# Multidimensional Data Visualization 

## Low-dimensional Data



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## Visualizing Relational Data



You have learned how to do this

## What about data attributes?



1 attribute
general graphs (color)

2 attributes (color, size)

## What about data attributes?



How to visualize this?

## Examples of trees with data attributes



Attributes (per node)

- type (color)
- image


Attributes (per node)

- type (color)
- multiple text tags


## The Visualization Pipeline



Direct vs Inverse Mapping Principles

$\xrightarrow[\text { direct mapping }]{\longrightarrow}$ inverse mapping
A. Telea, Data Visualization - Principles and Practice, $2^{\text {nd }}$ ed., CRC Press, 2014

## What are data attributes?

| ute type | operations | examples | K | $\left\lvert\, \begin{aligned} & \text { less } \\ & \text { general } \end{aligned}\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| categorical(1) | equality | gender |  |  |
| ordinal | above, <,> | weekdays | InfoVis data |  |
| discrete | above, +,- | \#persons |  |  |
| quantitative | above, *,/ | voltage | - SciVis data | $\downarrow$ general |

${ }^{1}$ relations can be seen as ordered pairs of categorical attributes

## Structure of a relational+attribute dataset

| node ID |  | attrib | s per |  |  | ges | most) | node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Node | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | ... | $\mathrm{A}_{\mathrm{n}}$ | $\mathrm{E}_{1}$ | $\mathrm{E}_{2}$ | $\ldots$ | $\mathrm{E}_{\mathrm{m}}$ |
| 1 | any kinds/values of all above four attribute types |  |  |  | 2 | 6 |  | 3 |
| 2 |  |  |  |  | 5 | 1 |  | 8 |

## What about 'big data'?

## Two independent things to measure

| node ID | $n$ attributes per node |  |  |  | $m$ edges (at most) per node |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Node | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}$ | ... | $\mathrm{A}_{\mathrm{n}}$ | $\mathrm{E}_{1}$ | $\mathrm{E}_{2}$ | $\ldots$ | $E_{m}$ |  |
| 1 |  |  |  |  | 2 | 6 |  | 3 |  |
| 2 |  |  |  |  | 5 | 1 |  | 8 |  |
| $\ldots$ |  |  |  |  |  |  |  |  |  |

## Number of nodes $N$

- each node (table row) has the same type and number of attributes
- also called samples, observations, or data points

Number of attributes $n$

- each attribute (table column) is of a given type (ordinal, categorical, etc)
- also called dimensions or variables


## What about 'big data'?

## Quiz

What is harder to visualize?

1000 data points having each 1 numerical attribute

100 data points having each 10 numerical attributes

## Why?

## What about 'big data'?

What is harder to visualize

- $\mathrm{N}=1000$ data points having $\mathrm{n}=1$ single numerical attribute each
- $\mathrm{N}=100$ data points having $\mathrm{n}=10$ numerical attributes each

The number of values $\mathbf{n *} \mathbf{N}$ to show is the same, but...

1000 samples $\mathbf{x} 1$ attribute
100 samples $\times 10$ attributes

1D graphs/charts work pretty well :)

many chart kinds, many problems (not scalable, cluttered, abstract, ...)

## Solution: Aggregation

Aggregating the samples (N)


Simple idea: reduce \#data points (N) by grouping related samples (e.g. averaging) Does this always work?

## Sample Aggregation Challenges

Continuous, numerical, spatial data

bone dataset, 80 K points

bone detail, 88 polygons

Discrete, non-numerical, non-spatial data


- we throw away $75 \%$ of the data
- the semantics stays the same
- interpolation: simple
- resampling: Cauchy-continuous ©
- we throw away one single character
- the semantics becomes fully different!
-interpolation: often not possible
- resampling: not Cauchy continuous $;$


## Solution: Aggregation

## Aggregating the dimensions ( n ) by selection



## Advantages

- very easy to do


## Problems

- which (few) dimensions to select to visualize?
- what to do with the other (tens..hundreds of) dimensions?


## Solution: Aggregation

## Aggregating the dimensions (n) by synthesis

2D projection


## Advantages

- visualization encodes all the data (samples, attributes)


## Problems

- what do the visual variables mean?
- how to decode the $n$ attributes from them?


## 1. Scatterplots



## Scatterplots

- show the correlation of $2 . .5$ ordinal/quantitative variables (measured at the same points)



Correlation between life expectancy, fertility rate and population of some world countries (2010)


Five variables (x/y position, size, color, label)

## Scatterplot challenges

## Size coding vs occlusion

- stock data (dot $=$ stock, $x=$ traded volume, $y=$ percentage change, color $=$ industry sector)

disk size $=$ market capitalization shows more data, but too much occlusion!

disk size = constant
shows less data (3 variables), but less occlusion

Color coding vs occlusion

color = 1-year performance (quantitative) less clutter than when mapping categorical variable industry sector to color

color $=$ constant
shows less data (2 variables) - classical scatterplot
easiest plot to interpret from all

## Encoding density

## Take a very large scatterplot

- tens..hundreds of thousands of data items (points)
- how to handle overplotting (occlusion)?

standard scatterplot
we have no idea what happens inside the big blob!



## Transparency coding

- very simple technique
- emphasizes data-rich regions, suppresses outliers
- in the limit: scatterplots become continuous density fields (for millions of points)


## Encoding density

Going from discrete scatterplots to continuous fields

color-coded density field we see local point density well

color-coded density field
we see local point density well

color-and-height coded density field we see local point density even better

## Main idea

- interpret the point-density as a continuous 2D scalar field
- this transforms the discrete scatterplot into a continuous field
- visualize this continuous field using classical field visualization methods
- continuous color coding (e.g. using rainbow colormap)
- 3D height plots, contours
- abstract from details, show overview (good for big data)


## Scatterplot matrices (SPLOMs)

- consider all pairs of columns $c_{i}, c_{j}$
- construct scatterplots $P_{i j}$ for all pairs $c_{i}, c_{j}$ over all rows
- arrange $P_{i j}$ in a (symmetric) matrix; shows correlation of any $c_{i}$ with any $c_{j}$
- nice, but does not scale to tens..hundreds of columns



## 2. Data Tables



## Tables

- one of the most ubiquitous types of (InfoVis) data
- table: set of rows (observations) and columns (dimensions)
- columns can have different types
- rows and columns are not uniquely ordered
- drawing a large table (> 10 columns or 50 rows) becomes useless...


## How to visualize big tables?



## Tables

First enhancement: overlay bar charts on columns

## Added value

- quickly scrolling through the table pre-attentively highlights minima, maxima, and large changes
- this allows us to explore large tables easily



## Tables

Second enhancement: the 'table lens' technique [Rao et al, '94]

- shows the table at any user-chosen level of detail

| 國Table: sif |  |  |  |  |  |  | [ x |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| id | date | time | open | high | low | close | $\wedge$ |
| 472 | 2005-02-15 | 11:00 | 1.480000 | $=1.480000$ | 1.480000 | 1.480000 |  |
| 473 | 2005-02-14 | 15:00 | 1.490000 | +1.490000 | 1.480000 | 1.480000 |  |
| 474 | 2005-02-14 | 14:00 | 1.500000 | +1.500000 | 1.470000 | 1.470000 |  |
| 475 | 2005-02-14 | 13:00 | 1.500000 | *1.52000 | 1.500000 | 1.520000 |  |
| 476 | 2005-02-14 | 12:00 | 1.470000 | -1.500000 | 1.470000 | 1.50000 |  |
| 477 | 2005-02-14 | 11:00 | 1.510000 | $=1.510000$ | 1.510000 | 1.510000 |  |
| 478 | 2005-02-10 | 14:00 | 1.340000 | -1.340000 | 1.330000 | 1.330000 |  |
| 479 | 2005-02-10 | 13:00 | 1.310000 | \$1.360000 | 1.310000 | 1.360000 |  |
| 480 | 2005-02-10 | 12:00 | 1.300000 | -1.310000 | 1.300000 | 1.310000 |  |
| 481 | 2005-02-10 | 11:00 | 1.300000 | $=1.300000$ | 1.300000 | 1.30000 |  |
| 482 | 2005-02-09 | 16:00 | 1.190000 | -1.22000d | 1.190000 | 1.220000 |  |
| 483 | 2005-02-09 | 15:00 | 1.090000 | = 1.090000 | 1.090000 | 1.09000 |  |
| 484 | 2005-02-09 | 14:00 | 1.100000 | = 1.100000 | 1.100000 | 1.10000 |  |
| 485 | 2005-02-09 | 13:00 | 1.177000 | -1.170006 | 1.130000 | 1.130000 |  |
| 486 | 2005-02-09 | 12:00 | 1.250000 | -1.25000d | 1.200000 | 1.20000 |  |
| 487 | 2005-02-07 | 15:00 | 1.290000 | -1.290000 | 1.280000 | 1.280000 |  |
| 488 | 2005-02.07 | 14:00 | 1.280000 | $=1.280000$ | 1.280000 | 1.280000 |  |
| 489 | 2005-02-07 | 13:00 | 1.280000 | $=1.280000$ | 1.280000 | 1.280000 |  |
| 490 | 2005-02-07 | 12:00 | 1.230000 | -1.260000 | 1.230000 | 1.260000 |  |
| 491 | 2005-02.04 | 15:00 | 1.300000 | -1.300000 | 1.290000 | 1.29000 |  |
| 492 | 2005-02-04 | 14:00 | 1.280000 | -1.200000 | 1.280000 | 1.200000 |  |
| 493 | 2005-02.04 | 13:00 | 1.350000 | +1.350000 | 1.310000 | 1.310000 |  |
| 494 | 2005-02-04 | 12:00 | 1.350000 | = 1.350000 | 1.350000 | 1.350000 |  |
| 455 | 2005-02-03 | 15:00 | 1.320000 | -1.330000 | 1.320000 | 1.330000 |  |
| 496 | 2005-02-03 | 14:00 | 1.340000 | -1.340000 | 1.310000 | 1.310000 |  |
| 497 | 2005-02-03 | 13:00 | 1.310000 | = 1.310000 | 1.310000 | 1.310000 |  |
| 498 | 2005-02-03 | 12:00 | 1.300000 | \$1.310000 | 1.300000 | 1.310000 |  |
| 499 | 2005-02-02 | 15:00 | 1.290000 | +1.290000 | 1.270000 | 1.270000 |  |
| 500 | 2005-02-02 | 14:00 | 1.230000 | -1.24000 | 1.230000 | 1.240000 |  |
| 501 | 2005-02.02 | 13:00 | 1.210000 | \$1.22000 | 1.210000 | 1.220000 |  |
| 502 | 2005-02-02 | 12:00 | 1.190000 | -1.24000 | 1.190000 | 1.240000 |  |
| 503 | 2005-02-01 | 16:00 | 1.190000 | $=1.190000$ | 1.190000 | 1.19000 |  |
| 504 | 2005-02-01 | 15:00 | 1.180000 | \$1.190000 | 1.180000 | 1.19000 |  |
| 505 | 2005-02-01 | 13:00 | 1.160000 | $=1.160000$ | 1.160000 | 1.160000 |  |
| 506 | 2005-02-01 | 12:00 | 1.150000 | $=1.150000$ | 1.150000 | 1.150000 |  |
| 507 | 2005-02-03 | 16:00 | 1.130000 | $=1.130006$ | 1.130000 | 1.130000 |  |
| 508 | 2005-02.03 | 15:00 | 1.120000 | $=1.120000$ | 1.120000 | 1.120000 |  |
| 509 | 2005-02-03 | 14:00 | 1.110000 | $=1.110000$ | 1.1110000 | 1.111000 |  |
| 510 | 2005-02-03 | 13:00 | 1.100000 | \$1.11000 | 1.100000 | 1.110000 |  |
| 511 | 2005-02.03 | 12:00 | 1.100000 | =1.100000 | 1.100000 | 1.100000 |  |
|  |  |  |  |  |  |  |  |

text opacity=1 font size=12pt

text opacity $\downarrow$ font size $\downarrow$

text not drawn bar opacity $\uparrow$

bar opacity=1
simplification=on

[^0]
## Tables

Third enhancement: single-column sorting

- sort table on values of user-selected column (attribute)
- zoom-out mode shows distribution and correlation of column values


## Stock data example



## Tables

Fourth enhancement: multiple-column sorting and row grouping

- sort table on multiple user-selected column values
- emphasize same-value column ranges with cushions ${ }^{1}$

Show stock data grouped by industry, company, and date


Sorting has two roles

- group rows having same value in an attribute
- show how other attributes vary within a group


## Tables

Multiple sorting: generates on-the-fly a hierarchy (tree) from the table

- one tree-level per sort
- one node per group of sorted rows having the same value

We'll see soon how to visualize this tree!


## Design Question: Trees or Tables?

Consider this simple table:

| Name | Age | Salary | Function |
| :--- | :--- | :--- | :--- |
| John Doe | 47 | $65 K$ | management |
| Bill Smith | 35 | 40 K | IT |
| Fanny Mae | 37 | 35 K | administration |



## Conclusion

- both designs are possible
- 'right' one depends on type of task
- compare items per category: use tree design
- compare all items across categories: use table design



## Icicle plots

## Basic idea

- nodes: rectangles; edges: not drawn explicitly, but shown by node positions
- one level per vertical band (root at left, leaves at right)
- siblings stacked vertically within a band
- compact display, no clutter; node size = sum of subtree importance


We shall see several variations of the icicle plot further!
¹J. Kruskal, J. Landwehr, Icicle plots: Better displays for hierarchical clustering, JSTOR, 1983

## 3. Treemaps



## Treemaps [Shneiderman '92]



Basic idea: ‘slice and dice’ layout

- 1 node $=1$ rectangle
- child node rectangles: nested in the parent node rectangle (recursive subdivision)
- leaf rectangle size and color show data attributes
- edges: not drawn explicitly!
- very compact: tens of thousands of nodes on one screen! no pixel wasted
- aspect ratios are not very good; hierarchy depth unclear

How can we improve the basic idea?

## Squarified Cushion Treemaps

## Two extensions of basic treemaps

- enforce near-square aspect ratios during rectangle subdivision ${ }^{1}$
- use shading: cushion profiles to convey hierarchical structure ${ }^{2}$


For each rectangle $r_{i}$ (except root)

- define 2D parabola $h_{i}(x, y)$ with height $H_{i}=f^{d}$ ( $d=$ depth of $r_{i}$ in tree)

Compute global height $h=\Sigma_{\mathrm{i}} h_{i}$
For each image pixel $I(x, y)$

- $I(x, y)=\operatorname{shading}(h(x, y))$
- use an oblique light source for better results

[^1]${ }^{2}$ J. J. van Wijk, H. van de Wetering (1999) Cushion treemaps: Visualization of hierarchical information, Proc. InfoVis, 135-142

## Squarified Cushion Treemaps



- rectangle borders are not explicitly drawn $\rightarrow$ gain space
- borders are implicit in the shading discontinuities
- discontinuity strength conveys tree depth levels
- near-square aspect ratio of cells allows easier size (area) comparison


## Squarified Cushion Treemaps

## Comparison of methods

hard disk, $\sim 30 \mathrm{~K}$ files


Slice and dice layout

- unbalanced cell sizes
(hard to compare)
- tree structure is not very clear


Squarified layout

- balanced cell sizes (easier to compare)
- tree structure is extremely salient


## Squarified Cushion Treemaps

## Example 1: WinDirStat tool ${ }^{1}$

- visualize a file system (e.g. your hard disk)

${ }^{1}$ http://windirstat.info


## Squarified Cushion Treemaps

## Example 2: Map of the Market ${ }^{1}$

Visualize stock exchange data online

- hierarchy: nesting of companies within sectors
- rectangle size: market capitalization
- color: gain (green) ... loss (red)



## Squarified Cushion Treemaps

## Example 3: Another Map of the Market ${ }^{1}$

## Visualize stock exchange data online

- hierarchy: nesting of companies within sectors
- rectangle size: market capitalization
- color: gain (green) ... loss (red)



## Squarified Cushion Treemaps

## Treemaps from tables

- recall the multiple-sorting idea for tables?
- implicitly creates a hierarchy from a table (with just a few clicks to sort columns)
- visualize hierarchy with a treemap


Stock exchange table

- three levels:
- market sector
- companies
- prices / day over 1 month
- colors
- red: daily loss
- green: daily gain
- light blue: unavailable data

Discovered a small emerging company with steady growth!
A. Telea (2006) Combining Extended Table Lens and Treemap Techniques for Visualizing Tabular Data, Proc. EuroVis

## 4. Parallel coordinates



## Parallel coordinates

## Take again a table

- rows: car brands
- columns: car parameters (MPG, cylinders, horsepower, weight, acceleration, fabrication year)


## Parallel coordinates

- table columns: different $y$ axes
- table cells: points on their corresponding axes
- table rows: polylines connecting their points
- column correlations: 'bundles' of close lines

A. Inselberg (2009) Parallel Coordinates: Visual Multidimensional Geometry and its Applications. Springer

Implementations: Mondrian (theursus.de/mondrian), Prefuse (prefuse.org), Xdat (xdat.org), Xmdv (http://davis.wpi.edu/xmdv)

## Tables vs parallel coordinates

Table plot


- row: horizontal line
- column: vertical line
- column-column correlation: not easy to see
- no overdraw/clutter


## Parallel coordinates



- row: skewed polyline
- column: vertical line (permuted values)
- column-column correlation: easy to see
- overdraw/clutter present


## Parallel coordinates

## Selection

- use mouse to select attribute ranges on axes
- highlight all rows (lines) passing through selection
- supports queries such as
- show all cars with a low acceleration
- find what attributes (e.g. MPG, cylinders, weight, ...) low-acceleration cars have



## Parallel coordinates

## Enhancements

- permute axes (horizontally) and swap their direction (vertically) to minimize line crossings
- add histograms on axes to show \#rows per unit-data value



## Smooth parallel coordinates

## Enhancements

- use curves (splines) instead of polylines
- reduces visual clutter
- makes visually following a sample (curve) easier
classical parallel coordinates

smooth parallel coordinates

M. Rida et al (2006). Multivariate continuous data - Parallel Coordinates. Graphics of Large Datasets: Visualizing a Million. Springer


## Bundled parallel coordinates

## Enhancements

- use curves (splines) instead of polylines (as in smooth parallel coordinates)
- bundle the curves (as in graph bundling)
- massively simplifies the visualization, reduces clutter
- following groups of similar samples (close curves) is much easier
classical parallel coordinates

bundled parallel coordinates


[^2]
## Hierarchical parallel coordinates

- reduce clutter for very large datasets ( $10^{6} . .10^{9}$ rows)
- hierarchically cluster rows $r_{i}$
(1) create a cluster $C_{i}=\left\{r_{i}\right\}$ for each row. Set $S=\left\{C_{i}\right\}$
(2) find two most similar clusters $C_{i}, C_{j}$ using an Euclidean distance metric $d\left(r_{i}, r_{j}\right)=\Sigma_{k}\left(r_{i k}-r_{j k}\right)^{2}$
(3) build parent cluster $C=\left(C_{i}, C_{j}\right), S=S \backslash\left(C_{i} \cup C_{j}\right) \cup C$
(4) repeat from step 2 until $S=\{$ root cluster \}
- select a 'cut' $K$ in the cluster tree $S$ at desired level-of-detail
- visualize each cluster $\mathrm{C} \in K$ with an opacity band which encodes cluster size and diameter


root cluster

finer-level cut

[^3]
## Putting it all together

## Low-dimensional data visualization

- easy-to-use tool: SPLOMs, parallel coordinates, and projections (next module)
- Java implementation (runs anywhere), simple text input format



## Summary: Low-dimensional data visualization

For what

- datasets with many samples $N$ but few (2..10) dimensions $n$


## Main design idea

- allocate one visual variable for one..a few dimensions


## Techniques

- scatterplots, scatterplot matrices
- table lenses
- table-tree duality
- icicle plots, treemaps
- parallel coordinates

Open challenge: What to do with many dimensions?


[^0]:    R. Rao, S. Card (1994) The table lens: merging graphical and symbolic representations in an interactive focus + context visualization for tabular information, Proc. ACM CHI, 318-322

[^1]:    ${ }^{1}$ M. Bruls, K. Huizing, J. J. van Wijk (1999) Squarified Treemaps, Proc. VisSym, 322-330

[^2]:    G. Palmas et al (2014). An Edge-Bundling Layout for Interactive Parallel Coordinates. Proc. PacificVis

[^3]:    * Y. Fua, M. Ward, E. Rundensteiner, Hierarchical Parallel Coordinates for Exploration of Large Datasets, IEEE InfoVis, 1999

    See also the Xmdv tool, http://davis.wpi.edu/xmdv

