Programming with dependent types

Wouter Swierstra

Dependent types

- Two 45 minute talks on two dependently typed systems (Coq & Agda).
- My goal is **not** to teach all the details of these systems;
- I want to give you a taste of what's out there. I've added pointers to further reading throughout the slides.

QuickCheck

- You’ve already seen how useful QuickCheck can be to find bugs.
- But is QuickCheck always right?

Example

Random testing

- QuickCheck is a fantastic tool, capable of finding *many* bugs.
- "Program testing can be used to show the presence of bugs, but never to show their absence!" – *Edsger Dijkstra*

A challenge problem

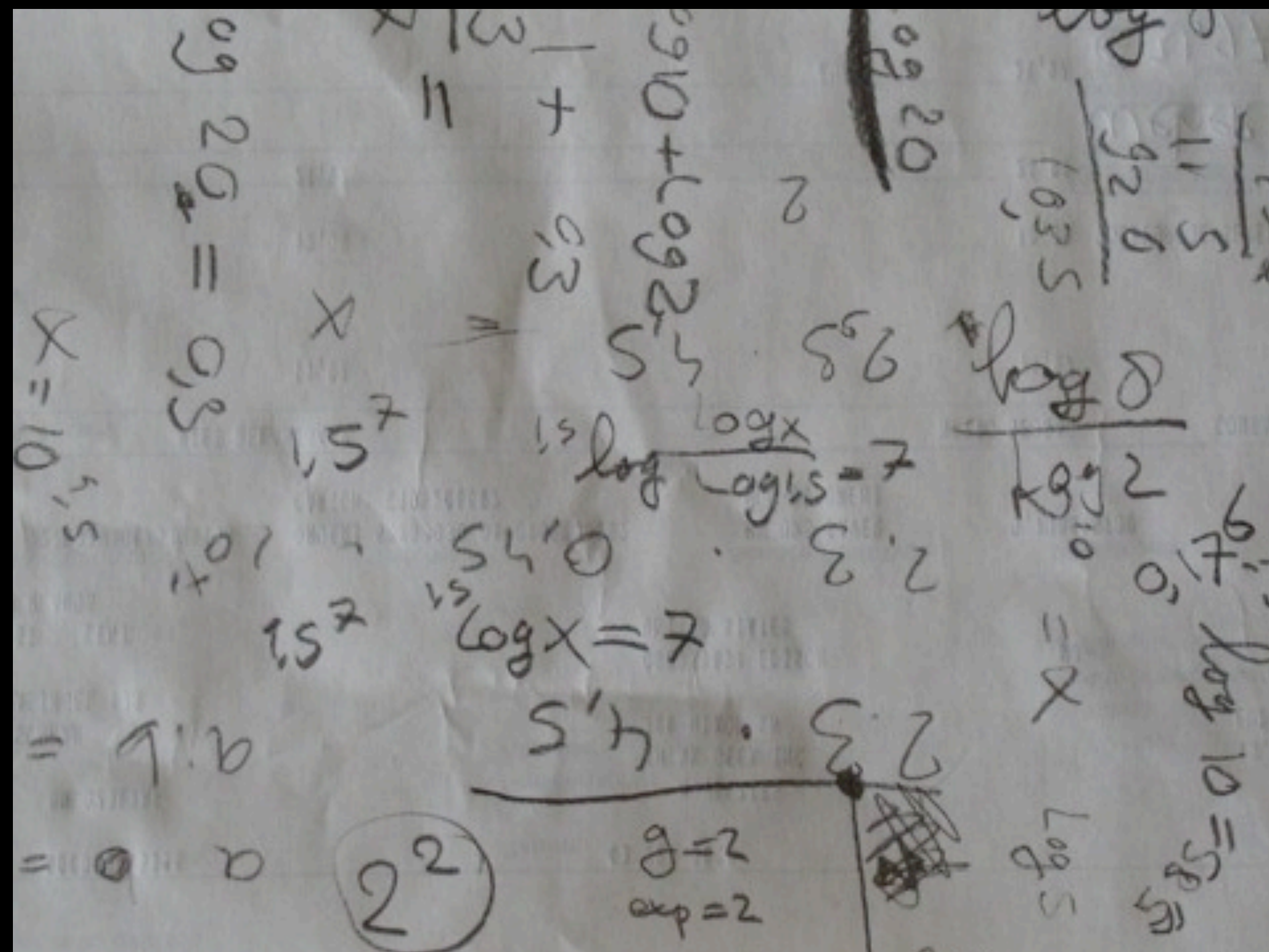
Prove that for all lists xs, ys, and zs:

```
xs ++ (ys ++ zs) = (xs ++ ys) ++ zs
```

Given the following definition for append:

```
[] ++ ys = ys

(x : xs) ++ ys = x : (xs ++ ys)
```



Maths

Equational reasoning

Let’s try a proof by induction on the list `xs`

In the base case we need to show that:

```
[] ++ (ys ++ zs) = ([] ++ ys) ++ zs
```

In the inductive case we need to show that:

```
(x : xs) ++ (ys ++ zs) =
((x : xs) ++ ys) ++ zs
```

Base case

```
[] ++ (ys ++ zs) =

 {  definition of ++ }

ys ++ zs =

 { definition of ++ }

([] ++ ys) ++ zs
```

Inductive case

```
(x : xs) ++ (ys ++ zs) =

  { def of ++ }

x : xs ++ (ys ++ zs) =

  { induction hypothesis }

x : (xs ++ ys) ++ zs =

  { def of ++ }

((x:xs) ++ ys) ++ zs
```

Equational reasoning

- It’s ‘easy’ to do proofs about pure functional programs.
- And once we have a proof, we know for sure that a property holds. Right?

```
1
  = { def const }
const 1 (head [])
  = { def head }
error “Exception: head []”
  = { def head }
const 2 (head [])
  = { def const }
2
```

Total functions

- Equational reasoning is only valid on *total* functions, i.e. those functions that are guaranteed compute an output for all possible inputs. Non-examples include:
  - The `head` function is not total (it do not have a branch for the empty list);
  - Nor is `dropWhile` (it may never terminate).

Coq

An interactive theorem prover

Coq

A total functional programming language

Programming in Coq

```
Inductive List (a : Type) : Type :=
  | Nil : List a
  | Cons : a -> List a -> List a.
Fixpoint append (xs ys : List a) : List a
  := match xs with
     | Nil => ys
     | Cons x xs => Cons x (append xs ys)
     end.
```

Tactics

- Coq proofs are (usually) written using *tactics*.
  - reflexivity
  - simpl
  - rewrite
  - induction

Example

Back to Haskell

- You can *extract* Haskell programs from your Coq developments.
- This discards any proofs that you've done, but leaves you with *verified code*.
- This works 'reasonably well' – even for larger Haskell projects like xmonad.

Tactics

- There are many more tactics (http://coq.inria.fr/refman/tactic-index.html)
- You'll need many other tactics to complete complex proofs...
- ... but the tactics you've seen so far should be enough to formalize any equational proof.

More about Coq...

- If you want to learn more about Coq, there are numerous tutorials and books online:
  - Coq in a hurry (Bertot)
  - Software Foundations (Pierce et al.)
  - Coq’Art (Bertot & Castéran)
  - Certified programming with dependent types (Chlipala).

Agda

Data.Word

- There are several different types for fixed-length bit words:
  - `Word8`
  - `Word16`
  - `Word32`
  - `Word64` – see a pattern?

From HaskellDB

```
data N1 = N1

data N2 = N2

...

data N255 = N255
```

From HaskellDB

```
data N1 = N1

data N2 = N2

...

data N255 = N255

class LessThan a b

instance N1 LessThan N2

.....
```

Haskell's limitations

- You can define algebraic data types and GADTs in Haskell.
- Data types are not always so simple...
- But how can you define the type of sorted lists? Or balanced trees? Or a number between 12 and 43?

Agda

- Agda is a dependently typed functional language;
- Just as in Coq, you can prove properties about functional programs (although there is no separate tactic language).
- But it supports programming with a advanced data types.

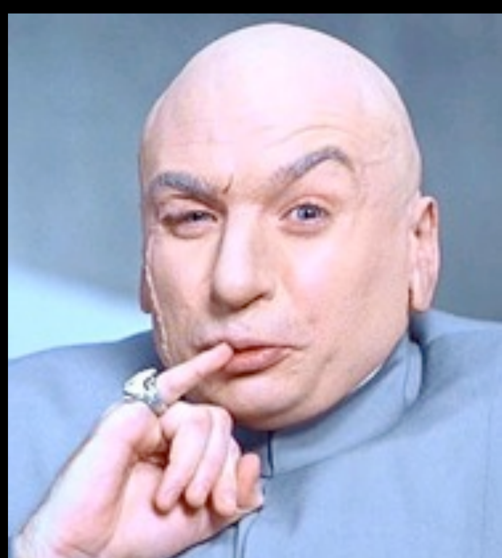
Dependent types

- In Haskell, you can write new types that abstract over other types, e.g., `List a`.
- But types cannot depend on values.
- In Agda you can define types that depend on values, such as numbers, booleans, or any other data type.

Demo

Why dependent types?

Why dependent types?



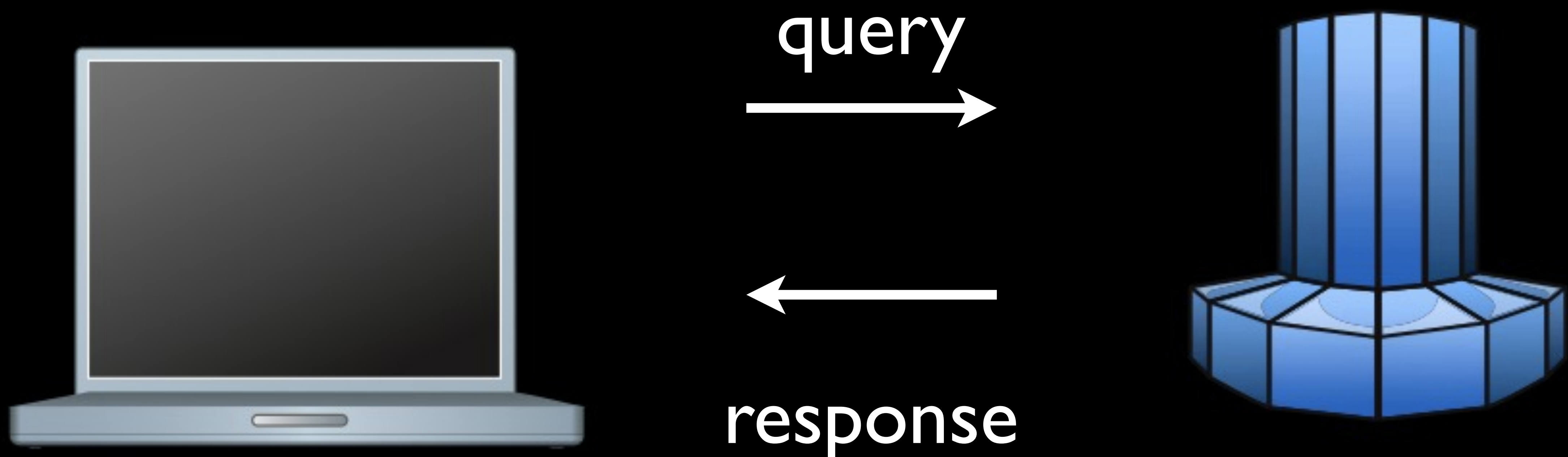
1011...

Evil real world

Computing types

- Sometimes you need to *compute* (static) types 'just-in-time' from (dynamic) data.
- This is 'impossible' in Haskell...
- ... but easy in Agda.

Example: database



Example: data base

Example: data base

“DESCRIBE ...”

```
“NAME     TYPE
--------------
UserID   INT32
...
```

Computing types

```
fromString :: String -> Set

fromString “INT32” = Int32

fromString “BOOLEAN” = Bool

fromString “DATE” = Date

...
```

Further reading

- The Agda wiki: wiki.portal.chalmers.se/agda
- Dependently typed programming with Agda (Norell)
- The Power of Pi (Oury & Swierstra)
- List of publications using Agda is maintained on the Agda wiki.

Conclusions

- Dependent types can be used for the verification of functional programs;
- Dependent types can describe precise data types;
- Dependent types can compute new types on the fly – 'just in time static typing'.

Dutch Hug

- Tomorrow night we'll have a meeting of the Dutch Haskell User's Group.
- Talks by myself and possibly a myster guest.
- Pizza!
- Drinks afterwards in the Basket!