain deceleration. Furthermore, fluctuations of the light intensity can cause instabilities on this surface [28].

FIGURE CAPTIONS

Figure

Schematic diagram of apparatus for beam slowing, including optical and electronic arrangement. The tapered magnetic field is produced by layers of varying length on the solenoid. A plot of B vs z, with and without the turns in the small shaded area of the solenoid, is shown. Note the regions of large dB/dz (steep slope on broken curve). The decelerating laser is chopped and fluorescence from the probe laser is recorded by a boxcar integrator during short windows to measure the velocity distribution. The timing of the boxcar gates A and B relative to the chopping of the decelerating laser is shown at the lower right-hand side. (Figure from Ref. [8]).

Figure

Laser-compressed velocity distribution (solid line). The unmodified distribution is shown dotted. The broad feature with apparent velocity near 2000 m/s is actually from atoms in the $F = 1$ ground state having velocities around 1000 m/s. The half-width of the cooled distribution Δv near 800 m/s is only 80 m/s, a factor of 10 times narrower than the thermal width. (Figure from Ref. [9]).

Chapter 2

Optical Molasses

2.1 Introduction

We now consider experiments using the radiative force on atoms from more than just a single laser beam. For example, if two identical laser beams of low intensity are directed opposite to one another (e.g., by retroreflection of a single beam from a mirror), the scattering forces of the two beams discussed in the previous section obviously cancel each other for atoms at rest. However, atoms moving slowly along the light beams will experience a net force proportional to their velocity whose sign depends on the laser frequency. If the laser is tuned below atomic resonance, the frequency of the light in the beam opposing the atomic motion will be Doppler shifted toward the blue in the atomic rest frame, and will therefore be closer to resonance; similarly, the light in the beam moving parallel to the atom will be shifted toward the red, further out of resonance. Atoms will therefore interact more strongly with the laser beam that opposes their velocity and they will slow down. This is illustrated in Fig. 1.3.2.

The slowing force is proportional to velocity for small enough velocities, resulting in viscous damping[29, 30] that gives the technique the name "optical molasses" (OM). By using three intersecting orthogonal pairs of oppositely directed beams, one can severely restrict the movement of atoms in the intersection region and thereby collect and cool many atoms in a small volume. OM has been demonstrated at several laboratories[31], in some cases with the use of low cost diode lasers[32].

Note that OM is not a trap for neutral atoms because there is no restoring force on atoms that have been displaced from the center. Still, the detainment times of atoms caught in OM of several mm diameter can be remarkably long.

2.2 Low Intensity Theory for a Two Level Atom in One Dimension

It is straightforward to estimate the force on atoms in OM from hier Eq. (1.2). We begin by restricting the discussion to the case where the light intensity is low enough so that stimulated emission is not important. We thereby eliminate consideration of excitation of an atom by light from one beam and stimulated emission by light from the other, a sequence that can lead to very large, velocity-independent changes in the atom's speed. In the low intensity case we can simply add the forces from the two light beams to find $\vec{F}_{OM} = \vec{F}_+ + \vec{F}_-$ where

$$\vec{F}_\pm = \pm\frac{\hbar\vec{k}\gamma}{2}\frac{s}{1+s+\Delta_\pm^2} \quad , \quad \Delta_\pm \equiv 2(\delta \mp |\omega_D|)/\gamma. \tag{2.1}$$

The sum of the two forces is

$$\vec{F}_{OM} \cong 8\hbar k^2 \delta s \vec{v}/\gamma(L+s)^2 \equiv \beta\vec{v} \tag{2.2}$$

where $L \equiv 1 + (2\delta/\gamma)^2$ and the approximation is valid for $v^4 << (\gamma/k)^4$. The other quantities are defined immediately following Eq. (1.1).

If $\delta < 0$, this force opposes the velocity and therefore viscously damps the atomic motion. $\vec{F}_{OM}$ has maxima near $v = \pm(\gamma'/2k)(X/\sqrt{3})$ and decreases rapidly for larger velocities. Here $\gamma' \equiv \gamma\sqrt{s+1}$ is the power broadened linewidth, X is the numerical factor $\sqrt{x-1+2\sqrt{x^2+x+1}}$, and $x \equiv (2\delta/\gamma')^2$. For $x \gg 1$ these maxima appear at $v = \pm\delta/k$ as expected, but for the usual realm of OM, $x \sim 1$ and $X\sqrt{3}$. Since $\vec{F}_{OM}$ is nearly linear with velocity for $|v| < \gamma/2k$ when $x \sim 1$ (i.e., $s \leq 1$ and $\delta = -\gamma/2$), it is convenient to define a capture velocity $v_c \equiv \gamma/2k$. However, the range of this damping force can be increased considerably beyond v_c by using curved wavefronts [33], a properly arranged inhomogeneous magnetic field [34], or a variety of other tricks [35, 36], some of which are discussed in section 6.1 below.

If there were no other influence on the atomic motion, all atoms would quickly decelerate to $v = 0$ and the sample would reach $T = 0$, a clearly unphysical result. We must consider that there is also some heating caused by the light beams and it derives from the discrete size of the momentum steps the atoms undergo with each emission or absorption. Since the atomic momentum changes by $\hbar k$, their kinetic energy changes on the average by the recoil energy $E_R \equiv \hbar^2k^2/2M \equiv \hbar\gamma_R$, so that the average frequency of each absorption is $\omega_{abs} = \omega_A + \gamma_R$ and the average frequency of each emission is $\omega_{emit} = \omega_A - \gamma_R$. Thus the light field loses an average energy of $\hbar(\omega_{abs} - \omega_{emit}) = 2\hbar\gamma_R$ for each scattering at a rate $2\Gamma_{sc}$ (two beams), and this energy becomes atomic kinetic energy because the atom recoils from each event. The atomic sample is thereby heated because these recoils are in random directions.

The competition between this heating with the damping force of Eq. (2.2), results in a non-zero kinetic energy in steady state. For atoms in OM at steady

state, the rates of heating and cooling are equal. We therefore equate the cooling rate, $\vec{F} \cdot \vec{v}$ to the heating rate and calculate the steady state kinetic energy to be $(\hbar\gamma/8)(2\delta/\gamma + \gamma/2\delta)$. This result is dependent on δ, but it has a minimum at $|2\delta/\gamma| = 1$ whence $\delta = -\gamma/2$. The temperature found from the kinetic energy is then $T_D = \hbar\gamma/2k_B$ where k_B is Boltzmann's constant and T_D is called the Doppler temperature or the Doppler cooling limit. For ordinary atomic transitions T_D is below one mK, and for the Na transition at $\lambda = 589$ nm, T_D= 240 μK.

Another instructive way to calculate T_D is to note that the average momentum transfer of many spontaneous emission is zero, but the rms scatter of these about zero is finite. One can imagine these decays as causing a random walk in momentum space with step size $\hbar k$ and step frequency $2\Gamma_{sc}$. The random walk results in diffusion in momentum space with diffusion constant $D_0 \equiv 2(\Delta p)^2/\Delta t = (\hbar k)^2(2\Gamma_{sc})$ where the factor of 2 arises because of the two beams. Then Brownian motion theory (Fokker-Planck Eq.) gives the steady state temperature in terms of the damping constant β to be $k_B T = D_0/\beta$. This turns out to be $\hbar\gamma/2$ as above for the case $s \ll 1$ when $\delta = -\gamma/2$. There are many other independent ways to derive this remarkable result that predicts the final temperature of atoms in OM to be independent of the optical wavelength, atomic mass, and laser intensity (as long as it is not too large).

2.3 Atomic Beam Collimation

When an atomic beam crosses a one-dimensional OM as shown in Fig. 2.4, the transverse motion of the atoms is quickly damped while the longitudinal component is essentially unchanged. This transverse cooling of an atomic beam is an example of a method that can actually increase its brightness (atoms/sec-sr-cm^2) because such active collimation uses dissipative forces to compress the phase space volume occupied by the atoms. This is outside the usual realm of beam focusing or collimation techniques because all optical beams and most particle beams, up to now, have been subject to only selection by apertures or conservative forces that preserve the phase space density of atoms in the beam (brightness theorem).

2.3.1 Low Intensity Case

At low intensity the velocity dependence of the optical force that collimates atomic beams using transverse OM derives from the Doppler shift of the transverse velocity $v_t \equiv v \sin\theta$, where typically $v = \sqrt{3k_B T/M}$. The damping constant β is maximum for $\delta = \gamma/2$ and $s = 2$, and for $v_t < v_c$ the maximum value of this damping force is approximately $\hbar k^2 v_t/2$ (see Fig. 2.4). For $v_t > v_c = \gamma/2k$, this force decreases approximately as $1/v_t$ from its maximum value of $\hbar k\gamma/2$, just as in OM. By contrast, for faster atoms the force from

one of the beams dominates because of the Doppler shift, and for high enough speeds (v_t a few times larger than v_c) the Doppler shift can take an atom almost completely out of resonance with both beams.

One can estimate this velocity compression at low intensity in one dimension for two-level atoms as follows. The narrowest momentum distribution after cooling has an energy width determined by the Doppler limit $M(\Delta v)^2/2 = \hbar\gamma/4$, corresponding to $\Delta v = \sqrt{\hbar\gamma/2M}$. The relevant transverse velocity width before optical collimation is about twice the capture range v_c. Therefore the one-dimensional momentum space compression is about $\sqrt{\gamma/\gamma_R}$. For an atomic beam, this compression can be done in the two transverse directions reducing the occupied volume of transverse momentum space by γ/γ_R. Furthermore, longitudinal cooling of an atomic beam as described above can compress the width of the longitudinal velocity distribution from its original thermal value of $\sqrt{3k_BT/M}$ to about the same Δv. For Rb $\Delta v = 12$ cm/s, $v_c \simeq 2.3$ m/s, $\gamma_R \simeq 2.3 \cdot 10^4 \mathrm{s}^{-1}$, and $\gamma/\gamma_R \simeq 1600$. Thus the decrease in phase space volume from the momentum contribution alone for a Rb atomic beam can exceed 10^6.

Contributions to increasing the brightness of an atomic beam are not limited to momentum space compression. One must also consider the spatial expansion of the atomic beam in the transverse direction as it crosses the beam of collimating laser light. The minimal transverse expansion Δx for atoms emitted from a point source is of the order of the minimum stopping distance of atoms with transverse velocity $2v_c$. From this we find $\Delta x = 2v_c/a_{\max} = \gamma/2k\gamma_R \simeq 100\mu$m for Rb. Since this is typically smaller than the oven hole, this expansion is not of great consequence. In fact, spatial compression using atomic beam focusing as shown in Fig. 2.4, coupled with longitudinal cooling and further collimation, could compress the spatial extent of the beam to $\simeq \sqrt{\gamma/\gamma_R}/k$, several μm for Rb. This may be a factor of 50 in each direction, thus leading to total phase space compression of more than 10^9.

We see that optical techniques can create atomic beams 10^6 or more times intense than ordinary thermal beams, and also many orders of magnitude brighter. Furthermore, this number could be increased several orders of magnitude if the transverse cooling could produce temperatures below the Doppler temperature (see below). For atoms at the recoil temperature $T_R \equiv \hbar\gamma_R/k_B$ where $\Delta p = \hbar k$ and $\Delta x = \lambda/\pi$, the brightness increase is 10^{17}.

2.3.2 Experiments in One and Two Dimensions

Optical collimation has been demonstrated in a thermal beam of natural Rb that was produced by an oven at $T \simeq 150^oC$ with aperture $\simeq 330$ μm diameter[37, 38]. The beam was mechanically collimated by a defining aperture of diameter $\simeq 330\mu$m about 24 cm away (see Fig. 2.4). The laser light ($\lambda = 780$ nm) was tuned to select either of the stable Rb isotopes, and experiments were done with each of them. The atomic beam profile was measured with a scanning hot tungsten wire, 25 μm in diameter, 1.3 m away from the region of interaction

with the laser beam.

For two-dimensional collimation in the plane perpendicular to the atomic beam, a single hot wire scan would not provide enough information. Instead of scanning both a vertical and a horizontal hot wire, a new method[37, 38] was devised for observing the spatial distribution of atoms in the plane perpendicular to the beam using a heated mesh instead of a hot wire (see Fig. 2.4). Ions emitted from the hot grid are accelerated into a pair of multichannel plate electron multipliers whose output electrons are accelerated onto a phosphor coated screen which is viewed by a standard TV camera. The camera's output is fed to a frame grabber in a computer where the image can be analyzed.

This device has been used for viewing the atomic beam with both one and two dimensional collimation. Figure 2.4 shows the "ridge" created through the middle of the circular beam spot image made by this device when the laser was tuned red of resonance (collimation). When the laser was tuned blue, the transverse one dimensional heating produced a readily visible "valley" in the beam profile.

With about half of the molasses laser beam split off to produce a vertically oriented molasses in addition to the horizontal one, full two-dimensional collimation of the atomic beam is achieved. Fig. 2.4 shows the results of this experiment. Here the intensity of each beam was only about half of that in Fig. 2.4, both light beams were linearly polarized normal to the atomic beam direction, and about 0.4 Gauss was applied along that direction. (The B-field and both polarization vectors were mutually orthogonal.)

The collimated atomic beam spot size shown in Fig. 2.4 of about 1.25 mm diameter has been reduced from its original 6 mm diameter (with the lasers blocked). This corresponds to an increase of *both* brightness *and* intensity by a factor of more than 20. Furthermore, this increase may well be hundreds of times larger, but limitations imposed by the resolution of the imaging system may prevent its measurement.

2.3.3 Optical Molasses at High Intensity

The low intensity description above is appropriate when excited atoms return to the ground state preferentially by spontaneous emission because the rate of stimulated emission is much smaller than γ. When this is not the case, there is a fundamental change in the nature of the optical force.

The force at high intensity is most easily described in the dressed atom picture of the atom-light interaction [39]. Here we choose to write a three part Hamiltonian: the ordinary atomic part normally used in the Schroedinger equation, the radiation part $\hbar\omega_L(a^\dagger a + 1/2)$ whose eigenvalues are $E_n = (n + 1/2)\hbar\omega_L$, and the atom-field interaction. The energy level diagram of the first two parts consists of the ordinary atomic energies repeated for each value of n and vertically displaced by $\hbar\omega_L$ each time, as shown schematically in Fig. 2.4. We are primarily interested in the two atomic states coupled by the laser light,

and we observe that they form closely spaced pairs of one excited state and one ground state separated by $\hbar(\omega_L - \omega_A) = \hbar\delta$, as shown in Fig. 2.4. These pairs become degenerate for $\delta = 0$.

The third term in the Hamiltonian, the interaction between the atom and the radiation field, couples the ground and excited states that form each of these pairs through the off-diagonal matrix elements $< g|e\vec{\epsilon}_L \cdot \vec{r}|e >$. The eigenvectors of the Hamiltonian including this interaction are called the "dressed states" of the atom in the field [39]. This interaction splits the energy levels further apart to $\hbar\Omega \equiv \hbar\sqrt{\gamma^2 s/2 + \delta^2}$ where s is the saturation parameter defined in Eq. (1.1). Ω is independent of the sign of δ, and the shift $\hbar(\Omega - |\delta|)/2$ is the light shift of each dressed state. The light also mixes the states by an amount expressed in terms of a mixing angle ϕ given by $\cos(2\phi) \equiv -\delta/\Omega$, so that each ground state is mixed with a component of excited state and vice versa.

The connection between the dressed atom energy levels and the optical force derives from the spatial dependence of the energy level shifts caused by the standing wave field [40]. Since the light intensity in a standing wave varies between zero at the nodes and a maximum at the antinodes, the energy levels of the atom are sinusoidally modulated in space as shown in Fig. 2.4. Moving atoms experience this spatially dependent energy as a potential, and exchange kinetic with potential energy as if moving up and down hills while they move through the optical field. The possibility for a velocity dependent force arises because the atoms can undergo spontaneous decays from a sinusoidal potential in one manifold to one in another manifold, and those potential curves may not necessarily be the same ones.

Consider an atom moving as shown in Fig. 2.4. We choose the case $\delta > 0$ so that the state $|1>$ is a ground state in the node where Ω is a minimum and its energy is lowest. A moving atom cannot undergo spontaneous emission from such a state in the node where it is a pure ground state, but must decay where there is some excited state mixed in by the atom-field interaction, as shown in Fig. 2.4. If the decay is to another state $|1>$ in a different manifold, the atomic motion in the potential is unaffected, but if the decay is to a state $|2>$ in a different manifold, the phase of the hill climbing and descending is reversed.

By contrast, an atom in state $|2>$ is more likely to undergo a spontaneous decay near a node where it is a pure excited state than at an antinode where its wavefunction has an admixture of ground state. If the decay is to a state $|1>$, again the phase of motion in the sinusoidal potential is reversed. The result of such a sequence of decays is that atoms climb more hills than they descend on the average, radiating light of average frequency higher than ω_L from the tops of the hills. Thus they convert kinetic energy into potential energy, and then radiate away the potential energy. Their motion is thereby damped by the light field, and the atoms are cooled. This cooling scheme works only for the case of $\delta > 0$ which is exactly opposite of the detuning needed for laser cooling in low intensity light as discussed earlier for OM.

The force derives from exchange of momentum between the atoms and the

light field, but in this case the atomic processes are entirely stimulated. Atoms are excited by light from one beam and stimulated to emit into the other, thus exchanging $2\hbar k$ with the field for each cycle. Such processes produce optical coherences between the ground and excited states, manifest by the mixing of the atomic dressed states. The rate of these processes is not limited by γ, but increases with the light intensity. This force can therefore be very much larger than the $\hbar k\gamma/2$ limit associated with spontaneous emission processes.

This "blue cooling" at high intensity has been demonstrated for several atoms [41, 42, 11, 43]. More quantitative calculations show how the force reverses sign as the light intensity is increased from low to high intensity[44] and describe the phenomenon in terms of virtual bosons called Dopplerons[45]. The cooling mechanism is reminiscent of the Stark cooling of Rydberg states described above in section 1.2.1[26], where atoms move uphill more than downhill because of spontaneous emission between different states.

2.4 Experiments in Three Dimensional Optical Molasses

Optical molasses experiments can also work in three dimensions at the intersection of three mutually orthogonal pairs of opposing laser beams (see Ref. [6] and Fig. 2.4). Even though atoms can be collected and cooled in the intersection region, it is important to stress again that this is *not* a trap. That is, atoms that wander away from the center experience no force directing them back. They are allowed to drift freely and even escape, as long as there is enough time for their very slow movement to allow them to reach the edge of the region of the intersection of the laser beams. Because the atomic velocities are randomized during the damping time $1/\gamma_R$, atoms execute a random walk with a step size of $\sqrt{\hbar\gamma/M}/\gamma_R \cong$ few μm. To diffuse a distance of 1 cm requires about 10^7 steps or about half a minute[46, 47].

Three dimensional OM was first observed in 1985[30]. Preliminary measurements of the average kinetic energy of the atoms were done by blinking off the laser beams for a fixed interval. Comparison of the brightness of the fluorescence before and after the turnoff was used to calculate the fraction of atoms that left the region while it was in the dark. The dependence of this fraction on the duration of the dark interval was used to estimate the velocity distribution and hence the temperature. The result was not inconsistent with the two level atom theory described above.

Soon other laboratories had produced 3-D OM. The photograph in Fig. 2.4 shows OM in Na at the laboratory in the National Bureau of Standards in Gaithersburg. The phenomenon is readily visible to the unaided eye, and the photograph was made under ordinary snapshot conditions. The three mutually perpendicular pairs of laser beams appear as a star because they are viewed

along a diagonal.

This group developed a more accurate ballistic method to measure the velocity distribution of atoms in OM[48]. The limitation of the first measurements was determined by the size of the OM region and the unknown spatial distribution of atoms[30]. The new method used a separate measuring region composed of a 1-D OM about 2 cm below the 3-D region, thereby reducing the effect of this limitation. When the 3-D lasers were shut off, the atoms dropped into the 1-D region and the time-of-arrival distribution was measured. This was compared with calculated distributions for T_D and 40 μK as shown in Fig. 2.4. Using a series of plots like Fig. 2.4 it was possible to determine the dependence of temperature on detuning, and that is shown in Fig. 2.4 along with the theoretical calculations.

It was an enormous surprise to observe that the ballistically measured temperature of the Na atoms was as much as ten times *lower* than T_D= 240 μK[48], the temperature minimum calculated from the theory. This breaching of the Doppler limit forced the development of a new picture of OM that accounts for the fact that in 3-D, a two level picture of atomic structure is inadequate. The multilevel structure of atomic states, and optical pumping among these sublevels, must be considered in the description of 3-D OM.

These experiments also found that OM was less sensitive to perturbations and more tolerant of alignment errors than was predicted by the 1-D, two level atom theory. For example, if the intensities of the two counterpropagating laser beams forming an OM were unequal, then the force on atoms at rest would not vanish, but the force on atoms with some non-zero drift velocity would vanish. This drift velocity can be easily calculated by using Eq. (2.1) with unequal intensities s_+ and s_-, and following the derivation of Eq. (2.2). Thus atoms would drift out of an OM, and the calculated rate would be much faster than observed by deliberately unbalancing the beams in the experiments[31].

FIGURE CAPTIONS

Figure

Velocity dependence of the optical damping forces for one-dimensional optical molasses. The two dotted traces show the force from each beam, and the solid curve is their sum. The straight line shows how this force mimics a pure damping force over a restricted velocity range.

Figure

Overall schematic of the apparatus used for one-dimensional transverse cooling. (Figure from Ref. [37]).

Figure

Scheme for optical brightening of an atomic beam. First the transverse velocity components of the atoms are damped out by an optical molasses, then the atoms are focused to a spot, and finally the atoms are recollimated in a second optical molasses. (Figure from Ref. [37]).

Figure

Schematic diagram of the neutral-atom camera showing the repeller grid, the hot grid, the multichannel plates, and the phosphor screen. Atoms are ionized at the hot grid, directed toward the MCP's by the field between it and the repeller, and accelerated toward the MCP's by the voltage between them and the hot grid. The output electrons excite the phosphor, which is viewed by the TV camera. PC is a personal computer. (Figure from Ref. [38]).

Figure

Collimation of an atomic beam in one dimension by optical molasses as pictured by the arrangement of Fig. 2.4. The outline of the circular beam spot represents a 6 mm diameter image on the phosphor, the saturation parameter $S = 8$, and the laser detuning was (a) -27 MHz and (b) +27 MHz. The intensity on the phosphor screen was recorded during 1/30 s and no image averaging was performed. (Figure from Ref. [37]).

Figure

Image formed by the neutral atom camera of Fig. 2.4 with two-dimensional molasses acting on the atomic beam. The outline of the circular beam spot represents a 6 mm diameter image on the phosphor. The 7 mW molasses laser beam was nearly uniformly intense and rectangular, about 8x20 mm. Its detuning was about -30MHz for (a) and about +30 MHz for (b). Note the collimation for the red detuning and the divergence for the blue detuning. Again the recording time was 1/30 s and no image averaging was performed. (Figure from Ref. [37]).

Figure

Energy level diagram for the atom plus field Hamiltonian. In each vertical column there is the familiar level scheme of a typical atom, but the columns are vertically displaced by $\hbar\omega_{laser}$ because of the addition of one laser photon per column. The nearly degenerate pairs are indicated

Figure

The nearly degenerate pairs of energy levels of Fig. 2.4. In the presence of the coupling interaction, each of these is a mixture of ground and excited states,

so each can decay by spontaneous emission as indicated. (Figure from Ref. [40]).

Figure

The spatial dependence of the energy levels of Fig. 2.4. Two level atoms moving in such a light field can decay as shown, and the most probable decay point is from the tops of the hills. Since atoms may fall to the bottoms of hills, energy is lost on the average. (Figure from Ref. [40]).

Figure

Schematic diagram of the arrangement of laser beams for 3-D optical molasses. Three mutually perpendicular standing waves are formed by reflecting 3 laser beams from mirrors. Because of the red-detuned laser light, atoms experience a friction force in all directions and are therefore confined in a viscous medium, the optical molasses.

Figure

Photograph of optical molasses in Na taken under ordinary snapshot conditions in the lab at N.I.S.T. The upper horizontal streak is from the slowing laser while the three beams that cross at the center are on mutually orthogonal axes viewed from the (111) direction. Atoms in the optical molasses glow brightly at the center. (Figure from Ref. [49]).

Figure

Data from dropping atoms out of optical molasses into a probe beam about 18 mm below. The calculated time-of-flight spectra are for 240 μK and 40 μK. The shaded area indicates the range of error in the 40 μK calculation from geometric uncertainties. The width of the data is slightly larger than the calculation, presumably because of shot-to-shot instabilities. (Figure from Ref. [48]).

Figure

Temperature vs detuning determined from time-of-flight data for various separations d between the optical molasses and the probe laser. The solid curve represents the measured molasses decay rate; it is not a fit to the temperature data, but its scale (shown at right) was chosen to emphasize its proportionality to the temperature data. The dashed line shows the temperature expected on the basis of the two-level atom theory. (Figure from Ref. [48]).

Chapter 3

Cooling Below the Doppler Limit

3.1 Introduction

In response to the surprising measurements of temperatures below T_D, two groups developed a model of laser cooling that could explain the lower temperatures[50, 51]. The single most important feature of these models is the non-adiabatic response of a moving atom to the light field. Atoms at rest in a steady state have ground state orientations reflecting the local light field caused by optical pumping processes that distribute the populations over the different ground state sublevels. In the low light intensity regime the orientation of the atoms is completely determined by the ground state distribution: the optical coherences and the exited state population follow the ground state distribution adiabatically.

For atoms moving in a light field that varies in space, optical pumping will act to adjust the atomic orientation to the changing conditions of the light field. In a weak pumping process the orientation of moving atoms will always lag behind the orientation that would exist in steady state. It is this phenomenon of non-adiabatic following that is the essential feature of the new cooling process. Implicit in this process is the requirement that the ground state have multiple sublevels, and an unexpected characteristic is that such atoms can be cooled more effectively than two level atoms[49].

Production of spatially dependent optical pumping processes can be achieved in several different ways. As an example we consider two counter propagating laser beams that have orthogonal polarizations. The superposition of the two beams will cause the resulting light field to have a polarization that varies over a wavelength along the direction of the laser beams. Laser cooling by such a light field is called polarization gradient cooling. In a three-dimensional optical molasses the transverse wave character of light requires that the light field always

has polarization gradients.

Another way to make optical pumping spatially dependent is to use a standing wave of constant polarization and an additional field such as a uniform DC magnetic field. Since the standing wave light field has nodes and anti-nodes, the rate of the optical pumping compared with the rate of Larmor precession of atoms in the magnetic field changes dramatically over a wavelength. The resulting cooling process is called Magnetically Induced Laser Cooling[52] or Magnetic Orientational Cooling[53].

The cooling process that derives from this non-adiabatic following is effective over a limited range of atomic velocities. The force optimizes for atoms that travel a distance $\lambda/2$ during one optical pumping process. If atoms travel at a lower velocity, they will not have reached a very different part of the optical field before the pumping process occurs; if atoms travel faster they will already go beyond the largest change in the field before being pumped towards another sublevel[54]. Of course the velocity where this force is effective scales with the characteristic distance over which the optical field changes. This can be much larger than λ for two light waves with oblique $\vec{k}$ vectors.

Note that the nature of the new cooling process is fundamentally different from the cooling process discussed in the previous chapter. In that case, the differential absorption from the laser beams was caused by the Doppler shift of the laser frequency, and the process is therefore known as Doppler cooling. In the cooling process described in this chapter, the force is still caused by differential absorption of photons from the two laser beams, but the the velocity-dependent differential rates, and hence the cooling, relies on the non-adiabaticity of the optical pumping process. Since lower temperatures can be obtained with this cooling process, this process is called sub-Doppler cooling[48, 52, 49].

3.2 Linear $\perp$ Linear Polarization Gradient Cooling

One of the most illustrative models for discussion of sub-Doppler laser cooling was introduced by Dalibard and Cohen-Tannoudji[50] and we follow their work in this section. They considered the case of orthogonal linear polarization of one pair of counter propagating laser beams that damps atomic motion in one dimension. The polarization of this light field varies over half of a wavelength from linear at 45^o to the polarization of the two beams, to σ^+, to linear but perpendicular to the first direction, to σ^-, and then it cycles. To study the effects of this polarization gradient on the cooling process, they considered a $J_g = 1/2$ to $J_e = 3/2$ transition. This is one of the simplest transitions that shows sub-Doppler cooling.

In the place where the light field is purely σ^+, the pumping process drives the ground state population to the $M_g = +1/2$ sublevel. This optical pumping

occurs because absorption always produces $\Delta M = +1$ whereas the subsequent spontaneous emission produces $-1 \leq \Delta M \leq +1$. Thus $\Delta M \geq 0$ for each scattering event, and its average is positive. For σ^--light the population will be pumped toward the $M_g = -1/2$ sublevel. Thus in travelling through a half wavelength in the light field, atoms have to readjust their population completely from $M_g = +1/2$ to $M_g = -1/2$ and back.

3.2.1 Light Shifts

The interaction between nearly resonant light and atoms not only drives transitions between atomic energy levels, but also shifts their energies. These shifts are essentially caused by the Stark effect from the electric field of the light. This light shift of the atomic energy levels plays a crucial role in this scheme of sub-Doppler cooling, and the changing polarization has a strong influence on the light shifts. In the low-intensity limit of two laser beams each of intensity sI_s, the light shift ΔE_g of the ground magnetic substates are given by

$$\Delta E_g = \frac{2\hbar\delta s C_{ge}}{1 + 4\delta^2/\gamma^2} \tag{3.1}$$

where C_{ge} is the Clebsch-Gordan coefficient that describes the coupling between the atom and the light field. The other quantities are defined immediately following Eq. (1.1). These coefficients are given in Fig. 3.9 for a $J_g = 1/2$ to $J_e = 3/2$ transition. Since C_{ge} depends on the magnetic quantum numbers and on the polarization of the light field, the light shifts are different for different magnetic sublevels. The ground state light shift is negative for a laser tuning below resonance ($\delta < 0$) and positive for $\delta > 0$.

In the present case of orthogonal linear polarizations, the light shift for the magnetic substate $M_g = 1/2$ is three times larger than that of the $M_g = -1/2$ substate when the light field is completely σ^+. On the other hand, when the light field becomes σ^-, the shift of $M_g = -1/2$ is three times larger. So in this case the optical pumping discussed above causes there to be a larger population in the state with the larger light shift. This is generally true for any transition J_g to $J_e = J_g + 1$. A schematic diagram showing the populations and light shifts for this particular case of negative detuning is shown in Fig. 3.9.

3.2.2 Origin of the Damping Force

To discuss the origin of the cooling process in this polarization gradient scheme, we consider atoms with a velocity v at a position where the light is σ^+-polarized. The light will optically pump such atoms to the strongly negative light-shifted $M_g = +1/2$ state. In moving through the light field, atoms must increase their potential energy (climb a hill) because the polarization of the light is changing and the state $M_g = 1/2$ becomes less strongly coupled to the light field. Atoms arriving at a position $\lambda/4$ away where the light field is now σ^--polarized will

be optically pumped to $M_g = -1/2$ which is now lower than the $M_g = 1/2$ state. Again the moving atoms are at the bottom of a hill and start to climb. In climbing the hills the kinetic energy is converted to potential energy, and in the optical pumping process the potential energy is radiated away since the spontaneous emission is at a higher frequency than the absorption. Thus atoms seem to be always climbing hills and losing energy in the process. This process brings to mind a Greek myth and is thus called "Sisyphus cooling". In this sense it is similar to the Stark cooling discussed near the end of section 1.2.1[26] and the high intensity "blue cooling" discussed in section 2.3.3[40, 41] above.

The cooling process described above is effective over a limited range of atomic velocities. The damping is maximum for atoms that undergo one optical pumping process while traveling over a distance $\lambda/4$. Slower atoms will not reach the hilltop before the pumping process occurs and faster atoms will already be descending the hill before being pumped towards the other sublevel. In both cases the energy loss will be smaller and therefore the cooling process less efficient.

We can estimate the damping force $F = \beta v$ from the distance dependence of the energy loss. We denote the optical pumping time by $\tau_p \equiv 1/\gamma_p$ and find the optimum speed to be $v_c = \gamma_p/k$. The force at this velocity v_c is $F = \Delta W/\Delta z \cong \mp \Delta E k \cong -\Delta E k^2 v_c/\gamma_p \equiv -\beta v_c$ whence $\beta = k^2 \Delta E/\gamma_p$. To find the order of magnitude of the friction coefficient β we have to estimate the light shift ΔE and the pumping rate γ_p. For a detuning $|\delta| >> \gamma$ we find from Eq. (1.1) that $\gamma_p s\gamma^3/2\delta^2$ (accounting for the presence of two laser beams), and from Eq. (3.1) we find that $\Delta E \hbar \gamma^2 s/2\delta$, whence $\beta \hbar k^2 \delta/\gamma$. It is interesting that β becomes larger when the optical pumping rate γ_p becomes smaller. Although this seems counter-intuitive, it happens because v_c becomes smaller when γ_p becomes smaller.

This result is of particular significance since it shows that the friction coefficient for this sub-Doppler process is larger by a factor $(2|\delta|/\gamma)$ than the maximum friction coefficient for Doppler cooling. It can be shown that the momentum diffusion coefficient of this process is of the same order of magnitude as that of Doppler cooling, so that the temperature will be smaller then the Doppler temperature by the same factor. Furthermore, it shows that the friction coefficient for this case is independent of intensity, since both ΔE and γ_p depend linearly on the intensity.

3.3 Magnetically Induced Laser Cooling

Although the first models to discuss sub-Doppler cooling relied on the polarization gradient of the light field, it was soon realized that a light field of constant polarization in combination with a magnetic field could cause sub-Doppler cooling[55]. In this process the atoms are cooled in a standing wave of circular polarized light.

A simple model was presented[52] where the authors also used the $J_g = 1/2$

to $J_e = 3/2$ transition. In the absence of a magnetic field, the σ^+ light field drives the population to the $M_g = +1/2$ sublevel. Since the $M_g = +1/2$ sublevel is more strongly coupled to the light field than $M_g = -1/2$, the light shift of this state is larger. Thus atoms travelling through this standing wave will descend and climb the same potential hills and will experience no average force.

The situation changes if a small transverse magnetic field is applied. Optical pumping processes determine the atomic states in the antinodes where the light is strong. But in the nodes, where the intensity of the light field is zero, the small transverse magnetic field precesses the population from $M_g = 1/2$ towards $M_g = -1/2$. Atoms that leave the nodes with $M_g = -1/2$ are optically pumped in the antinodes to $M_g = +1/2$ by the σ^+ light.

The cooling process is depicted in Fig. 3.9 for negative detuning. Potential energy is radiated away in the optical pumping process, and kinetic energy is converted to potential energy when the atoms climb the hills again into the nodes. Again the cooling process is caused by a 'Sisyphus' effect, similar to the case of lin $\perp$ lin. The whole process is repeated when the atoms travel through the next node of the light field. Since this damping force is absent without the magnetic field, it is called Magnetically Induced Laser Cooling (MILC).

Efficient cooling by MILC depends critically on the relation between the Zeeman precession frequency ω_Z and the optical pumping rate γ_p in the antinodes. It is clearly necessary that $\gamma_p >> \omega_Z$ in the antinodes where the light is strong. But as in any cooling process that depends on non-adiabatic following, there is a limited velocity range where the force is effective. For MILC the velocity can not be too small compared to ω_Z/k or atoms will undergo many precession cycles at the nodes and no effective cooling will result. On the other hand if the velocity is large compared to γ_p/k, then atoms will pass through the antinodes in a time to short to be optically pumped to $M_g = +1/2$ and no cooling will result either. Thus there are two conditions on the experimental parameters that can be combined to give

$$\omega_Z < kv < \gamma_p. \tag{3.2}$$

Sub-Doppler cooling has been observed for MILC as shown in Fig. 3.9 for rubidium atoms cooled on the $\lambda = 780$ nm transition in one dimension[52]. The spread of the cooling peak around $v = 0$ is as low as 2 cm/s, much lower than the one dimensional Doppler limit $v_D = \sqrt{7\gamma/20k_B}$, which is 10 cm/s for Rb.

3.4 $\sigma^+ - \sigma^-$ Polarization Gradient Cooling

Dalibard and Cohen-Tannoudji[50] discussed another model for sub-Doppler cooling where the polarization vectors of the two laser beams are also orthogonal, but in this case circularly polarized. The resulting polarization of the light field is linear everywhere, but the direction rotates through an angle 2π over one optical wavelength[56]. In the basis where the quantization axis is along the

direction of the linear polarization, the light shifts are independent of position. However, the quantization axis of this basis is not fixed in space, but rotates around the direction of the laser beams.

The models discussed in sections 3.2 and 3.3 both relied on optical pumping between states that have spatially varying light shifts. Since the pumping process always preferentially pumps the atoms towards states with larger light shift for a J_g to $J_e = J_g + 1$ transition, a negative detuning will always pump the atoms to states with the lowest energy and therefore cool the atoms by dissipating energy. However, it is clear that the sub-Doppler force in this $\sigma^+ - \sigma^-$ case cannot rely on a 'Sisyphus' effect because there are no "hills" and "valleys". Still, there is a non-adiabaticity present for atoms moving in such a light field. Sub-Doppler cooling in such a light field requires at least a $J_g = 1 \Leftrightarrow J_e = 2$ transition.

For atoms at rest in the light field, optical pumping tends to redistribute the populations among the magnetic substates according to the local direction of the linearly polarized light, so the $M_g = 0$ sublevel will be populated most strongly and the sublevels with $M_g = \pm 1$ will be populated less. By contrast, moving atoms experience a rotation of the quantization axis, and must be optically pumped in order to follow it. Thus the population of the ground magnetic substates will always lag behind the steady state distribution appropriate to the local field, i.e., the polarization direction.

Dalibard and Cohen-Tannoudji showed that this non-adiabatic following will populate the state with $M_g = +1$ more then the state with $M_g = -1$ for atoms travelling toward the laser beam with σ^+ polarization, and vice versa for atoms travelling in the opposite direction. Even a small imbalance in the population can produce a very large damping force. This is because the $M_g = 1$ sublevel scatters the σ^+ light 6 times more efficiently than the σ^- light because of the different Clebsch-Gordon coefficients. Since the atoms remain in the $M_g = 1$ sublevel after absorption of a σ^+ photon followed by spontaneous emission, atoms travelling towards the σ^+ beam scatter many photons from it and experience a large momentum change in the direction opposite to their motion. Atoms travelling towards the σ^- beam are preferentially pumped toward the $M_g = -1$ sublevel from which they strongly scatter photons from the σ^--beam and also recoil in the opposite direction.

The atomic motion is clearly damped, and in this case the cooling mechanism also relies on a differential scattering of light from the two laser beams. However, in this case the differential scattering is NOT caused by the difference in Doppler shifts of the two laser beams as in Doppler cooling, but by the imbalance in the populations caused by the time lag in the following of the atomic orientation to the local field.

It is difficult to assess from the discussion above how large the friction coefficient and the diffusion coefficient for the $\sigma^+ - \sigma^-$ cooling process will be. Dalibard and Cohen-Tannoudji show that both coefficients remain smaller by approximately the same amount compared to the lin $\perp$ lin configuration, so

comparable temperatures are to be expected for the two cases[50]. However, the reduction of the friction coefficient can be important in experiments, for cooling times will become longer and the effect of perturbations to the cooling process will have a larger impact.

3.5 Cooling in a Strong Magnetic Field

The sub-Doppler cooling model called MILC discussed in section 3.3 was restricted to the case where the Larmor precession rate ω_Z is small compared to the optical pumping rate γ_p in the antinode. Thus the perturbation of the transverse magnetic field on the magnetic substates could be neglected and the precession causes only a redistribution of the population at the nodes of the light field. At larger magnetic field this picture breaks down and there are new phenomena in sub-Doppler laser cooling.

With $\omega_Z \gg \gamma_p$ it is convenient to choose the quantization axis along the magnetic field direction and describe the optical pumping as a small perturbation. The Optical Bloch Equations (OBE) for the atomic density matrix can then be solved by transforming to a frame rotating with a frequency ω_Z around the magnetic field direction[57]. Neglecting terms that oscillate at twice the rotation frequency, an approximate solution to the OBE can be found that shows resonances for $2\vec{k} \cdot \vec{v}_r = \pm\omega_Z$[57]. The nature of these resonances can be inferred from Fig. 3.5. By choosing the quantization axis along the magnetic field direction, the ground states are split by an amount $\hbar\omega_Z$ and the light field can now induce both σ and π transitions. When the opposite Doppler shifts of the two laser beams combine to match the difference in ground state energies, one can expect the Raman transitions between the two ground states to become resonant. The condition for this resonance is $(\vec{k}_1 - \vec{k}_2) \cdot \vec{v} = \omega_Z$.

The force for this case can be written as

$$F = \frac{-\beta(v - v_r)}{1 + \left(\frac{v - v_r}{v_c}\right)^2} \tag{3.3}$$

where both the friction coefficient β and capture velocity v_c are comparable to the corresponding parameters for other sub-Doppler processes[50]. However, the cooling no longer drives the atoms towards $v = 0$, but to $v = v_r = \omega_Z/2k$, and this resonance velocity depends only on the magnetic field strength. Since the cooling process relies on the velocity of the atom to shift the Raman transition into resonance, the model is called the Velocity Selective Resonance (VSR) model. Note that the laser frequency can be detuned far from atomic resonance and thus the Doppler shift of the laser light does not produce any appreciable difference in the absorption rate of the two beams. However, the Doppler shift is important for the resonance in the Raman transition between the ground states, which can be much narrower than the natural width γ[58].

Measurements in a one-dimensional molasses of rubidium atoms have verified this model[57]. Fig. 3.9 shows the result when the magnetic field is increased from small values up to 1 Gauss. There is cooling towards zero velocity at small magnetic field, as discussed in section 3.3. However, at larger magnetic field the central peak starts to split into two peaks, symmetric around the center of the profile. The splitting of the two peaks Δv_p is plotted in Fig. 3.9 as a function of the magnetic field for various values of the detuning and the laser intensity for the two isotopes of rubidium. The straight lines are given by the condition $\Delta v_p = 2v_r = \omega_Z/k$ with the appropriate g_F-factor for each isotope. There is good agreement between the data and the VSR model.

Figure 3.9 also shows the results for blue detuning of the laser in the lower traces. Since the force vs. velocity curve is reversed when the detuning is changed from negative to positive, there is heating for positive detuning at small B field. However, at large B field there is clearly sub-Doppler cooling at zero velocity. This is related to the cooling by blue-tuned optical molasses discussed in section 2.3.3, where a coherence is established between the ground and excited states of a two level atom[40, 41, 44, 45]. In the present case, however, the coherence is established between two magnetic sublevels by the strong magnetic field, and the laser intensity is always very low.

3.6 VSR and Polarization Gradients

The VSR picture can be extended to include the effects of polarization gradient cooling processes[59], and Fig. 3.9 shows some of the most interesting cases. To establish the Raman resonance condition, one has to examine carefully the possible two photon couplings between different states. In the case of a strong magnetic field these rely solely on the direction of the magnetic field and of the polarization vectors of the two laser beams. From the energy difference in the splitting of the coupled ground states one can then obtain the resonance velocities. Since the light can drive only transitions with $\Delta M = 0, \pm 1$, the total number of resonances in each case is limited to five, namely $\Delta M_g = 0, \pm 1$, and ± 2.

One of the simplest examples of the VSR picture is the case of one-dimensional laser cooling with the $\sigma^+ - \sigma^-$ polarization gradient scheme in a strong magnetic field pointing along the σ^- laser beam direction. The resonance condition can then only be fulfilled between two ground state levels with $\Delta M_g = +2$ (see Fig. 3.9a). Since one laser beam has a σ^+-polarization and the other beam has a σ^--polarization, there is no cooling toward the opposite resonance velocity $v_r = -\omega_Z/k$ because the Raman transition is Doppler tuned out of resonance at this velocity. Cooling in this scheme will be toward $v_r = +\omega_Z/k$, which is twice the resonance velocity in the case of MILC.

Figures 3.9 and 3.9a show the experimental results for this configuration. The experiments clearly show cooling to one velocity, and at zero magnetic field

it is to $v_r = 0$. Increasing the magnetic field leads to a deflection of the peak towards the right, decreasing the magnetic field leads to a deflection to the left. Note that the atoms are not simply deflected, but also cooled towards this resonance velocity, and the width of the peaks in all cases is below the Doppler limit.

Another interesting example of VSR is the case of orthogonal linear polarizations, where the magnetic field is directed along the polarization vector of one of the laser beams. This beam therefore induces $\Delta M = 0$ transitions, whereas the other beam induces $\Delta M = \pm 1$ transitions (Fig. 3.9b). Thus the selection rules can only be satisfied for $\Delta M_g = \pm 1$, or equivalently $v_r = \pm\omega_Z/2k$. Since the problem is not symmetric with respect to reversing the atomic velocity, cooling towards $v_r = +\omega_Z/2k$ is different from cooling towards $v_r = -\omega_Z/2k$.

Typical experimental results are shown in Fig. 3.9b. At zero magnetic field one observes the cooling towards $v = 0$ as described in section 3.2. Increasing the field leads to two distinct peaks whose splitting is proportional to the magnetic field strength. Note that the two peaks are not symmetrical. Changing the direction of the magnetic field from parallel to one of the polarization vectors to parallel to the other one changes the resonance from $v_r = +\omega_Z/2k$ to $v_r = -\omega_Z/2k$ and vice versa. This is also observed in the experiments, and the velocity of atoms in each peak satisfies the VSR condition $v_r = \pm\omega_Z/k$ as shown in Fig. 3.9.

Finally we discuss another case where the atoms are cooled in the lin $\perp$ lin polarization gradient configuration, but now with the magnetic field direction at an angle of 45^o with respect to both polarization vectors. Each laser beam can induce transitions $\Delta M = 0, \pm 1$ in a quantization frame along $\vec{B}$, so all five resonance conditions can be fulfilled. The resonance condition for $v_r = 0$ is particularly interesting (see Fig. 3.9c). The force in this case derives from a redistribution process that transfers photons from one laser beam to the other in stimulated processes that returns atoms to their original ground state.

The experimental results of Fig. 3.9c at strong magnetic field show three of the five resonances discussed at the beginning of this section. Numerical calculations show that the resonances for $\Delta M_g = \pm 2$ are small compared to the others so they cannot easily be detected. The force for the three other resonances is also weaker than in the previous configurations, consistent with the low signal-to-noise ratio of the data. However, the peak at $v_r = 0$ can clearly be distinguished at a strong magnetic field.

3.7 Theory of sub-Doppler laser cooling

The results discussed in the preceding sections all apply to cases where the number of evolution equations for the density matrix elements are small, but real atoms have a much richer structure than these simple cases. For example, cooling of the alkalis Na, Rb and Cs is achieved on the $F = 2 \Leftrightarrow 3$, $F =$

$3 \Leftrightarrow 4$ and $F = 4 \Leftrightarrow 5$ transitions respectively. It is commonly accepted that the principles of sub-Doppler laser cooling discussed for the simple cases are applicable to these more complicated transitions. An operator description of sub-Doppler laser cooling for any given transition for any given polarization of the laser beams in the presence of external fields has recently been published[60].

The theory considers the case of an atom moving through a monochromatic radiation field of frequency ω. The electric field is assumed to be classical and is given by

$$\vec{E}(\vec{R},t) = \vec{E}_+(\vec{R})e^{-i\omega t} + \vec{E}_-(\vec{R})e^{+i\omega t}. \tag{3.4}$$

In principle the theory is applicable to laser cooling in three dimensions, but we will describe only the case of one-dimensional cooling, where the velocity $\vec{v}$ is in the direction of the laser beams. The atom-field coupling is given by the Rabi operator $\mathcal{R} = \vec{\mu}_{eg} \cdot \vec{E}_+/\hbar$ which has the magnitude of the Rabi frequency. Also, the force operator $\vec{\mathcal{F}}$ in the radiation field can then be written as the commutator of the momentum operator with the Hamiltonian, or

$$\vec{\mathcal{F}} = \vec{\nabla}\left(\vec{\mu}\cdot\vec{E}\right) = \hbar\left(\vec{\nabla}\mathcal{R} + \vec{\nabla}\mathcal{R}^\dagger\right). \tag{3.5}$$

In the low-intensity limit, where the lowest temperatures for sub-Doppler cooling are observed, the optical coherences and the excited state can be eliminated from the evolution equations of the atomic density matrix because they follow the ground state adiabatically. The force in the steady state can then be written in terms of the ground state density matrix $\sigma_{gg}(t)$

$$\vec{F}(t) = \mathrm{Tr}\{\sigma_{gg}(t)\vec{\mathcal{F}}_{eff}(t)\} \tag{3.6}$$

with

$$\vec{\mathcal{F}}_{eff}(t) \equiv \frac{-i\hbar}{\gamma/2 + i\delta}\mathcal{R}^\dagger\vec{\nabla}\mathcal{R} + \frac{i\hbar}{\gamma/2 - i\delta}(\vec{\nabla}\mathcal{R}^\dagger)\mathcal{R}. \tag{3.7}$$

After solving the evolution equations for the ground state density matrix, Eq. (3.6) can then be used to calculate the force.

It is interesting to split the Rabi operator in two parts $\mathcal{R}(R) = \theta(R) \times \Omega(R)$ with $\Omega(R)$ the Rabi frequency containing the position dependent strength of the electric field and $\theta(R)$ the coupling of the atom to the field. This can also be position dependent because of the changing polarization vector of the local field. We can expand Eq. (3.7) as $\vec{\mathcal{F}}_{eff} = \vec{\mathcal{F}}_1 + \vec{\mathcal{F}}_2 + \vec{\mathcal{F}}_3 + \vec{\mathcal{F}}_4$ with

$$\vec{\mathcal{F}}_1 = \frac{i\hbar\gamma/2}{(\gamma/2)^2 + \delta^2}\theta^\dagger\theta\left\{(\vec{\nabla}\Omega^*)\Omega - \Omega^*\vec{\nabla}\Omega\right\} \tag{3.8}$$

$$\vec{\mathcal{F}}_2 = \frac{-\hbar\delta}{(\gamma/2)^2 + \delta^2}\theta^\dagger\theta\left\{(\vec{\nabla}\Omega^*)\Omega + \Omega^*\vec{\nabla}\Omega\right\} \tag{3.9}$$

$$\vec{\mathcal{F}}_3 = \frac{i\hbar \mid \Omega \mid^2 \gamma/2}{(\gamma/2)^2 + \delta^2}\left\{(\vec{\nabla}\theta^\dagger)\theta - \theta^\dagger\vec{\nabla}\theta\right\} \tag{3.10}$$

$$\vec{\mathcal{F}}_4 = \frac{i\hbar \mid \Omega \mid^2 \gamma/2}{(\gamma/2)^2 + \delta^2} \left\{ (\vec{\nabla}\theta^\dagger)\theta + \theta^\dagger \vec{\nabla}\theta \right\} \tag{3.11}$$

The force operator $\vec{\mathcal{F}}_1$ depends on the gradient of the Rabi frequency and is the well known radiation pressure, proportional to the phase of the field. The operator $\vec{\mathcal{F}}_2$ is the dipole force operator, determined by the amplitude gradient of the field. The force operators $\vec{\mathcal{F}}_3$ and $\vec{\mathcal{F}}_4$ both arise from a gradient in the polarization direction, and they are related to the forces discussed in the preceding sections describing polarization gradient cooling. Note, that $\vec{\mathcal{F}}_3$ depends on the phase gradient of θ, corresponding to a radiative force, whereas $\vec{\mathcal{F}}_4$ depends on the amplitude gradient of θ, corresponding to a redistribution force.

All force operators $\vec{\mathcal{F}}_i$ have a common prefactor given by

$$\vec{\mathcal{F}}_i \propto \frac{\hbar\vec{k} \mid \Omega \mid^2 \gamma/2}{(\gamma/2)^2 + \delta^2}, \tag{3.12}$$

where the vector $\vec{k}$ derives from the gradient operator $\vec{\nabla}$ on the right side of Eq's. (3.8a-d). This factor is identical to the radiation force in the limit of low intensity for the case of Doppler cooling of a two level atom in section 1.2. Since the remaining factor is of the order 1, the force in sub-Doppler cooling can never exceed the Doppler force. This is to be expected, since the force still derives from the scattering of photons from the two laser beams that form the optical molasses. However, the velocity dependence of the force for the sub-Doppler case, which derives from taking the trace over the density matrix in Eq. (3.6), is much stronger in this case then for the case of Doppler cooling. This leads to an increase in the damping coefficient and, since the diffusion coefficient in the two cases are comparable, leads to lower temperatures as observed in experiments.

After finding the ground state density matrix by solving the evolution equation, Eq. (3.6) can be used for calculation of the force on an atom. Nienhuis et al.[60] studied the case where the atomic velocities are assumed to vary slowly over an optical wavelength. This corresponds to atomic kinetic energies large compared to the potential energy variations caused by the light shift. The periodicity of the problem suggests expansion of the density matrix elements in a Fourier series, and the resulting linear relations for the Fourier coefficients can then be solved numerically. The force is obtained by substitution of the Fourier series in Eq. (3.6), and then averaging over a wavelength.

In Fig. 3.9 we show the force for the case of orthogonal linear polarization for transitions from J_g to $J_e = J_g + 1$ for different J_g's. There is an increase of the damping coefficient when J_g is increased that can easily be understood because, for atoms with larger values of J_g, the states with $M_g = \pm J_g$ are less strongly coupled to the light field. For these atoms the optical pumping towards the other states proceeds at a lower rate. For the optimum in the force the atomic velocity should therefore be lower, whereas the energy loss (the difference in

the light shift) remains the same. Since the damping coefficient depends on the ratio of the energy loss to the velocity, it will increase.

The operator formalism[60] has been extended to the case of laser cooling in stronger magnetic fields by finding operators and calculating forces for the cases described in sections 3.5 and 3.6[61]. An example of the results of such a calculation for the $\sigma^+ - \sigma^-$ configuration described in section 3.6 is shown in Fig. 3.9. For $B = 0$ there is strong damping to $v = 0$, but for larger B the atoms are damped to a non-zero velocity, in this case ω_Z/k, corresponding to the $\sigma^+ - \sigma^-$ VSR illustrated in Fig. 3.9a. Reference[61] shows good agreement between the calculations and the data for all cases, thus putting the VSR view of sub-Doppler laser cooling on firm theoretical grounds.

The momentum diffusion of the atoms can also be obtained using this semi-classical model[60]. It is calculated from the time correlation of the force operator and consists of three terms: (1), the contribution from the random direction of spontaneous emission; (2), the contribution from stimulated processes on a fast time scale caused by the decay of the optical coherences; and (3), the contribution from stimulated processes on a slow time scale caused by the optical pumping among ground states. The first two terms can easily be evaluated at each instant from the local steady state density matrix. The third term depends on the evolution matrix of the ground state density matrix and involves an integration over time.

Substitution of both the force and diffusion as a function of velocity in the Fokker-Planck equation allows us to calculate the temporal evolution of the velocity distribution. The results must be cautiously interpreted however, because both the force and the diffusion calculated with the procedure described above assumes that the atoms are in a steady state, i.e., the interaction time is long compared to the optical pumping time.

The operator description can be used to show that lower light intensity results in lower diffusion, but not in a lower damping coefficient. The temperature in the steady state is given by the ratio of the diffusion to the damping coefficient so lower light field intensity (or proportionally lower external field) lowers the final temperature. Of course, lowering the intensity also lowers the range for which the semi-classical theory is valid. Furthermore, at very low temperatures, where the recoil of the atom by one photon absorption is comparable to the atomic velocity, the Fokker-Planck equation is no longer valid and a quantum theory is necessary.

3.8 Sisyphus Cooling

The reader may note that there are several apparently different cooling schemes described in sections 1.2.1, 2.3.3, 3.2.2 and 3.3 that all share a similar energy loss mechanism. In this section we try to emphasize the similarities of these schemes, and explain the underlying physical principles.

The motion of atoms in a spatially varying potential causes an exchange between kinetic and potential energy. If the potential is periodic in space, so is this exchange. The simplest and most dramatic example is for the light shift of a two level atom in a standing wave detuned from atomic resonance. In this case, the sinusoidal potential for the ground state is exactly out of phase with that of the excited state because their light shifts have opposite signs as shown in Fig. 2.4. For a multilevel atom, different sublevels may be subject to different potentials as shown in Fig. 3.9.

The presence of multiple potentials enables a velocity dependent energy loss mechanism when the populations of the states of moving atoms can be manipulated in some appropriate way. In general, if atoms can be in states that give up large amounts of kinetic energy as they move *up* the potential, and then be switched to states that gain back a smaller amount of kinetic energy as they move *down* the potential, the net effect of multiple cycles is to extract energy from the atoms. In some cases when the multiple potentials are out of phase with one another, the switching can be arranged in such a way that atoms are always moving up a potential hill. Of course, that energy must be accounted for, and it is easily seen that if the switching is done by optical pumping, then the fluoresced light is bluer than the absorbed light, and the energy is radiated away. Thus there is a cyclic refrigeration process that converts kinetic energy into potential energy, and eventually radiates away the potential energy.

Any cooling process must be both dissipative and irreversible in order to satisfy thermodynamics. In the examples discussed above the dissipative aspect comes from the velocity dependence of the force. Atoms moving through a light field are driven to adjust their internal state to the changing conditions. However, this adjustment can not be instantaneous and a certain time lag in their internal state arises. Of course, this non-adiabatic character depends on the atomic velocity. For low velocities the time lag is small and small energy losses result. For higher velocities the time lag increases and higher losses occur, until the velocity is too high for atoms to have a significant response to the light field changes. Thus there is a range of velocities where atoms experience a damping force that is proportional but opposite to their velocities.

The irreversible aspect is the optical pumping process. As long as there are spontaneous emission events, light is radiated into the unoccupied modes of the radiation field and lost from the atomic system. Since this radiation is generally of higher frequency than the laser light, energy is taken out of the atomic system and the atoms are cooled. Since the phase of the spontaneously emitted light is random, information from the atomic system is lost and the process is therefore irreversible. By contrast, stimulated emission puts light back into the radiation field that is driving the atoms, and leaves the atomic wave function with a fixed phase relation to the optical field. This process is responsible for the light shift that produces the spatially varying potential in many of the examples discussed below.

The major differences between the cooling schemes of the four sections cited

above are in the way the optical pumping is tailored to obtain the desired effect. In the case of atoms excited into Rydberg states described near the end of section 1.2.1, excitation occurs in the small region of strong electric field where the laser beams are focussed between a pair of small electrodes. Moving atoms gain potential energy climbing out of the field region and when they undergo spontaneous decay outside this region where the atomic potential energy is higher, they radiatively dissipate the potential energy they gained. Slow atoms decay before they travel very far so they lose little kinetic energy, but fast ones go further uphill and thus lose more energy. By contrast, ground state atoms moving into the field have much smaller Stark shifts, and undergo negligible kinetic energy changes. In summary, the atoms thus move along a level potential until they are excited in the field, then they climb a hill until they decay, which may not be until they reach the flat region at the top of the hill, and then they move along a level potential and repeat the process.

In the second case, the high intensity standing wave of section 2.3.3, atoms can only decay from a point where the excited state component of their wave functions is significant, and this is at the top of potential hills for blue-tuned light. Thus they also dissipate their potential energy into the radiation field. In the third case, described in section 3.2.2, a given pair of atomic ground state magnetic sublevels experiences a spatially varying light shift as atoms move through a polarization gradient. Atoms are optically pumped between sublevels as they move through the regions of varying polarization in just the right way to keep them always moving uphill. The fourth case comes from section 3.3 where there is a low intensity standing wave and a weak $\vec{B}$ field perpendicular to its $\vec{k}$ vector. Atoms are shifted between their ground state sublevels by Larmor precession at the nodes, and by optical pumping at the antinodes, in just the right way to produce a cyclic energy loss as they move through the light field. The Larmor precession is not irreversible, but the optical pumping is, thus satisfying thermodynamic criteria.

The physical notions described in these four examples are often categorized as "Sisyphus cooling" because of the obvious connection with an ancient Greek myth[50, 49]. Furthermore, the concept can be extended to include magneto-optical cooling effects to a finite velocity as described in sections 3.5 and 3.6. Velocity selective resonances are also closely related to Sisyphus cooling.

3.9 Optical Molasses In Three Dimensions

The theoretical models and experimental results discussed so far in this chapter are all for the case of one dimension. The theoretical models are not easily extended to more dimensions and do not provide the same kind of analytical solutions as does 1-D. To be able to compare with theory directly, 1-D experiments are required and these experiments have been discussed in sections 3.5 and 3.6. One of the limitations of 3-D experiments is that they are not able

to study cooling schemes without polarization gradients, since the transverse nature of electromagnetic radiation prevents the construction of 3-D radiation fields with all polarizations parallel.

One of the outcomes of the theory presented in sections 3.3 and 3.5 is that the final temperature $T_{\rm lim}$ in polarization gradient cooling scales with the light shift ΔE_g of the ground states, i.e.

$$k_B T_{\rm lim} = c\Delta E_g, \tag{3.13}$$

where ΔE_g is given by Eq. (3.1). The value of the coefficient c depends on the polarization scheme used and is 0.125 for lin $\perp$ lin and 0.097 for $\sigma^+ - \sigma^-$. Note that lowering the temperature can easily be achieved by lowering the light shift, either by increasing the detuning δ or decreasing the intensity s (see Eq. 3.1). Since this is a result of the semi-classical theory, the temperature will always be limited by the recoil temperature, as discussed at the end of section 3.7.

In the experiments reported by Salomon et al.[62] the temperature was measured in a 3-D molasses under various configurations of the polarization. All beams were linearly polarized, but in one configuration the polarization of two counterpropagating beams was chosen to be parallel to one another and in another configuration they were chosen to be perpendicular. Temperatures were measured by a ballistic technique, where the flight time of the released atoms was measured as they fell through a probe a few cm below the molasses region. The sensitivity of the technique was increased by pushing most of the atoms out of the molasses just before they were released with a specially tailored laser beam. In this way the initial vertical position of the remaining atoms was determined more accurately and therefore the fall time was a better measure of their initial vertical velocity.

Results of their measurements are shown in Fig. 3.9 where the measured temperature is plotted for different detunings as a function of the intensity. For each detuning the data lies on a straight line through the origin. The lowest temperature obtained is 3 μK, which is a factor 40 below the Doppler temperature and a factor 15 above the recoil temperature of Cs. If the temperature is plotted as a function of the light shift (see Fig. 3.9b) all the data is on a single universal straight line. The slope of the line is 0.45 for the parallel configuration and 0.35 for the perpendicular configuration. Both slopes are a factor of about 3 higher then the theoretical estimates of 1-D and the authors ascribe this discrepancy to the 3-fold increase of the number of laser beams.

However, there are a number of differences between the theoretical and experimental situations studied. First, the theory is 1-D, whereas the experiments are 3-D. Second, the level scheme used in the theory is $J_g = 1/2 \Leftrightarrow J_e = 3/2$ (lin $\perp$ lin) or $J_g = 1 \Leftrightarrow J_e = 2$ ($\sigma^+ - \sigma^-$), whereas the cooling transition in Cs is a $F_g = 4 \Leftrightarrow F_e = 5$ transition. Third, the polarization gradient in the 1-D theory is well-defined, whereas in the 3-D experiment atoms see different gradients in different directions and the gradients could change dramatically during the atoms' diffusive movement in the molasses.

In an experiment by Gerz et al.[63] the effect of the angular momentum of the transition on the temperature was studied by exploiting the two isotopes of Rb. In ^{85}Rb the cycling transition is a $F_g = 3 \Leftrightarrow F_e = 4$ transition, whereas in ^{87}Rb the $F_g = 2 \Leftrightarrow F_e = 3$ transition is used. The authors then studied the temperature as a function of the light shift and found a small effect of the slope on the isotope used. The temperatures for ^{85}Rb are 10% lower under the same laser parameters compared to ^{87}Rb indicating an increases damping for higher F as predicted by theoretical results[60].

Simulations of the behavior of alkali atoms in a molasses were performed by both Molmer[64] and Javanainen[65]. Both authors found that in most cases, relation (3.10) holds even in two and three dimensions. However, Javanainen[65] showed that in 3-D the temperature is not just given as the ratio of the diffusion averaged over a wavelength and the damping averaged over a wavelength, since the trajectories of the atom are not straight, but severely altered by the cooling process. These aspects have led to the departure from the semi-classical treatments and toward quantum treatments, which are discussed in section 6.4.3.

FIGURE CAPTIONS

Figure

Squares of the Clebsch Gordan coefficients for the various components of the transition between a ground state with $J = 1/2$ and an excited state with $J = 3/2$.

Figure

The spatial dependence of the light shifts of the ground state sublevels of the $J = 1/2 \Leftrightarrow 3/2$ transition shown in Fig. 3.9 for the case of the lin $\perp$ lin polarization configuration. The arrows show the path followed by atoms being cooled in this arrangement. Atoms starting at $z = 0$ in the $m = +1/2$ sublevel must climb the potential hill as they approach the $z = \lambda/4$ point where the light becomes σ^- polarized, and there they are optically pumped to the $m = -1/2$ sublevel. Then they must begin climbing another hill toward the $z = \lambda/2$ point where the light is σ^+ polarized and they are optically pumped back to the $m = +1/2$ sublevel. The process repeats until the atomic kinetic energy is too small to climb the next hill. Each optical pumping event results in absorption of light at a lower frequency than emission, thus dissipating energy to the radiation field.

Figure

The spatial dependence of the light shifts of the ground state sublevels of the $J = 1/2 \Leftrightarrow 3/2$ transition shown in Fig. 3.9 for the case of magnetically induced cooling in the absence of any polarization gradient. The arrows show the path followed by atoms being cooled in this arrangement. Atoms starting at $z = 0$ in the strongly light-shifted $m = +1/2$ sublevel must climb the potential hill as they approach the node at $z = \lambda/4$. There they undergo Zeeman mixing in the absence of any light and may emerge in the $m = -1/2$ sublevel. They will then gain less energy as they approach the antinode at $z = \lambda/2$ than they lost climbing into the node. Then they are optically pumped back to the $m = +1/2$ sublevel in the strong light of the antinode, and the process repeats until the atomic kinetic energy is too small to climb the next hill. Each optical pumping event results in absorption of light at a lower frequency than emission, thus dissipating energy to the radiation field.

Figure

Typical data of beam collimation using circularly polarized light and a weak magnetic field on a beam of ^{85}Rb atoms (see Fig. 2.4). The scanning hot wire was 1.3 m downstream from the interaction region. The laser parameters are defined as in Eq. (1.1). (Figure from [66]).

Figure

Optical excitation for a $J = 1/2 \Leftrightarrow 3/2$ transition for VSR in σ^+ polarized light with a strong $\vec{B}$-field perpendicular to the $\vec{k}$ vectors. In this case $\vec{B}$ is the appropriate choice for the quantization axis. The energy levels are split by the Zeeman interaction. The dark arrows indicate absorption from one laser beam, light arrows from the other. Stimulated emission processes follow the

same arrows downward, but are not shown. The resonance condition is satisfied for both sets of arrows (each set has one dark and one light), but for one set π absorption is from one laser beam and σ stimulated emission is into the other, and vice versa for the other set.

Figure

The change in atomic-beam profile of ^{85}Rb 1.3 m downstream from the molasses as measured by the hot wire for negative (top) and positive (bottom) detuning. The laser parameters are $s = 0.25$ and $\delta = \pm 0.67\gamma$ and the magnetic field is (a) 0.057 G, (b) 0.114 G, (c) 0.23 G, (d) 0.40 G, (e) 0.57 G, and (f) 1.14 G. The solid lines are experimental data and the dashed lines are the results from the model. (Figure from Ref. [57]).

Figure

The separation between the peaks for many data sets, including those of Fig.3.9, vs magnetic-field strength for the $F = 3 \Leftrightarrow 4$ transition in ^{85}Rb and the $F = 2 \Leftrightarrow 3$ transition in ^{87}Rb. Symbols denote experimental points for various intensities ($0.25 \leq s \leq 10$) and detunings ($1 \leq |\delta| \leq 10\gamma$), where the average longitudinal velocity ($v \sim 350$ m/s) was used to convert the deflection angle into a transverse velocity. The solid lines are for the resonance condition. The laser parameters are defined as in Eq. (1.1). (Figure from Ref. [57]).

Figure

Schematic diagram of atomic transitions at the resonance condition in VSR. (a) The $\sigma^+ - \sigma^-$ case where the ground state energies are split by a magnetic field. VSR between them requires the light frequencies to be different, and in the rest frame of a moving atom this is provided by the Doppler shift. The energy splitting could be much larger (e.g., hyperfine structure) and the light have different laboratory-frame frequencies. (b) The scheme for the linear $\perp$ linear case. (c) The case for $v_r = 0$ when a magnetic field is applied that splits the sublevels by more than γ'_p. Different polarizations at different places cause either $\Delta M_F = \pm 1$ or $\Delta M_F = 0$ VSR. (d) The degenerate case where cooling is to $v = 0$.

Figure

Atomic beam profile of velocity selected ^{85}Rb 1.3 m away from a $\sigma^+ - \sigma^-$ optical molasses with various values of $\vec{B}$ along the $\vec{k}$ vectors. The laser parameters are $s = 3$ and detuning $\delta/2\pi = -12$ MHz. (Figure from Ref. [59]).

Figure

Velocity distributions of ^{85}Rb atoms after passing through optical molasses in a strong $\vec{B}$ field, as determined from the spatial beam profile measurements such as those in Fig. 3.9. The upper trace of each pair is for $\vec{B} \neq 0$ and the lower one is for $\vec{B} = 0$. (a) $\sigma^+ - \sigma^-$ molasses with the $\vec{B}$ field along the laser axis as in Fig. 3.9, (b) lin $\perp$ lin with $\vec{B}$ along one of the laser polarizations, and (c) lin $\perp$ lin with $\vec{B}$ at 45^o to the polarization vectors. (Figure from Ref. [61]).

Figure

The deflection of the peaks similar to those shown in Fig. 3.9b vs $\vec{B}$ for the

lin $\perp$ lin configuration for many data sets on the $F = 3 \Leftrightarrow 4$ transition in ^{85}Rb and the $F = 2 \Leftrightarrow 3$ transition in ^{87}Rb. Symbols denote experimental points for various intensities ($0.25 \leq s \leq 10$) and detunings ($1 \leq |\delta| \leq 10\gamma$), where the average longitudinal velocity ($v \sim 350$ m/s) was used to convert the deflection angle into a transverse velocity. The solid lines are for the resonance condition. Note that the deflection reverses when $\vec{B}$ is reversed. The straight lines are $v = \pm\omega_Z/2k$. The laser parameters are defined as in Eq. (1.1) (Figure from Ref. [59]).

Figure

The calculated force vs velocity curve for the lin $\perp$ lin configuration adapted from Ref. [60].

Figure

The calculated force vs velocity curve for the $\sigma^+ - \sigma^-$ polarization configuration for the case of $B = 0$ and $B \neq 0$[57]. When $B \neq 0$ the force has a dispersion shape centered at a finite $v_r = \omega_Z/k$ where it vanishes. Atoms are cooled to this velocity.

Figure

Temperature as a function of laser intensity and detuning for Cs atoms in an optical molasses from Ref. [62]. a) Temperature as a function of the detuning for various intensities. b) Temperature as a function of the light shift. All the data points are on an universal straight line.

Chapter 4

Magnetic Trapping of neutral Atoms

4.1 Introduction

Electromagnetic trapping of neutral atoms is a relatively new phenomenon in physics that has potential for use in very many areas, including high resolution precision spectroscopy. It has the possibility of leading towards the spectroscopic ideal of an isolated atom at rest, in the dark, available for interaction with electromagnetic field probes. Although ion trapping, laser cooling of trapped ions, and trapped ion spectroscopy have been known for many years[67], it was only in 1985 that neutral atoms were first trapped[68]. Confinement of neutral atoms depends on the interaction between an inhomogeneous electromagnetic field and an atomic multipole moment. Although Earnshaw's theorem prohibits an electrostatic field from stably trapping a charged particle (similarly for magnetic fields and monopoles), dipoles may be trapped by a local field minimum (local maxima are forbidden[69]). Unperturbed atoms do not have electric dipole moments because of their inversion symmetry, and therefore electric traps require induced dipole moments that must be produced by mixing states of opposite parity. This is often done with nearly resonant optical fields, thus producing the optical traps discussed in the next chapter. On the other hand, many atoms have ground or metastable state magnetic dipole moments that may be used for trapping them.

In order to confine any object it is necessary to exchange kinetic for potential energy in the trapping field, and in neutral atom traps, the potential energy must be stored as internal atomic energy. There are two immediate and extremely important consequences of this requirement. First, the atomic energy levels will necessarily shift as the atoms move in the trap, and these shifts will affect the precision of spectroscopic measurements, perhaps severely. Since one of

the potential applications of trapped atoms is in high resolution spectroscopy, such inevitable shifts must be carefully considered. Although research groups in many laboratories are studying various metrological applications of neutral atom traps, there are presently no well defined schemes for overcoming such limitations.

Second, practical traps for ground state neutral atoms are necessarily very shallow compared with thermal energy because the energy level shifts that result from convenient size fields are typically much less than 1 K. Neutral atom trapping therefore depends on substantial cooling of a thermal atomic sample, and is often connected with the cooling process.

The small depth of neutral atom traps also dictates stringent vacuum requirements because an atom can not remain trapped after a collision with a thermal energy background gas molecule. Since these atoms are vulnerable targets for thermal energy background gas, the mean free time between collisions *must* exceed the desired trapping time. The cross section for destructive collisions is quite large because even a gentle collision (i.e., large impact parameter) can impart enough energy to eject an atom from a trap.

4.2 Magnetic Traps

The Stern-Gerlach experiment in 1924 first demonstrated the mechanical action of inhomogeneous magnetic fields on neutral atoms having magnetic moments, and the basic phenomenon was subsequently developed and refined, for example, into the use of magnetic hexapole lenses for focusing and state selecting atoms in beams in the 1950's. An atom with a magnetic moment $\vec{\mu}$ can be confined by an inhomogeneous magnetic field because of an interaction between the moment and the field. This produces a force given by $\vec{F} = -\vec{\nabla}(\vec{\mu} \cdot \vec{B}) = -|\vec{\mu}|(\vec{\nabla}|\vec{B}|)\cos(\vec{\mu}, \vec{B})$.

W. Paul originally suggested a quadrupole trap comprised of two identical coils carrying opposite currents. This trap clearly has a single center where the field is zero, and is the simplest of all possible magnetic traps (see Fig. 4.4.4). The trap has equal depth in the radial ($x - y$ plane) and longitudinal (z-axis) directions when the coils are separated by 1.25 times their radius. Its experimental simplicity makes it most attractive, both because of ease of construction and of optical access to the interior.

The field is zero at the center of this trap, and increases in all directions as $|\vec{B}| = A\sqrt{\rho^2 + 4z^2}$ where $\rho^2 \equiv x^2 + y^2$ and the field gradient A is constant. The field gradient is constant along any line through the origin, but has different values in different polar directions. Therefore the force $\vec{F} = -\vec{\nabla}(\vec{\mu} \cdot \vec{B})$ that confines the atoms in the trap is neither harmonic nor central, and angular momentum is not conserved. Such a trap has been used in the first neutral atom trapping experiments at N.I.S.T. on laser cooled Na atoms for times exceeding one second, and that time was limited only by background gas pressure[68].

Other trap configurations have been studied in some detail, and the general features of a large class of possible traps has been presented[70].

Because of the dependence of the trapping force on the angle between the field and the atomic moment, the orientation of the magnetic moment with respect to the field must be preserved as the atom moves about in the trap. Otherwise the atom may be ejected instead of confined by the fields of the trap. This requires velocities low enough to assure that the interaction between the atomic moment $\vec{\mu}$ and the field $\vec{B}$ is adiabatic, especially when the atom's path passes through a region where the field magnitude is small and therefore the energy separation between the trapping state and non-trapping state is small. Therefore energy considerations are not sufficient to determine the stability of a neutral atom trap: orbit calculations and their consequences must also be considered.

4.3 Classical Motion of Atoms in a Magnetic Quadrupole Trap

There are several motivations for studying the motion of atoms in a magnetic trap. Knowing their positions may be important for trapped atom spectroscopy [71, 72]. Cooling atoms once they are trapped may depend upon knowing both their positions and velocities so that laser beams of proper polarization and direction can be applied. Simply studying the motion for its own sake has turned out to be an interesting problem because this particular distorted conical potential does not have analytic solutions, and its bound states are not well known.

4.3.1 Simple Picture of Classical Motion in a Trap

For the two-coil quadrupole magnetic trap, stable circular orbits of radius ρ in the $z = 0$ plane can be found classically by setting $\mu\nabla B = Mv^2/\rho$, so $v = \sqrt{\rho a}$, where $a \equiv \mu\nabla B/M$ is the centripetal acceleration supplied by the field gradient (cylindrical coordinates are appropriate). Such orbits have a period of $T \equiv 2\pi/\omega_T = 2\pi\sqrt{\rho/a}$. For traps of few cm size and a few hundred Gauss depth, we have $a \sim 250$ m/s^2, and the fastest trappable atoms in circular orbits have $v_{max} \sim 1$ m/s and $T \sim 50$ ms. Because of the anharmonicity of the potential, the orbital periods depend on the orbit size, but in general, atoms in lower energy orbits have shorter periods.

In order for the quadrupole trap to work, the atomic magnetic moments must be oriented so that they are repelled from regions of strong field. This orientation may be produced by the optical pumping process that occurs during Zeeman compensated laser deceleration and cooling of atoms, but it must be preserved while the atoms move around in the trap even though the trap fields change directions in a very complicated way. The condition for adiabatic motion can

be written as $\omega_Z >> |dB/dt|/B$, where $\omega_Z = \mu B/\hbar$ is the Larmor precession rate in the field. The orbital frequency for circular motion is $\omega_T = v/\rho$, and since $v/\rho = |dB/dt|/B$ for a uniform field gradient, the adiabaticity condition is $\omega_Z >> \omega_T$. More general orbits must satisfy a similar condition.

For the two-coil quadrupole trap, the adiabaticity condition can be easily calculated. Using $v = \sqrt{\rho a}$ for circular orbits in the $z = 0$ plane, the adiabatic condition for a practical trap ($A \sim 1$ T/m) requires $\rho >> (\hbar^2/M^2 a)^{1/3} \sim 1$ μm as well as $v >> (\hbar a/M)^{1/3} \sim 1$ cm/s. We note that violation of these conditions (i.e., $v \sim 1$ cm/s in a trap with $A \sim 1$ T/m) results in the onset of quantum dynamics for the motion (deBroglie wavelength orbit size).

Since the non-adiabatic region of the trap is so small (less than 10^{-12} of the trap volume in experiments to date) nearly all the orbits of most atoms are restricted to regions where they are adiabatic. Therefore most atoms stay trapped for many thousands of orbits corresponding to several minutes. At laboratory vacuum chamber pressures of typically 10^{-10} Torr, the mean free time between collisions that can eject trapped atoms is 2 minutes, so the transitions caused by non-adiabatic motion are not likely to be observable.

4.3.2 Numerical Calculations of the Orbits

Atoms near the center of a two-coil quadrupole trap see the potential $U = \mu A\sqrt{4z^2 + ^2\ \rho}$[70] that is appropriate as long as an atom stays in a fixed Zeeman sublevel so its interaction with the field is determined. In any direction U rises linearly with distance from the origin, but the force is not central anywhere except along the lines $z = 0$ and $\rho = 0$, and therefore orbital angular momentum is not conserved. This asymmetric anharmonic potential does not yield analytical solutions, but numerical integration of Hamilton's classical equations of motion have been reported[73]. In addition to the circular orbits discussed above, there are other closed orbits in the planes containing the symmetry axis. Some of these are shown, along with the potential contours, in Fig. 4.4.4.

In addition to these closed orbits, there are unclosed, bounded orbits that can not be adequately described by plots such as those shown in Fig. 4.4.4. Instead, Poincaré sections were made for each crossing of the $z = 0$ plane[73], and some of these are shown in Fig. 4.4.4. These show relatively smooth island chains that break into successively smaller islands at higher energy. As the energy of the trapped particle is increased toward the depth of the trapping potential, the motion becomes more and more erratic, and the islands of stability begin to break up and proliferate.

When the atomic kinetic energy approaches the trap depth, the motion clearly becomes chaotic, as shown in Fig. 4.4.4. However, there are regions where the potential is higher than in the saddle points at the trap threshold, and there are orbits where atoms bounce off these high points as shown in Fig. 4.4.4. Therefore the surface of section plots show islands even for energies above the trap depth.

4.4 Quantum Motion in a Trap

Modern techniques of laser cooling certainly have the capability to cool atoms to recoil energies where their deBroglie wavelengths are on the micron scale. Such cold atoms may be readily confined to micron size regions in magnetic traps with easily achievable field gradients, and in such cases, the notion of classical orbits is inappropriate. The motional dynamics must be described in terms of quantum mechanical variables and suitable wave functions.

4.4.1 Heuristic Calculations of the Quantum Motion of Magnetically Trapped Atoms

In order to consider the behavior of extremely slow (cold) atoms in the two-coil quadrupole trap, we begin with a heuristic quantization of the orbital angular momentum using $Mr^2\omega_T = n\hbar$ for circular orbits. We readily find a spectrum where the total energy is $E_n = (3/2)E_1 n^{2/3}$ where $E_1 = [M(a\hbar)^2]^{1/3} \sim h\times$ (5 kHz), and that for velocities of trapped atoms achieved as of 1993 of a few m/s, $n \sim 10^7$. We also readily find that $\omega_Z = n\omega_T$ so the adiabatic condition is satisfied only for $n >> 1$. The lower lying (small n) bound states, whose orbits are confined to a region near the origin where the field is small, are strongly coupled to unbound states as a result of the motion (dynamic coupling). On the other hand, the large n bound states are less coupled because they spend most of their time in a stronger field, and thus satisfy the condition of adiabaticity of the orbital motion relative to the Larmor precession. In this case the separation of the rapid precession from the slower orbital motion is reminiscent of the Born-Oppenheimer approximation for molecules.

4.4.2 One Dimensional Quantum Calculations

There is some insight to be gained by studying the one dimensional motion of atoms with only two spin states that move along the z-axis or on a line in the $x-y$ plane in the potential of a quadrupole trap. In this case the atoms see a potential of a vee or an inverted vee, depending on their spin state, and the one dimensional solutions to the Schroedinger equation are Airy functions that have been truncated and reflected. Several of these are illustrated in Fig. 4.4.4. As we saw above, the energy eigenvalues increase as $n^{2/3}$.

Some continuum wave functions (those that oscillate to infinity) strongly overlap the bound ones (those that decay in the classically forbidden region outside the potential) for small n levels, and less strongly for larger n. Their overlap integrals (transition moments) decrease with n and show behavior similar to the Franck-Condon factors of molecular transitions. The orbital motion results in a mixing of the degenerate bound and continuum states, so that eigenfunctions of the full Hamiltonian are superpositions of both kinds of states. Thus all atoms eventually escape from the trap and there are no rigorously bound states. For

sufficiently high n, the escape rate is slow enough that atoms remain trapped for long enough to do experiments.

One approach to the problem of the transitions between bound and free states is to consider atoms moving on either of two potential ramps of opposite slopes. The ramps cross at the origin, and their upper parts combine to form the trapping potential while their lower parts form the potential for unbound states. Then we have the simple one-dimensional mathematical problem of a Hamiltonian with potential

$$V = \begin{bmatrix} x & \alpha(x) \\ \alpha(x) & -x \end{bmatrix}, \tag{4.1}$$

i.e., two linear potentials of positive and negative slope respectively, coupled by some function $\alpha(x)$. The solution to the Schroedinger equation is a wave spinor $\Psi(x) \equiv [\psi_1(x), \psi_2(x)]$. Though this motion on a bi-potential does not correspond to an actual physical problem, it does display some of the features of quantum motion in a trap, such as resonances and Airy-type wavefunctions.

We take the case of a wave of energy E incident on the "trap" from the left. In the asymptotic limit, the potentials are linear, therefore the limiting solutions of the Schroedinger equation consist of Airy functions. Using the asymptotic properties of Airy functions, we obtain

$$\psi_1(x) \longrightarrow_{x \Rightarrow -\infty} \sim (E-x)^{-1/4}(e^{-iq_-} + re^{iq_-}) \tag{4.2}$$

and

$$\psi_2(x) \longrightarrow_{x \Rightarrow +\infty} \sim t(E+x)^{-1/4} e^{iq_+} \tag{4.3}$$

where $q_\pm \equiv (2E \pm 2x)^{3/2}/3 + \pi/4$, and r and t are the complex values of the reflection and transmission coefficients. The phases δ_r and δ_t are defined by $r = |r|e^{i\delta_r}$ and $t = |t|e^{i\delta_t}$, and $\delta_t - \delta_r = \pi/2$. A phase thus represents a shift in the oscillating part of the Airy functions, caused by the trap potential.

For three dimensional scattering problems with asymptotically constant potentials, it is well known that the time-delay τ for an incident wavepacket is given by $\tau = 2\hbar(d\delta/dE)$. For the case of 1-D scattering with asymptotically linear potentials, solutions give the analogous expression $\tau = \hbar(d\delta_r/dE)$. We define a resonance to occur at an energy where there is a maximum time-delay, corresponding to a wavepacket that is trapped. Thus resonances occur when there is a sharp jump in the phase as E is increased.

Sporn and Bergeman considered a Gaussian coupling function of the form $\alpha(x) = \beta e^{-\gamma x^2}$ where β and γ are constants. The Schroedinger equation for this case can only be solved numerically and the solution is a wave spinor. Their numerical calculations for the phase δ_r and their time-delay as a function of energy is plotted in Fig. 4.4.4 for six resonance peaks[74]. As before, resonance energies increase roughly as $n^{2/3}$. Fig. 4.4.4 is a plot of the two spatial components of the wave spinor at the lowest resonance energy. There is maximum amplitude

near the origin as expected for a trapped state that provides an intuitive idea of the appearance of a trapping resonance. A more detailed description of these calculations is in preparation[74].

4.4.3 Three Dimensional Quantum Calculations

The quantum mechanical description of atomic motion in a two-coil quadrupole begins with the Hamiltonian $\mathcal{H} = p^2/2M + V$ where $V = -\vec{\mu} \cdot \vec{B}$ and has off diagonal elements that arise from the inhomogeneity of the $\vec{B}$ field. Thus we begin a solution of the Schroedinger equation $\mathcal{H}\Psi = E\Psi$ by making a transformation Λ that diagonalizes V, namely, $\Lambda V \Lambda^{-1} \equiv V_d$. We define $\Phi \equiv \Lambda\Psi$, $E' \equiv \Lambda E \Lambda^{-1}$, and then the Schroedinger Eq. $(-\Lambda p^2/2M + V_d\Lambda - \Lambda E)\Psi = 0$ becomes

$$\frac{1}{2M}[\Lambda, p^2]\Lambda^{-1}\Phi + \frac{p^2}{2M}\Phi + (V_d - E')\Phi = 0. \tag{4.4}$$

Eq. (4.3) is an eigenvalue equation for Φ that can be solved for its eigenfunctions by leaving out the first term and then treating it as a perturbation. By exploiting the cylindrical symmetry of V, the radial and angular parts of the remaining unperturbed equation can be separated, and the angular solutions are combinations of spherical harmonics. The remaining radial equation has a centrifugal repulsion term proportional to $1/r^2$ that helps reduce the wave function at the origin for all but the lowest orbital angular momenta. The numerical solutions for the unperturbed equation have been discussed in some detail for the case of atoms with two spin states[75].

The perturbation that is the commutator of the first term in Eq. (4.3) is non-zero because the spatially dependent terms of V, and hence of Λ, do not commute with p. Its diagonal matrix elements represent energy shifts and its off-diagonal elements drive transitions between the eigenfunctions of the unperturbed equation. These transitions couple bound and continuum unperturbed states, and thus correspond to the Majorana transitions that can cause atoms to be ejected. Thus it becomes clear how the dynamics of the atomic orbits cause atoms to escape from the trap. This first term has a complicated functional dependence upon the angular and spin variables. Its effect on the widths and energies of the bound and unbound quantum states of atoms in the trap field has been carefully described by numerical calculations[75].

Present optical cooling methods have the capability of cooling atoms to within a few times the recoil limit of kinetic energy corresponding to $k_B T_R \cong \hbar\gamma_R$ where typical values of γ_R are 10 kHz. Thus optical cooling will always leave population in several quantum levels of a trap with a few T/m. However, there have been many proposals of non-optical cooling methods that reach much below T_R, and there have been at least two experiments that produced an atomic beam with sub-recoil spread in one dimension[76, 77]. Furthermore, permanent magnets may be used to make traps of much higher field gradients than a few

T/m[78]. Thus there are definite prospects for populating just a few quantum levels of such a trap, and then doing spectroscopy on them[79].

4.4.4 Experiments on Magnetic Trapping of Neutral Atoms

There have been a number of successful magnetic trapping experiments. The first electromagnetic trapping of neutral atoms[68] used an atomic beam slowed by the Zeeman compensation technique described in section 1.2.1 above. The atoms were allowed to drift out of the solenoid and into the center of a two-coil trap. Then a short pulse of nearly resonant light brought the drift velocity to zero, and the field coils were quickly turned on. Atoms could be trapped for 1 second, but this value was limited only by the vacuum of 10^{-8} Torr. The density of trapped atoms was estimated to be $10^3/\text{cm}^3$, several orders of magnitude below the density of the background gas. This first demonstration showed that ideas of neutral atom trapping could indeed be put into practice.

Later, similar experiments in a much better vacuum achieved two minute trap time[16]. In these experiments the authors used a multi-section superconducting solenoid magnet whose first section had a decreasing field to serve as a Zeeman-compensated atom slower and whose second section formed a carefully designed magnetic trap (Fig. 4.4.4[72]). This Ioffe trap, which has been discussed by Pritchard[79] and others[70], has a bias field to discriminate against transitions that can eject atoms from it, as discussed in sections 4.4.2 and 4.4.3 above.

The trap has been used for the first successful experiments in both optical and rf spectroscopy[71] as well as laser cooling[72, 80] of trapped neutral atoms. The optical absorption spectrum of atoms in this trap is shown in Fig. 4.4.4[72]. A low intensity probe beam is directed through the trap on axis, and the transmission through the trapped atoms is recorded. The Zeeman splitting caused by the dc bias field isolates the transition between the sublevels with $M_F = 2$ of the ground state and $M_F = 3$ of the excited state, which tunes linearly with magnetic field. Thus the absorption spectrum reflects the number of atoms subject to a particular value of B.

The spectrum can be used to establish the spatial distribution of the trapped atoms since the field profile is known. As the spectrum shows, there is no absorption below the bottom of the trap, and the highest absorption appears where the trap's field gradient is smallest. The relative strength of the two absorption peaks, as well as the shape of the curve between them, can be used to determine that most of the atoms lie in the deeper portion of the trap (shading in Fig. 4.4.4). From this one can estimate the average energy of the trapped atoms, and extract a temperature. The initial experiments indicated that all energies below the trap depth were equally likely to be populated[72] indicating that the temperature is higher than the trap depth. Later experiments by the same authors[80] demonstrated laser cooling of these magnetically trapped atoms, and the use of optical absorption spectroscopy to measure the lowered

temperature (see Fig. 4.4.4a and Fig. 4.4.4b). Laser cooling of magnetically trapped hydrogen atoms has also been reported[81], and these same authors have also developed a laser-assisted evaporative cooling technique for further cooling of their hydrogen sample.

The ground state hyperfine structure of Na allows for rf spectroscopy on magnetically trapped atoms[71]. The inhomogeneous magnetic field results in a position sensitive resonance condition for the rf transition, and therefore enables further study of the spatial distribution of the trapped atoms[72, 80], much like NMR zeugmatography in medical diagnosis. This, too, can be used to extract the energy distribution of the atoms, and has also been used to observe laser cooling of magnetically trapped atoms[80, 81] (see Fig. 4.4.4c).

In other experiments, magnetic traps have been built directly onto the end of the slowing solenoid[82] and have been loaded by atoms in optical molasses[83]. Traps have been built with permanent magnets for use in trapping atomic hydrogen for precision spectroscopy[78], and for Li[84]. To date there has been no observation of quantized states of magnetically trapped atoms.

FIGURE CAPTIONS

Figure

Schematic diagram of the coil configuration used in the quadrupole trap and the resultant magnetic field vectors. Because the currents in the two coils are in opposite directions, there is a $B = 0$ point at the center. (Figure from Ref. [75]).

Figure

Orbits of atoms in a magnetic trap found from solving the classical equations of motion. These closed trajectories are shown superposed on the potential contours of the trap. The orbit on the upper left clearly has enough energy to escape, but is confined by bouncing off the field close to the coils, which are located outside the corners of each plot. Very small changes in the initial conditions lead to open orbits.

Figure

Poincaré plots of open orbits for magnetically trapped atoms. The same calculations that led to the plots of Fig. 4.4.4 are used for these. In the upper left plot, an orbit similar to the upper left of Fig. 4.4.4 is started with slightly different parameters, and an energy low enough to stay trapped. The orbit is quasi-regular. In contrast to the other three, the plot on the upper right is for non-zero angular momentum. The two lower plots are for different energies, showing the breakup of the island chains.

Figure

Airy wave functions for bound and free atoms in the potential of the quadrupole magnetic trap. Both trapped and anti-trapped wave functions are plotted, but only the potential for trapping is shown. The lowest energy state is on the lower left and the energy increases from the bottom. The even wave functions are on the left, odd ones on the right.

Figure

The time delay of the "scattered" state is plotted vs energy. There are six clear peaks that broaden and diminish with increasing energy.

Figure

Plot of the real and imaginary parts of the two wave functions ψ_1 and ψ_2 at the lowest resonance energy. The amplitude peaks near the origin because this is a trapped state.

Figure

Plot of the magnetic field profile for the Ioffe trap used in the experiments of Ref's. [71], [72], and [80]. (Figure from Ref. [72]).

Figure

Distribution of atoms in the magnetic trap of Ref. [68] (shaded area). The corresponding optical absorption spectrum is shown on the right. (Figure from Ref. [72]).

Figure

Optical absorption spectrum of magnetically trapped atoms (a) before and (b) after laser cooling. Also similar information using rf spectroscopy (c). (Figure from Ref. [80]).

Chapter 5

Optical Traps for Neutral Atoms

5.1 Dipole Force Traps

Since inversion symmetry requires that atoms do not have permanent electric dipole moments, optical trapping of neutral atoms by electrical interaction must proceed by inducing a dipole moment either by nearly resonant optical frequency fields or by electrostatic fields. There are several types of optical traps that employ various configurations of laser beams[5, 6]. These can produce both strong field gradients appropriately arranged for trapping, and adequate mixing of atomic states of opposite parity to provide dipole moments for interaction with the field. However, these traps may require additional cooling to offset the radiative heating. Inducing dipole moments using dc fields can be accomplished in atoms that have a sufficiently close-lying energy state of opposite parity (this excludes most atomic ground states but favors Rydberg states).

For optical traps, the oscillating electric field of a laser induces an oscillating atomic electric dipole moment that interacts with the laser field. If the laser field is spatially inhomogeneous, the interaction and associated energy level shifts of the atoms (ac Stark shift or light shift) varies in space and therefore produces a potential, just as in the sub-Doppler cooling schemes described in section 3.1 above. The force from this potential is sometimes called the dipole force. When the laser frequency is tuned below atomic resonance the sign of the interaction is such that atoms are attracted to the maximum of laser field intensity, while if it is tuned above resonance, the attraction is to the minimum of field intensity. Atoms may even be captured on the nodes or antinodes of a standing wave, thereby making a microscopic optical trap.

5.1.1 Macroscopic Optical Traps

One kind of laser trap combines the dipole or gradient force with the radiation pressure force discussed in section 1.2 above. In this design, shown schematically in Fig. 5.2.3, two slightly focused laser beams with Gaussian transverse intensity profiles are directed coaxially and oppositely, with their foci slightly separated[85]. The frequency is below resonance, so the dipole force produces transverse confinement (the ground state light shift is negative so atoms are drawn toward the axis where the intensity is highest). The scattering force produces axial confinement because atoms moving away from the equilibrium point midway between the two foci experience increased intensity in one beam and decreased intensity in the other. The unbalance results in a net scattering force that pushes them back to the equilibrium point.

Such a trap both cools and heats the atoms. Although Doppler cooling reduces the kinetic energy of the trapped atoms, two associated heating mechanisms necessarily destabilize such laser traps. One is the heating or momentum diffusion arising from the random direction of both absorption and spontaneous emission of light (fluctuations in the scattering force). More important at high intensity is the heating associated with fluctuations in the dipole force that are best discussed in the dressed atom picture described in section 2.3.3. Fluorescent decay from an excited state may land atoms in either of the two types of states shown in Fig. 2.4. Since the optical forces in these states have opposite signs, atoms experience a fluctuating force that has no correlation with their motion, and are therefore heated. The fluctuations of the force do not saturate with intensity, and hence can not be compensated by making a deeper trap using high intensity light. The result is that the steady state kinetic energy of atoms in such a trap, resulting from equilibrium between the heating and cooling mechanism, is always about equal to the trap depth. Atoms are thus continuously boiled out of the trap.

The characteristics of such optical dipole force traps have been studied by Gordon and Ashkin[54]. To obtain a trap that is sufficiently deep (20 mK) the saturation parameter should be as high as 2×10^8 and the detuning as large as 10^6 times the natural width (10^7 MHz for Na). The damping of the velocities by Doppler cooling is then about 100 times weaker than the heating by diffusion. Such a trap is inherently instable, but since the escape time of the atoms can be as large as 5 s, such dipole traps can work provided they are accompanied by effective cooling.

Variations of this trapping scheme have been discussed that include damping from auxiliary light beams[86], alternating light beams to avoid standing waves and thus large heating from dipole force fluctuations[87], and optical molasses[88]. The first reported optical trap used an alternation on the μs time scale between trapping fields that both confined and heated the atoms, and optical molasses that cooled them before they could escape very far[88]. The trapping light had the simplest possible configuration - a single tightly focused

laser beam whose bright focal spot attracted atoms[86]. Since these first experiments in optical traps, there have been several others directed at the study of collisions at very low energies. These are discussed later in chapter 6 below.

5.1.2 Microscopic Optical Traps

In a standing wave the light intensity varies from zero at a node to a maximum at an antinode in a distance of $\lambda/4$. This provides the opportunity for confining atoms in very tiny regions of space if they can be cooled and loaded into such a trap. Such effects have been observed in very careful experiments with optical molasses. The experimental discovery of sub-Doppler laser cooling discussed at the end of chapter 2[48] led to the new theories described in chapter 3[50, 51] and to more careful measurements. The ballistic technique used for measuring the sub-Doppler atomic velocity distribution had reached its limit of utility, and so the group at N.I.S.T. developed a new method. They were able to superpose the fluorescent light from atoms in optical molasses upon the light directly from the laser on a fast photodetector, and extract a beat signal that carried information about the Doppler shift of the atoms in their 3-D optical molasses[89]. These Doppler shifts were expected to be in the sub-MHz range for atoms with the previously measured 50 μK temperatures. Such features were observed and confirmed the previous measurements done by atomic ballistics[48].

The experiments also showed a much narrower peak atop the sub-MHz signal whose width corresponded to velocities much less than a single atomic recoil. Since there was no evidence to support the existence of such a narrow velocity distribution, the narrow peak was attributed to Dicke narrowing[90], a suppression of Doppler shifts when radiators or scatterers are confined to a space smaller than a wavelength. It is not surprising to expect atoms to be confined to wavelength-size regions in optical molasses because the light shifts associated with antinodes of the standing waves can be considerably larger than the kinetic energy of μK atoms. Atoms are confined for varying periods of time in the single cells of a lattice formed by the planes of the three-dimensional standing wave.

5.1.3 Quantum States of Atomic Motion and Optical Crystals

The momentum of very cold atoms trapped in optical potential wells is so small that their deBroglie wavelengths are comparable to the optical wavelength, and hence to the trap size. In fact, the deBroglie wavelength equals the size of the optical traps ($\lambda/2$) when the momentum is $2\hbar$k. Thus the atomic motion in the trapping volume is not classical, but must be described quantum mechanically. Even atoms whose energy exceeds the trap depth must be described as quantum mechanical particles moving in a periodic potential that display energy band structure[91].

Trapped atoms occupy vibrational states similar to those of molecules. The optical spectrum can show Raman-like sidebands that result from transitions among the quantized vibrational levels[92, 93] as shown in Fig. 5.2.3. The two sidebands correspond to transitions that raise or lower the energy of atoms in the wavelength size traps by one vibrational quantum. Their unequal strength reflects the unequal populations of the vibrational levels, and allows extraction of the "temperature". The dependence of this measured temperature on the laser trap parameters is consistent with the theories of laser cooling described in chapter 3 above. These quantum states of atomic motion can also be observed by stimulated emission[92, 94] and by direct rf spectroscopy[95, 96].

In 1993 there appeared the first descriptions of optical confinement of atoms in wavelength-size regions in two and three dimensions, and the beginning of a new area of work called optical crystals. In two dimensions, atoms pre-cooled in an MOT were subjected to two perpendicular crossed standing waves whose parallel polarizations are perpendicular to the plane of their $\vec{k}$ vectors, and whose relative temporal phase could be varied[97]. The spatial character of such a light field depends strongly on this relative phase. The transmission of a weak a probe beam was used to determine the spatial and velocity distributions of the atoms, and these depend strongly on the relative optical phase.

Because tiny mirror vibrations can cause strong changes in the relative phase of the optical fields, and thus change its character by, for example, converting linearly polarized light to circular, a new scheme of two and three dimensional fields was established[98]. Instead of producing optical wells in two dimensions with four beams (two standing waves), these authors used only three. The $\vec{k}$ vectors of the co-planar beams were separated by $2\pi/3$, and they were all linearly polarized in their common plane (not parallel) as shown in Fig. 5.2.3a. In this case, vibrations or other phase changes would displace the optical wells in space, but would not make major changes in the character of the optical field[97].

The same immunity to vibrations could be established for wavelength sized optical potential wells in three dimensions by using only four beams arranged in a quasi-tetrahedral configuration. The three linearly polarized beams of the 2-D arrangement described above were directed out of the plane toward a common vertex, and a fourth circularly polarized beam was added (Fig. 5.2.3b). All four beams were polarized in the same plane[98]. The authors showed that such a configuration also produced potential wells, but this time in 3-D. Concurrent experiments in both the 2-D and 3-D configurations showed strong confinement of the atoms in the potential wells, and the presence of quantized vibrational states of their motion[98]. Similar experiments at N.I.S.T. with a somewhat different configuration of laser fields have also demonstrated the existence of these optical crystals[99].

Progress in laser cooling and trapping of neutral atoms has evolved toward quantization of the atomic center of mass motion or external coordinates. The

classical description of atomic motion which assumes that atoms have arbitrary position and momentum is outmoded. In this new quantum picture of laser cooling, atomic position and momentum are considered as quantum mechanical variables. Laser cooling then becomes a process of optically pumping atoms to discrete states of lower mechanical energy[100]. Of course, laser cooling always depends on populating the desired internal states because it is these internal states of atoms that determine the magnitude and nature of the electromagnetic forces on them.

Entire atoms must be described by a deBroglie wave field occupying all allowed states of a region of space that may have a spatially varying potential which defines modes of the field. Clearly, occupation of particular modes of this field can result in spatial interference, and the field of atom interferometry emerges as a subset of this way of thinking. Atoms can only "interfere" if they occupy both the same internal and external states, and thus are indistinguishable.

5.2 Radiation Pressure Traps

One of the basic limitations of dipole traps comes from the large saturation parameters needed for confinement. To overcome this problem, traps have been proposed that rely on the scattering force to cool and trap atoms[101]. These designs include either four or six focused Gaussian beams that converge on a small volume where atoms are trapped. However such traps can not be stable as long as the trapping force is proportional to light intensity[101]. This can be simply understood by considering that the flow of optical energy cannot be directed inwards everywhere on the surface of the trapping volume, and thus the force can not be directed inwards everywhere. Since this is similar to the Earnshaw Theorem for electrostatics it is called the Optical Earnshaw Theorem. However, for atoms that have multiple ground states whose absorption probability can be altered by external fields, various configurations of laser beams can be used to make stable traps[102].

5.2.1 Introduction to Magneto-Optical Traps

One very important example uses both optical and magnetic fields to make a magneto-optical trap (MOT). This widely used and most important hybrid trap was first demonstrated in 1987[103]. The operation of an MOT depends on both the inhomogeneous magnetic fields and radiative selection rules to exploit both optical pumping and the strong radiative force[103, 104].

In order to see how it works, consider slowly moving atoms in a linearly varying magnetic field $B = B(z) \equiv Az$ and neglect the Doppler shift for the moment. A simple atomic transition ($J = 0$ to $J = 1$) has three Zeeman components, excited by each of three polarizations, whose frequencies tune with

field (and therefore with position) as shown in Fig. 5.2.3a. Two oppositely directed laser beams of opposite circular polarization, each detuned below the zero field atomic resonance by $|\delta| >> \gamma$, are incident as shown.

Atomic resonance can then only occur near the two points $z = \pm z'$ where the Zeeman tuning of each transition corresponds to the laser frequency, and because of the polarizations, it can only occur with the σ^+ beam at $z = -z'$ and with the σ^- beam at $z = +z'$. Consequently, atoms at $+z'$ will be driven toward $-z'$ and conversely, so that atoms bounce back and forth between these turning points, which can be moved either by tuning the laser or by changing A. If the magnitude of δ is decreased to the neighborhood of γ, atoms see a quasi-harmonic force. This scheme is readily extended to three dimensions with two opposed magnet coils forming a magnetic spherical quadrupole field as discussed in section 4.2 above. It can also be readily generalized to atomic transitions with more complicated angular momentum schemes.

5.2.2 Capturing and Cooling Atoms in a MOT

The Doppler effect for moving atoms changes Fig. 5.2.3a to Fig. 5.2.3b by shifting the optical frequencies in the atomic rest frame. The two dotted lines on the figure correspond to the two different frequencies of the counterpropagating beams seen by atoms moving near z''. For such atoms moving to the right in Fig. 5.2.3b, the σ^- beam is shifted much closer to resonance and the σ^+ beam much further from resonance (detuning of σ^+ shown as Δ). The Zeeman and Doppler shifts now combine to produce resonance with the σ^- beam when $\delta = (|k|v + \mu' Az/\hbar)$ where μ' is the Zeeman tuning coefficient. (Setting $v = 0$ returns the resonance condition described above.) As before, a fixed δ produces resonance for one beam and large detuning for the other, but now the Doppler shift contributes to the resonance condition so that it occurs in the interior region between $\pm z'$, for example at z''.

This enables resonance over an extended distance and velocity range because the changing Doppler shift of decelerating atoms can be compensated by the changing Zeeman shift as atoms move in the inhomogeneous magnetic field, just as in the laser deceleration discussion in section 1.2.1 above[104]. Of course, it will only work this way if the field gradient A does not demand an acceleration larger than a_{max}. Thus atoms are subject to the optical force over a distance that can be as long as the trap size, and can therefore be slowed considerably.

We can estimate the very large velocity capture range of an MOT by using $F_{max} = \hbar k\gamma/2$ and choosing a maximum size of a few cm for the beam diameters. Thus the energy change can be as large as several K[104]. The number of atoms in a vapor with velocities below v_c in the Boltzmann distribution scales as v_c, and there are enough slow atoms to fall within the large MOT capture range even at room temperature, because a few K includes 10^{-4} of the atoms. A more conservative estimate of the capture range might cost another factor of 10, but this is still a very large number of atoms for most room temperature vapors.

For example, at a temperature of 300 K, the vapor pressure of Cs is 10^{-5} Torr so the density is a few times 10^{11} atoms/cm^3 leaving 10^7 atoms/cm^3 within the capture range of an MOT. Thus a Cs MOT in a modest size cell can be filled with 10^9 atoms in less than a second from the room temperature vapor. Such a scheme was first demonstrated in 1990 with the trap shown in Fig. 5.2.3 using diode laser light[83], and has since been repeated in many laboratories for Na, Rb, He*, Ne*, and many other atoms.

Note that an MOT not only traps but also cools atoms optically[104]. It is possible to show that atoms confined in an MOT are cooled to a temperature of order $T_\gamma \equiv (\gamma/\gamma_R)T_D$ when δ is a few times larger than γ. Because of the red tuning, fast atoms moving toward their turning point lose more KE than they regain when rebounding from it, back toward $z = 0$. The rebound speed of an atom from this inelastic "collision" is approximately $v = \gamma/\mathrm{k}$. This is because atoms at rest at the turning points z' are optically accelerated toward the origin until their speed results in a Doppler shift larger than the natural width. This speed corresponds to temperature $M\gamma^2/3k_Bk^2T_\gamma$, and even a purely magnetic trap less than 5 Gauss deep can confine such cold atoms[83, 105]. When $\delta \sim \gamma$, the temperature in an MOT is even lower than T_γ because the atomic motion is damped, almost like in optical molasses[105, 106, 107, 108].

5.2.3 Variations on the M.O.T. Technique

Because of the wide range of applications of this most versatile kind of atom trap, a number of careful studies of its properties have been made[104, 105, 106, 107, 108, 109, 110, 111, 112], and several variations have been developed. One of these is designed to overcome the density limits achievable in an MOT. In the simplest picture, loading additional atoms into an MOT produces a higher atomic density because the size of the trapped sample is fixed. This size is approximately $\hbar\delta/\mu' A$ for $\delta >>$ few γ, or else it is the region accessible to atoms of energy somewhat below T_γ/k_B for $\delta \sim \gamma$.

However, the density can not increase without limit as more atoms are added. The atomic density is limited to 10^{11}/cm^3 because the fluorescent light emitted by some trapped atoms is absorbed by others, and this diffusion of radiation presents a repulsive force between the atoms[109, 110]. Adding atoms to a MOT thus increases the density up to some point, but adding more atoms then expands the volume of the trapped sample. In some cases the radiation pressure may cause the sample to break up into a central cloud surrounded by an orbiting ring[109, 110] driven by asymmetries in the magnetic field or laser beam profiles. Photographs of some of these atomic clouds are shown in Fig. 5.2.3. In addition, certain kinds of collisions among the trapped atoms may also play a role in limiting the density to a similar value.

One way to overcome this limit is to have much less light in the center of MOT than at the sides. Simply lowering the laser power is not effective in reducing the fluorescence because it will also reduce the capture rate and

trap depth. But those advantageous properties can be preserved while reducing fluorescence from atoms at the center if the light intensity is low only in the center.

The ground state hfs of alkali atoms provides a ideal way of implementing this idea[113]. In order to maintain optical absorption by atoms with a ground state hfs, the light usually contain two frequencies, each tuned for excitation out of one of the hfs levels. But if the light beam at one of those frequencies is tailored to have zero intensity at the center, then atoms trapped near the center of the MOT are optically pumped into the other hfs and stop fluorescing. They drift freely in the "dark" at low speed through the center of the MOT until they emerge on the other side into the region where light of both frequencies is present and they begin absorbing again. Then they feel the trapping force and are driven back into the "dark" center of the trap. Such a MOT has been operated at MIT[113] with densities close to $10^{12}/\text{cm}^3$, and the limitations may be from collisions rather that multiple light scattering.

Another variation of the MOT is designed to produce spin-polarized atoms. In a usual MOT, the orientation of the atomic spins varies throughout the trap volume because of the varying direction of the quadrupole magnetic field and the different optical polarizations. However, a new trap design has been built where two of the three pairs of laser beams are misaligned in the "racetrack" arrangement as shown in Fig. 5.2.3, and more coils have been added to change the field symmetry[114, 115]. In this case the trap can work adequately even when the two beams in the third pair have the same polarization and one pair of coils produces a uniform field. Atoms are therefore subject to a strong optical pumping toward a particular alignment, and the total sample has a 75% spin alignment[115].

In a third variation, the number of laser beams has been reduced from six to four and arranged in tetrahedral symmetry similar to Fig. 5.2.3b[116, 117]. There are several advantages to this arrangement apart from the simplicity of fewer laser beams. First, capturing atoms from a slowed atomic beam is enormously simplified because there is no laser light copropagating with the atoms. Second, the absence of standing waves precludes certain types of heating. Third, the restrictions on polarization purity may be relaxed. Of course, it is a bit more difficult to produce such a configuration of laser beams, but for certain applications, it is certainly advantageous.

Finally, Emile et al.[118] reported on a new magneto-optical trap, in which they used orthogonal pairs of counterpropagating beams having polarization under 45^o. They interpreted the trapping as being a result from a new magneto-optical force observed by Grimm et al.[119]. This force arises from a redistribution of photons from one laser beam into the other beam by a stimulated process in the presence of a magnetic field. Since this force arises from a stimulated process, the magnitude of the force can be made much larger then the spontaneous force. Therefore one can expect that this trap can have a larger increase of the phase-space density compared to the traditional MOT.

FIGURE CAPTIONS

Figure

Focused laser beams of the simple light trap discussed in the text.

Figure

(a) Fluorescence spectrum in a 1-D lin $\perp$ lin optical molasses. Atoms are first captured and cooled in an MOT, then the MOT light beams are switched off leaving a pair of lin $\perp$ lin beams, and the $\vec{B}$ field is turned off, then the measurements are made with $\delta = 4\gamma$ at low intensity. (b) Same as (a) except the 1-D molasses is $\sigma^+ - \sigma^-$ which has no spatially dependent light shift and hence no vibrational motion. (Figure from Ref. [93]).

Figure

Arrangement for a stable lattice for optical crystals. In (a) the three beams all have the same polarization plane and propagate at 120^o to one another. In (b) the four beams still share the same polarization plane and the vertically travelling one is circularly polarized. The other three no longer have coplanar wave vectors. (Figure from Ref. [98]).

Figure

(a) Arrangement for an MOT. The horizontal dashed line represents the laser frequency seen by an atom at rest. (b) For a moving atom, the two shorter dotted lines represent the frequencies of the opposing laser beams seen by a moving atom. At $z = z''$, an atom moving to the right sees the σ^+ beam detuned by Δ, but the σ^-beam is almost resonant with the transition to $M =$ -1. (Figure from Ref. [104]).

Figure

Schematic diagram of an MOT. (Figure from Ref. [105]).

Figure

Spatial distribution of atoms trapped in a MOT whose beams are slightly misaligned. When there are less than 10^8 atoms they form a central clump, but with more than that there is an orbiting group of atoms as well. (b) and (d) show time exposures of this, but (c) shows the clump distinctly when the camera is strobed at 110 Hz. (e) and (f) show a full ring from the top and side. (Figure from Ref. [110]).

Figure

Arrangement for a MOT to make spin-polarized atoms. In the plane shown, the laser beams' Gaussian profiles are indicated by the fuzzy, broad arrows and small plots, while the direction of the $\vec{B}$ fields is shown by the thin arrows. In the third dimension, the light beams are also $\sigma^+ - \sigma^-$ but not spatially offset. Atomic motion in the z-direction (vertical) is coupled to the y-direction (horizontal) by the offset beams, to provide confinement in all three directions. The constant sign of B_z and the σ^+ polarization of both z-directed beams maintain the strong spin polarization. (Figure from Ref. [115]).

Chapter 6

Applications of Laser Cooling

6.1 Improvements to Atomic Beams

The laser cooling techniques discussed in the preceding chapters focused on obtaining cold samples of atoms, but laser cooling has also been applied to obtain atomic beams with enormously increased brightness. One important application of these beams is for collision experiments. Collisions between atoms in thermal beams were hampered in the past by the Maxwell-Boltzmann velocity distribution of effusive beams, so even in the simplest experiments the signals were always averaged over this distribution. Although some clever schemes have been devised to overcome this problem, they always suffer from loss of intensity. In addition to the longitudinal velocity compression discussed in section 1.2, laser cooling can also provide intensity enhancement by transverse velocity compression. Most important, it enables collision experiments in the new regime of ultra-cold temperatures (see section 6.3 below). Another application lies in the field of atom-surface scattering, where well-collimated atomic beams with large transverse deBroglie wavelengths can be used to study surface structures. Still another application is in the area of precision measurements and atomic clocks. Many of the important atomic beam measurements of the 1950's and 60's were limited by the brightness of the atomic beam. These include several of fundamental importance, such as the electrical neutrality of matter and the search for dipole moments of elementary particles. This section focuses on techniques to obtain monochromatic, well-collimated, high brightness atomic beams.

One of the first beam brightening experiments was performed by Nellesen et al.[120, 121] where a thermal beam of sodium was slowed with the chirp technique (see section 1.2.1 above). Then the slow atoms were deflected out of the main atomic beam at an angle of 22^o using a focused laser beam, while the

fast atoms (not captured in the slowing process) remained undeflected. This process not only deflected the atoms, but also transversely cooled them. The deflected beam had selectable final velocities between 50-200 m/s.

In a later experiment[122] this beam was fed into a two-dimensional MOT (section 5.2 above) where the atoms were cooled and compressed in the transverse direction by an optical molasses of $\sigma^+ - \sigma^-$ polarized light. For this compression the MOT field was produced by permanent magnets. To improve the capture range, these magnets were shaped to obtain an increasing field gradient from 50 G/cm to 500 G/cm as the atoms moved through the optical molasses. In this way a beam of a few mm diameter was compressed into a beam only 43 μm wide with a tiny divergence. The intensity of the beam is approximately 10^9 cold atoms/cm^3, which is about 1000 times the density of the initial effusive beam. Although the density is high, the beam is still optically thin from the sides which makes it easy to manipulate it even further.

Another approach was used by Riis et al. who directed a slowed atomic beam into a hairpin shaped coil that they called an "atomic funnel"[34]. The wires of this coil generated a two-dimensional quadrupole field that was used as a two-dimensional MOT as described before. Inside the trapping region the beam of atoms is further slowed in the longitudinal direction by two counter-propagating laser beams of different frequencies, thereby forming a moving optical molasses, so that atoms moving at a certain selectable velocity experience zero force. In this way a monochromatic beam with a velocity of 260 m/s, a diameter of 150 μm and a flux of 10^9 atoms/s was produced, leading to a density of 2×10^6 atoms/cm^3. This is an increase of the density over chirped-cooled atomic beams of 40.

These approaches yield intense beams when the number of atoms in the uncooled beam is already high. However, if the density in the beam is initially low, for example in the case of metastable noble gases or radioactive isotopes, one has to capture more atoms from the source in order to obtain an intense beam. Aspect et al.[33] have used a quasi-standing wave of converging laser beams whose incidence angle varied from 87^o to 90^o to the atomic beam direction, so that a larger solid angle of the source could be captured. In this case they used a few mW of laser light over a distance of 75 mm.

The most sophisticated approach to this problem has been developed for metastable Ne by Hoogerland et al.[36]. They use a three-stage process to provide a large solid angle capture range and produce a high brightness beam. The first stage of their beam brightener consists of two pairs of nearly parallel mirrors arranged so that multiply reflected beams of light cross the atomic beam at varying angles to provide a large capture range (see Fig. 6.5). The laser beams bounce between the mirrors 10 times, and the angle of the light with respect to the direction of the atomic beam increases by 0.5 mrad each time, so that at the end of the mirrors the light intersects at almost 90^o with respect to the atomic beam. By recycling the laser intensity the laser power consumption of this section is only 30 mW and the collimation of the beam could be extended

over a large distance of 150 mm. The solid angle captured in the process is about 0.1 rad.

This region is followed by a magneto optical lens to focus the atoms to a small space. This is required because the transverse collimation is necessarily accompanied by an increase in the diameter of the atomic beam to 20 mm in the last case. To overcome this problem a two-dimensional MOT as discussed before could be used, but the high longitudinal speed of the atoms would require a very long region for this. Instead they used only a short section to deflect the atoms towards the center of the beam, effectively forming a magneto-optic lens. After 70 cm the atoms are focused to nearly a point and a small section of optical molasses can then be used to cool the transverse velocities. In this way they obtained an increase of the metastable intensity of over 1600.

6.2 Applications to Atomic Clocks

Throughout history humans have tried to build devices for measuring time. There was a great increase in the development of clocks after the Europeans discovered the western hemisphere when good clocks became necessary for accurate navigation. Various governments offered large awards for the construction of clocks that could maintain accuracy through an ocean voyage. Many countries established Naval Observatories for this purpose, and several of them remain today as the principle national arbiters of time.

In the 16th century Galileo discovered the periodicity of the pendulum, and in the 17th century Huygens developed an escapement mechanism for both pendulum and spring-driven clocks that set the standard for 200 years. Time keeping experienced significant progress in the 19th century with the advent of the American railroads. The first clocks with accuracy much better than 1 sec/day were based on crystal quartz oscillators developed at the beginning of the 20th century.

The idea of atomic clocks grew out of the atomic beam research begun in the late 1930's. Rabi, Ramsey, Zacharias, and others promoted the idea after World War II, but it took 20 years more to become adopted. In 1967 the internationally accepted definition of the second changed from mechanical time pieces calibrated by the Earth's orbit to atomic time calibrated by the hyperfine structure splitting of the ground state of Cesium. By definition, 1 second is exactly 9,162,631,770 cycles of the $M_F = 0 \Leftrightarrow 0$ transition in ^{133}Cs, the natural stable isotope.

Motivation for accurate time keeping comes from very many sources. For the purposes of scientific research, very accurate comparison of frequencies is necessary for testing basic theories, including relativity, QED, quantum mechanics, etc. For the purposes of navigation, time keeping has been essential for hundreds of years. All commercial and military aircraft and spacecraft carry quartz or atomic clocks, and many even carry redundant systems. The new

Global Positioning System (GPS) that will revolutionize how we move about in the next decade depends on atomic time, as do computer systems, radio and television broadcasting, telephone and communication systems, and a host of other contemporary technologies. As long as we believe that all ^{133}Cs atoms are identical, we are confident that an atomic clock anywhere in the universe keeps the same time as the commercially available standards found in dozens of laboratories throughout the world.

The limitation to both the accuracy and precision of atomic clocks is imposed by the thermal motion of the atoms. Both the non-zero speed and the variation of the speeds of different atoms from an atomic sample provide the ultimate limitation on high precision laboratory measurements and on clocks. One cause of this problem arises from the broadening of a spectral line caused by the small interaction time between the measuring equipment and rapidly moving atoms. At thermal velocities of typically 500 m/s, there are only a few ms to interact with a free atom in an apparatus of reasonable size (i.e. a few meters).

The other source of this limit arises from a frequency shift caused by the relativistic time differences between reference frames in relative motion (sometimes called the second order Doppler effect; the first order Doppler effect is the familiar classical frequency shift between moving objects). If we knew the velocity of the atoms with respect to the measuring apparatus, this effect could be calculated and accommodated as well. But the atoms have a velocity distribution, characterized by the temperature of their source. Although this too can be calculated, the details of the distribution at the low velocity end depend very sensitively on the details of the source, and sometimes cannot be adequately known. Thus a sample of laser cooled atoms could provide a substantial improvement in atomic clocks and in spectroscopic resolution.

The first attempts at providing slower atoms for better precision or clocks were by Zacharias in the 1950's. The idea was to make an atomic fountain in which the slowest atoms emitted from a thermal source would rise only a small distance before gravity pulled them down. This failed, however, because atomic collisions with fast atoms in the source aperture always speeded up the slowest atoms. The advent of laser cooling changed this because the slow atoms far outnumber the faster ones instead of vice-versa. The first rf spectroscopy experiments in an atomic fountain using laser cooled atoms were reported in 1989 and 1991[123, 124], and soon after that some other laboratories also reported successes.

The best results to date were reported by Gibble and Chu[125, 126]. They used an MOT with laser beams 6 cm in diameter to capture Cs atoms from a vapor at room temperature. Their estimated capture velocity was 30 m/s, consistent with the estimates of section 5.2.2 above. These atoms were launched upward at 2.5 m/s by varying the frequencies of the MOT lasers to form a moving optical molasses, and subsequently cooled to below 3 μK. The atoms were optically pumped into one hfs sublevel, then passed through a 9.2 GHz microwave cavity on their way up and again later on their way down. The

number that were driven to change their hfs state were measured vs microwave frequency, and the signal showed the familiar Ramsey oscillatory field pattern. The width of the central feature was 1.4 Hz and the S/N was over 50 (see Fig. 6.5). Thus the ultimate precision was 1.5 mHz corresponding to $\delta\nu/\nu \cong 10^{-12}/\tau^{1/2}$ where τ is the number of seconds for averaging. Stability of the rf signals was maintained with a hydrogen maser.

The ultimate limitation to the accuracy of this experiment as an atomic clock was collisions between Cs atoms in the beam. Because of the extremely low relative velocities of the atoms, the cross sections are very large (see section 6.3.1 below) and there is a measurable frequency shift. By varying the density of Cs atoms in the fountain, the authors found frequency shifts of the order of a few mHz for atomic density of $10^9/\text{cm}^3$, depending on the magnetic sublevels connected by the microwaves. Extrapolation of the data to zero density provided a frequency determination of $\delta\nu/\nu \cong 4 \times 10^{-14}$. Thus the authors suggest possible improvements to atomic time keeping of a factor of 1000 in the near future.

Another important approach to atomic clocks uses an optical transition frequency instead of a microwave frequency. A group at N.I.S.T. is studying the 2 Hz wide transition in metastable Xe atoms driven by a two photon transition between the $1s_5$ and $1s_3$ levels[127]. The energy difference corresponds to $\lambda = 1.1$ μm, but the angular momentum J of these levels differs by 2 so single photon transitions are not allowed. Because the two photon process at $\lambda = 2.19$ μm is so weak, its natural width is very small, enabling very large spectral resolution. The atoms can be cooled and trapped on the allowed $1s_5 \Leftrightarrow 2p_8$ transition at $\lambda = 883$ nm, an easily accessible wavelength for diode or Ti:Sapphire lasers. The natural width of this transition is $\cong 5$ MHz.

6.3 Ultracold Collisions

The new technique of laser cooling provides opportunities to study collision physics in the previously inaccessible domain where quantum effects are important. For collisions between atoms with thermal velocities, the deBroglie wavelength is always small compared to the size scale of the interatomic potential. But for laser cooled atoms, this is not true and collisions can not be described classically. One of the new practical problems that arises because of the large cross sections for such low energy collisions is the limit imposed on the density obtainable in a trap.

Three new phenomena can be distinguished in collisions at such low energies. First, the deBroglie wavelength of the collision partners becomes comparable to the typical range of the interaction potential and results in the onset of quantum threshold behavior. Second, at very small collision velocities, an excited atom can decay during the collision time and the cross section for excited state collisions can thus be severely affected. And third, since the collision energy is

small, external fields can influence the collision dynamics considerably. Each of these topics is discussed below. (For a detailed review, see Ref. [128].)

6.3.1 Quantum Behavior

Some insight in these collisions can be obtained from the Langevin model which was first introduced in 1905 to calculate mobility and diffusion coefficients[129].

In the simplest case, the interaction potential between two colliding particles contains two terms:

$$V(R) = -\frac{C_n}{R^n} + \frac{\hbar^2 \ell(\ell+1)}{2mR^2} \tag{6.1}$$

where the first term on the right side is the molecular Born-Oppenheimer interaction potential and the second term describes the centrifugal barrier. For two colliding S-states there is a van der Waals interaction and $n = 6$; for a quadrupole-quadrupole interaction $n = 5$; and when two like atoms are coupled by an allowed dipole interaction there is a dipole-dipole interaction and $n = 3$. The centrifugal barrier prevents reactions from taking place for low velocity unless $\ell = 0$ (S-wave scattering).

The maximum of the interaction potential of Eq. (6.1) occurs at R_c found from setting $dV(R)/dR = 0$. Then (see Fig. 6.5a):

$$R_c = \left(\frac{mnC_n}{\hbar^2 \ell(\ell+1)}\right)^{1/(n-2)} \tag{6.2}$$

and

$$V(R_c) = \left(\frac{n-2}{2}\right)\left(\frac{\hbar^2 \ell(\ell+1)}{2m}\right)^{n/(n-2} C_n^{2/(n-2)} \tag{6.3}$$

Reactive collisions can only take place if the collision energy is larger than $V(R_c)$ so that atoms that can pass over this barrier where they are attracted to the small inner region by the Born-Oppenheimer potential (see Fig. 6.5b). In order to estimate the cross section for the process we have to find the maximum impact parameter b_c that contributes to the reaction. Classically we can write $L = m\vec{v} \times \vec{r} \cong mvb$. Taking the energy to be equal to $V(R_c)$ and putting $L^2 = \hbar^2 \ell(\ell+1)$ we arrive at

$$b_c = \frac{L}{\sqrt{2mE}} \propto \frac{E^{(n-2)/2n}}{E^{1/2}} = E^{-1/n} \tag{6.4}$$

The cross section σ is proportional to $b_c \propto E^{-2/n}$. At threshold σ depends only on the collision energy and all the details of the collision are unimportant. Measuring σ near threshold allows for a direct determination of the power of the interaction potential n.

Reaction	n	λ nm	T_D	T_R	T_Q	T'_Q	T_S
H+H	6	121	1.8 mK	1.3 mK	2 K	20 K	-
He(^{3}P) +He(^{3}P)	6	1080	40 μK	4.1 μK	10 mK	100 mK	1 mK
Na+Na	6	589	240 μK	2.4 μK	1 mK	10 mK	-
Na(^{2}P$_{3/2}$)+Na	3	589	240 μK	2.4 μK	0.05 nK	1 nK	64 mK
Na(^{2}P$_{3/2}$)+Na(^{2}P$_{3/2}$)	5	589	240 μK	2.4 μK	80 μK	740 μK	64 mK
Cs+Cs	6	852	130 μK	0.2 μK	34 μK	330 μK	-

Table 6.1: Temperatures for the onset of the quantum regime T_Q and T'_Q for different reactions currently studied in optical traps. T_Q indicates the temperature for which the Langevin model gives S-wave scattering. T'_Q indicates the temperature where the WKB approximation fails[130]. The temperatures are compared with the Doppler temperature T_D and the recoil temperature T_R. Also indicated is the temperature T_S for which spontaneous emission becomes important *during* a collision.

Equation (6.2a) also permits an estimate of the number of partial waves contributing to reactive collisions. For the highest partial wave contributing at a certain temperature T we use $V(R_c) = 3k_BT/2$ and find

$$\ell_c(\ell_c+1) = \left(\frac{nm_nC}{\hbar^2}\right)\left(\frac{3k_BT}{(n-2)C_n}\right)^{(n-2)/n} \tag{6.5}$$

The onset of quantum behavior can be defined as the temperature T_Q where only S-waves can contribute, i.e. the temperature for which $\ell_c = 1$ no longer contributes. Table 6.1 gives typical values for T_Q for several reactions which are studied in traps. Notice that T_Q is of the order of the Doppler temperature for atomic interaction with an $n = 6$ potential, but that much lower temperatures are needed to observe the quantum threshold for dipole-dipole interactions ($n = 3$).

Julienne and Mies[130] suggest that the onset of quantum behavior can be found more rigorously by looking at the temperature T'_Q, where the WKB-approximation fails. Defining the local wave vector of the particle's motion $k = 2\pi/\Lambda = \sqrt{2m(E-V(R))/\hbar^2}$ the validity criterion for the use of WKB-methods becomes $d\Lambda/dR \ll 1$. Using the interaction potential of Eq. (6.1) they obtain the same scaling of the maximum angular momentum ℓ_c with temperature, but the overall onset of quantum behavior occurs at a somewhat higher temperature (see Table 6.1).

6.3.2 Decay of Excited Atoms

Spontaneous emission during collisions becomes important when the characteristic time τ_c during which the interaction takes place becomes much larger then

the lifetime of the excited atomic state τ_a. In excited state collisions the atomic interaction shifts the molecular complex out of resonance. From [131] one can find that this happens at an interatomic distance $R_w = \lambda/2\pi$ where λ is the wavelength of the light. From this relation one can easily find a collision velocity v_s below which spontaneous emission becomes important. In Table 6.1 we have indicated for different reactions the temperature T_s corresponding to this velocity. At temperatures below T_S the theory should include either Optical Bloch Equations (OBE) to describe the evolution of the excited state of the atom[132] or a Quantum Monte-Carlo method.

The situation becomes different when another laser, a so-called catalysis laser, is used to excite the atoms off resonance as shown by the arrow in Fig. 6.5. In this way one can excite the transient molecule at a specific internuclear distance for which the survival rate is much larger. Gallagher and Pritchard [133] discuss a model for the case of trap losses for sodium. Losses in a sodium trap can occur through two excited state reactions: (1) Fine-structure changing collisions where the collision of an $Na(3P_{3/2})$ atom results in a transition to the $3P_{1/2}$-state thus liberating 17 cm^{-1} of energy (more then 20 K, see Fig. 6.5a), and (2) radiative redistribution, where the absorbed photon is spontaneously emitted at a lower frequency because atomic potential energy is converted to KE (see Fig. 6.5b). In both reactions enough energy is released to eject both atoms out of the trap.

For the sodium case Gallagher and Pritchard[133] find that the cross sections increase considerably when the catalysis laser is detuned to the red. The molecular system is then excited at a much smaller internuclear distance yielding an increase in the survival probability of the excited state of an atom during the collision time. The fine-structure changing collisions dominate over the radiative redistribution yielding rate constants independent of temperature. Their model is used by Sesko et al.[134] to compare the predictions with measurements done in a cesium trap. At low intensity of the trapping beams, the loss of atoms from the trap could mainly be ascribed to fine-structure changing collisions that release 9.2 GHz (440 mK). In the collision the atoms can acquire velocities of about 5 m/s, enough to be expelled from the trap at low trapping intensities. At higher intensities the loss becomes linearly dependent on intensity and was ascribed to fine-structure changing collisions. Comparison of the results of Ref. [134] with the Gallagher-Pritchard model[133] was very favorable indicating the importance of the model. Their data contradicted earlier measurements done by Prentiss et al.[135] who observed no dependence of the trap loss on intensity.

Despite its success in explaining the result of Sesko et al., recent experimental results on ^{85}Rb and ^{87}Rb indicates that modifications to the model are necessary. Hoffman et al.[136] reported large differences for radiative redistribution between Cs and ^{85}Rb although the model predicts only small differences. Also Wallace et al.[137] finds large differences in their case between ^{85}Rb and ^{87}Rb. They point out, that the collision dynamics is strongly altered by the ex-

cited state hyperfine structure, which was corroborated by experimental results of Feng et al.[138].

6.3.3 External Field Effects

In this section we discuss the modification of the collision dynamics due to external field, i.e. magnetic or laser fields. In the trap described in section 5.1.1 measurements were performed for associative ionization of sodium atoms. In such collisions between two sodium atoms excited to the 3P states, the Na_2 ion is formed and an electron is released. Results showed[139] that the cross section increase by more than a factor of 10^3 when the collision energy is lowered to sub-Doppler temperatures from thermal energies. In this experiment the optical fields are cycled through several stages. Atoms are trapped by a very strong radiation field in the first cycle and then cooled at low light intensities in an optical molasses in a second cycle. Detailed studies[140] revealed that the ions are mainly formed in the first cycle, although the number of excited atoms in both cases only changes by a factor of 2. This was corroborated by Julienne and Heather[141], who showed that due to the strongly detuned trapping light the excitation takes place at a much smaller internuclear distance thereby increasing the survival probability of the excited state considerably. The rate constant for associative ionization during the trapping cycle was estimated to be a factor 1000 larger compared to the rate constant during the cooling cycle.

6.4 deBroglie Wave Optics

One common aspect of recent progress in laser cooling of neutral atoms has been the evolution toward quantizing the atomic center of mass motion or external coordinates. Up to now the motion of atoms has been described in a perfectly classical way, assuming they had arbitrary position and momentum, and that both of these quantities could be known simultaneously. This classical picture of atoms moving as baseballs without regard to their overall wavelike character has been of great use, but recent experiments have become so sensitive that the center-of-mass motion of atoms must be viewed quantum mechanically. We have already seen the first discussion of such ideas in the calculations of quantum states in a magnetic trap in section 4.4. A collection of articles on related topics is to be found in Ref. [**?**].

It is now necessary to consider atomic position and momentum as quantum mechanical variables, replete with wave packet spreading and non-commuting operators. A deBroglie wave field occupies allowed states of a region of space that may have a spatially varying potential which defines modes of the field. These may be eigenstates in the optical potentials created by the laser fields. By contrast, if the potential is uniform, then it can be set to zero and a classical description of the motion may be used.

In analogy with optics, the occupation of particular modes of this field can result in spatial interference, and the entire field of atom interferometry emerges as a subset of this way of thinking. One important difference between this and the optical case arises because unlike photons, atoms are not all bosons. There will be cases where only a single atom can occupy a particular mode of the deBroglie wave field.

Thus there is a new view of optical control of atomic motion in terms of its quantum mechanical behavior. Of course, atoms at ordinary velocities are distributed over thousands of quantum states, so laser cooling is intimately involved in these studies. But the subject is evolving toward a quantum description optical control of atomic motion, and may also involve rf or microwave transitions to prepare desired internal states. After all, it is the internal states of atoms that determine the magnitude and nature of the electromagnetic forces on them.

6.4.1 Atom Optics

To control the motion of atoms requires the same devices as in optics, including mirrors, lenses, beam splitters, zone plates, etc. One of the first proposals for such a device was made by Cook and Hill[142], who suggested reflecting atoms from the evanescent wave of laser radiation leaking into the vacuum when light is totally internally reflected at a vacuum-dielectric interface. If the light is detuned blue from resonance, the atoms are repelled from the interface and thus reflected. This technique was demonstrated by Balykin et al.[143], who specularly reflected Na atoms off an internally illuminated quartz plate. The laser light was nearly resonant with one of the hyperfine components of the D_2-line of sodium, so only atoms impinging on the plate in the $F = 2$ ground state were reflected, whereas atoms in the $F = 1$ ground state were unaffected. In this way they could achieve a quantum-state selectivity of around 100. Balykin and Letokhov[144] suggested that a pair of concave mirrors would be the ideal arrangement for building an atomic cavity. One of the basic limitations of such a cavity, however, would be gravity. Since atoms would always be perturbed by the gravitational force, the lifetime of atoms in the cavity would be limited.

This problem can be overcome by dropping the atoms from an optical molasses held a few mm above a concave surface. Atoms released from the molasses fall down and are then reflected by the mirror. Although several bounces are possible, early experiments[145, 146] reported only one or two bounces. In an improved version of their earlier experiment, Aminoff et al.[147] showed that atoms can bounce as many as 8 times, before they are lost from the cavity. The losses were attributed to light scattering during reflection, collisions with background atoms, and scattering of stray light. Going to higher detunings reduced the losses due to scattering, but the signal-to-noise ratio decreased as well.

Besides atomic mirrors there have also been several attempts to build coherent atomic beam splitters. If the atoms undergo absorption and stimulated

emission in a transverse standing wave geometry, the wave packet of the atoms splits up coherently and such a beam splitter might serve in an atomic interferometer. To minimize the effects of spontaneous emission, one has either to reduce the number of spontaneous emissions by detuning far from resonance or reduce the interaction time to much shorter then the spontaneous lifetime. In a first experiment, Moskowitz et al.[148] showed that a beam of sodium atoms crossing a standing wave is deflected into two symmetric peaks, where the scattered atoms acquire momentum in multiples of $2\hbar k$ due to the combined absorption and stimulated emission of photon momenta $\hbar k$. Later Gould et al.[149, 150] showed that the r.m.s. momemtum gained by the atoms in the case of large detuning is proportinal to $\Omega_0\tau/\delta$, where Ω_0 is the Rabi frequency, τ is the interaction time and δ is the detuning.

Another demonstration of an atomic beam splitter was shown by Sleator et al.[151]. They created a large period standing wave by reflecting a laser beam at a small angle from a mirror. The interference field in front of the mirror forms a standing wave with a period much larger than λ, in their case 15 μm. As discussed in section 2.3.3, the proper description for a two level atom in a strong near resonant radiation field is in terms of dressed states. Since the dressed states have opposite light shifts, they feel opposite forces as atoms traverse the light field, and thus an incoming deBroglie wave packet will be split in two. They achieved a splitting of $4\hbar k$ in their experimental setup. In a similar arrangement they observed the focussing of an atomic beam, where they used the antinode of this large period standing wave[152]. Since the thickness of such a lens is given by the beam waist of the laser beam in the direction of the atomic beam (in their case only 40 μm) one can produce thin atomic lenses this way.

A significant increase of the transverse momentum can be obtained by using a novel magneto-optical force, as was first described by Grimm et al.[119]. In this scheme the atomic beam is crossed transversely by two counterpropagating laser beams, where both laser beams are linearly polarized and the polarizations make an angle ϕ with respect to each other. To obtain this new magneto-optical force, the authors discuss a V-type level scheme with one ground state g coupled to two excited states e_+ and e_-, which are split by a magnetic field along the laser beam direction. By matching the Larmor precession caused by the magnetic field with the Rabi oscillation, one can achieve the absorption of light from one beam followed by stimulated emission into the other beam. In this way a large number of photon momenta $\hbar k$ can be transferred from the light field to the atom. In such an experimental arrangement, Pfau et al.[153] showed that an atomic beam can be split in two, where the splitting between the two peaks was as large as 42 $\hbar k$.

6.4.2 Atom Interferometry

Atom optics and interferometry of deBroglie waves are an important part of this deBroglie wave field picture. Atoms can "interfere" if they occupy the same internal state while in a superposition of external states, and thus are indistinguishable. "Parts of an atom", i.e. atoms whose internal state is a superposition of eigenstates, certainly have some overlap and can thus interfere. In this same picture, Ramsey oscillations, spin and photon echoes, and quantum beats are simply interference in the time domain rather than in space.

There have been several demonstrations of atomic interference in both the temporal and spatial domain. The most important component of an interferometer is a device to either split a beam or recombine two beams while maintaining the appropriate coherence. This has been accomplished using mechanical gratings[154, 155, 156, 157] as well as by optical fields using diffraction[149], pulsed excitation[158, 159], or the optical Stern Gerlach effect[152].

Some of these experiments have been done in the spatial domain by deflecting the coherently split atomic beams away from one another and then recombining them (transverse), while others have been done in the time domain by delaying or phase shifting one of the two states that form an atomic superposition (longitudinal). The transverse experiments are much more similar to the familiar optical interferometers, and the most commonly used configuration resembles a Mach-Zehnder type. The longitudinal experiments are much more similar to the familiar Ramsey separated oscillatory fields method, and these experiments may be regarded simply as a reinterpretation of the Ramsey oscillations. Longitudinal interference experiments have been used in atomic fountains to study possible atomic clocks[125, 158]. Transverse experiments have been used to measure the gravitational acceleration[160] or photon recoil[161]. This measurement can be converted into a precision measurement of the ratio of the fundamental constants, h/m.

6.4.3 Energy Band Structure of Neutral Atoms

In some cases of atom cooling in optical standing waves, the residual atomic kinetic energies may be reduced to the same order as the light shifts (height of the potential hills). Castin and Dalibard[91] have presented a full quantum treatment of the motion of laser cooled atoms in a standing wave with a polarization gradient. They considered the quantized motion of atoms in the periodic potential and calculated the band structure of this system in one dimension. They found several very narrow low-lying bands that were bound in the valleys of the potential, and other higher energy ones where the atoms moved across the standing wave.

Their calculation has been extended from the simple $J = 1/2$ case to higher angular momentum states that correspond with experiments in Rb[100] and metastable helium (He*)[162]. In such cases the atoms move in a manifold

of sinusoidal potentials that arise from the different optical coupling strengths between the ground and excited states of different magnetic quantum number m.

Transverse atomic beam cooling experiments on the $2^3S \Leftrightarrow 2^3P$ transition of He* at $\lambda = 1.083$ μm have shown that the band structure manifests itself directly in the velocity distribution of the atoms[162]. An essential condition for the experimental observation of these effects is that the velocity resolution be appreciably less than one recoil velocity, and this is satisfied because of the high recoil velocity of He* resulting from its small mass.

In conjunction with theory, the measurements provide direct observations of transitions of the center of mass motion of atoms between different energy bands, or even between different Bloch states within the same energy bands. For example, the effect of reduced population in the lowest band of a potential well is a small dip in the velocity distribution at $v = 0$ because of the zero-point vibrational motion of the atoms. Subtracting the velocity distribution associated with the finite-energy zero-point motion from the broader distribution of cooled atoms results in a dip. This quantum effect, caused by the presence of zero-point energy, appears in both the observed and calculated velocity profiles.

6.4.4 Quantum Theory of Laser Cooling

In the semiclassical picture of laser cooling used up to now, the motion of atoms is treated as if they were point particles whose positions and velocities can be known simultaneously. The optical damping force on them is calculated from the force operator $\nabla\mathcal{H}$, and to average over the mixture of states created by the light field, we use $F = -\mathrm{Tr}(\rho\nabla\mathcal{H})$. Here ρ is the atomic density matrix found from its equation of motion (optical Bloch equations): $d\rho/dt = [\mathcal{H}, \rho]/i\hbar + \Gamma_\rho$, $\mathcal{H}$ is the atomic Hamiltonian and Γ_ρ contains the relaxation terms of ρ. The velocity distribution evolves in classical phase space according to the Fokker-Planck Equation (FPE).

It is clear that such a treatment is no longer appropriate when the deBroglie wavelength of the particle becomes comparable to the wavelength of the light. The view of a localized particle moving up and down hills whose size are comparable to the size of the atom becomes untenable. At the same velocities the use of the FPE becomes inappropriate, since the recoil momentum becomes comparable to the width of the momentum distribution.

By contrast, in the quantum mechanical view, the external states of motion are treated as quantum mechanical variables just as the internal atomic states. Laser cooling then becomes a process of optically pumping atoms to states of lower kinetic energy, dissipating the lost energy into the radiation field. This new theoretical approach requires a different explanation of laser cooling than that of a damping force competing with momentum diffusion, because stationary quantum states rather than classical trajectories are involved. Optical pumping and spontaneous emission deplete the more energetic quantum states faster than

the low energy states because the transition rates are asymmetric in kinetic energy. Thus laser cooling becomes an optical pumping process among external states of motion as well as among internal atomic states.

For example, cooling atoms in the MILC configuration of section 3.3 above is less effective for states deep in the wells which do not tunnel appreciably to the standing wave nodes and are therefore less affected by the magnetic field. Hence at relatively high intensity, the calculated cooling peak is broader with the quantum calculations than with the semiclassical approach. Furthermore, the progressive transfer to the lower states becomes so much slower for these lowest-lying levels that there is a population deficiency for the lowest state.

6.5 Non-linear Optics

One of the most widespread applications of the interaction between atoms and light is non-linear optics. Atomic absorption and scattering provide the appropriate interaction for multiple quantum effects, Raman processes, and other related phenomena. In order to avoid inter-atomic effects, such studies are often done in a vapor where collisions are negligible. Since many cases require that the detuning of the light from atomic resonance be large enough to avoid resonant excitation, this detuning must generally exceed the Doppler width associated with the motion of the atoms in the atomic vapor. At ordinary temperatures, such Doppler widths are a large fraction of a GHz, but laser cooling can provide atomic samples with much smaller values. With such samples, non-linear optics will enter a new domain where the Doppler widths are much smaller than the natural widths, and optical detunings can therefore be reduced to a few times the natural width. Thus one can expect enormous non-linear effects, allowing the exploration of previously known effects at much lower intensities, as well as the study of new effects that could not be observed at ordinary temperatures.

The earliest demonstration of non-linear optical spectroscopy in laser-cooled atomic vapors was in 1991[163, 164]. Both groups studied the Raman transitions of atoms trapped in an MOT (see section 5.2). The first experiments measured the transmission of an auxiliary probe laser beam through the atomic sample under conditions where it absorbed as much as 75% of the light. The experimenters observed three features in the spectrum, two broad and one narrow as shown in Fig. 6.5. The two broad features result from atomic fluorescence phenomena that are roughly independent of frequency, and hence are not affected by the Doppler shifts caused by atomic motion. But the narrow feature is a Raman transition involving light from each of the two laser beams, one from the probe and the other from the trap. Satisfying the Raman resonance condition for excitation by one beam and stimulated emission by the other demands that all atoms in the sample see both beams with the same frequency, and since they are not parallel, the atomic velocities must be small enough for the Doppler shifts to be negligible. This requires a sample of laser cooled atoms.

The signals arise from transitions between different magnetic sublevels of the ground state whose degeneracies have been lifted by the light shifts caused by the trap laser beams (see section 3.2.1), and whose populations differ as a result of optical pumping, again by the trap laser beams. In later stages of the experiments, additional laser beams were added to test this hypothesis[163]. This test, along with careful modeling of all the experimental parameters, confirmed the scenario described here.

Other Raman transitions have provided further experiments in non-linear optics with cold, trapped atoms[165, 166]. In these experiments an optical cavity was placed around a sample of Cs atoms in an MOT, and tuned near the frequency of the gain peak near ω in Fig. 6.5. Since there is a population difference between the two ground states coupled by the Raman resonance, stimulated emission of radiation can occur, resulting in gain in the cavity. When the cavity is resonant with the emitted light, the process is enhanced and a laser beam emerges from the excited mode of the cavity. Again, careful tests verified this description of the origin of the strong gain reported[165, 166].

This is a most curious laser configuration, because the gain width of the active medium is very narrow. In the case of condensed matter lasers, the spectral range of the gain curve is typically many orders of magnitude larger than the width of the laser's optical cavity (the cavity width is approximately its free spectral range divided by its finesse, or several MHz in most cases). Even for gas lasers, the spectral width of the gain medium is the Doppler width, typically a large fraction of a GHz. In this unique gas phase laser, however, the Doppler width is well below the cavity width, and also below the atomic natural width. In fact, the spectral width of the gain medium is determined by the inhomogeneous laser and magnetic fields, as well as by the optical pumping rate, just as in the Raman spectroscopy experiment described above. Clearly there will be very many new and interesting studies on this non-linear system.

Non-linear optical effects also play an important role in the experiments on atoms trapped in the periodic wells associated with the standing waves of optical molasses in one[92, 93, 94], two [97], or three dimensions[98, 99]. That subject has been discussed in section 5.1.2 on microscopic optical traps. Here it is only mentioned for completeness, along with the suggestion that there will surely be non-linear experiments performed on such samples of atoms. In Ref. [98] there is a considerable discussion of Bragg reflection and four wave mixing from atoms bound in a three-dimensional optical lattice. Further studies on this topic have been performed[167].

Still another multi-beam effect on laser cooled atoms is recoil-induced resonances. Here the resonance condition is calculated very precisely so that it includes both the energy *and* the momentum imparted to an atom that undergoes absorption followed by stimulated emission[168, 169, 170]. Since the recoil of the atom in the scattering process is included in the energy balance, the initial and final state are almost always non-degenerate, and a very sensitive dependence on atomic velocity enters the resonance condition. In many

ways it is similar to the Compton effect, and may be appropriately described as stimulated optical Compton scattering. Thus the method lends itself to very high resolution measurements of atomic velocities, well below the recoil velocity $\hbar k/M$. Since this corresponds to recoil energies in the kHz region, the limits of this spectroscopy will probably be dominated by interaction time and small field inhomogeneities.

One of the most interesting, and potentially useful, applications of non-linear optics is phase conjugate reflection. Two light beams prepare the atoms in a sample and a third incident beam is retroreflected, independent of its initial angle. The temporal phase of the beam is reversed, so that any aberrations or wave front distortions may be removed. This has applications in processing images from satellites, airborne cameras, or other sources. The use of a laser cooled sample for phase conjugate reflection will make enormous improvements in its sensitivity, and hence its utility. The first experiments of this kind have already demonstrated that it works[171], and further improvements are in progress.

FIGURE CAPTIONS

Figure

Schematic representation of a two-dimensional collimator. The incoming laser beam makes several bounces and at each bounce the angle of the laser light perpendicular to the atomic beam is reduced. In this way the light interacts with atoms coming from a much larger solid angle of the source. Furthermore due to the recycling of the light less laser intensity is required. (Figure from Ref. [36]).

Figure

The central Ramsey fringes of the microwave clock transition $F = 3 \Leftrightarrow 4, M_F = 0 \Leftrightarrow 0$ in a 15 cm high Cs fountain. Each open circle data point represents approximately 1 s of data collection time. (Figure from Ref. [126]).

Figure

(a) Interaction potential of two colliding particles interacting by a Born-Oppenheimer potential. (b) The trajectory of an atom in the potential of (a) for different impact parameters b. Indicated is the critical impact parameter b_c for which the collision leads to a reaction in the inner region.

Figure

The potential between ground state atoms and when one is in the excited state. For strongly red detuned light the excitation takes place at a much shorter internuclear distance for which the excited complex has a much larger probability to remain excited during the collision.

Figure

(a) Schematic diagram of the fine structure changing collision. Since after the collision a photon is emitted which is red detuned the atoms gain an amount of ΔE of kinetic energy, which is enough for ejection out of the trap. (b) Schematic diagram of the radiative redistribution process. Due to the attraction between the particles a photon is emitted which is red detuned compared to the photon that is initially absorbed. The energy difference ΔE can be enough to eject the atoms out of the trap.

Figure

The absorption spectrum of a probe beam transmitted through a sample of Cs atoms in an MOT. The large, broad absorption feature centered at the atomic absorption frequency ω_0^s, and the weak gain feature centered at $(2\omega - \omega_0^s)$ are not of interest here. But the narrow dispersion-shaped feature at ω corresponds to a stimulated Raman transition between ground state levels. Its 400 kHz width is less than 1/10 of the natural width, and is dominated by the Zeeman shifts from the inhomogeneous trap fields. The spectroscopic linewidth limit is determined by the transit time broadening, the spread of the light shifts from the inhomogeneous optical field, and the residual population of the atomic excited state. (Figure from Ref. [163].)

Chapter 7

Summary

It is clear that methods for controlling the motion of atoms are at hand, and that they will be considerably developed. Much progress has already been made, and several labs are actively engaged in both experimental and theoretical research. Although all of the methods for electromagnetic trapping strongly perturb the energy levels of the atoms, and thereby compromise precision spectroscopy, there may very well be new ideas for overcoming these difficulties. For example, these methods may find spectroscopic application by using the trap (or quasi-trap as in the optical molasses) as an accumulator and refrigerator for atoms which can later be released and examined, by alternating the trapping and spectroscopy times to keep the atoms trapped but provide perturbation-free times for observation, or by having a trap with perturbation free regions where the atoms can be observed. Since atoms at these low energies move so slowly (few cm/sec), their motion is strongly influenced by gravity, but perhaps the zero-gravity environment of space can be exploited for further work.

These methods of atom manipulation will lead to a new era in experimental techniques of unprecedented sensitivity, selectivity, precision, and accuracy. Experiments with Wigner crystals, Bose condensates, and single atoms will become common. For example, the ion beams in accelerators and the atoms emitted from an evaporator crucible can be controlled, refrigerated, and steered to an astounding degree. There will also be collision studies, perhaps leading to observation of collective effects. The long-sought Bose condensation may very well be accomplished with trapped atoms.

Also, study of the traps themselves will provide new applications of both classical and quantum mechanics. Calculations of the orbits on a classical mechanical scale have shown signs of an interesting form of chaos. Orbits on a quantum scale exhibit very close relations with the non-adiabatic condition discussed above because the atoms must necessarily be close to the origin where the field is zero. Finally, studies of the thermodynamics and statistical mechanics of laser cooling, either in or out of traps, may provide new insights into

non-equilibrium processes that can be tested in the laboratory.

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