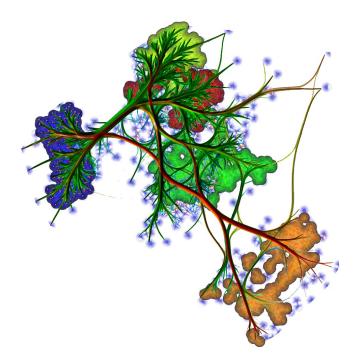
Multidimensional Data Visualization

Low-dimensional Data



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Department of Information and Computing Science Utrecht University, the Netherlands



Alexandru Telea

Professor Visual Data Analytics, Utrecht University, Netherlands

- full professor (UU, since 2019; RUG 2007-2019)
- 30 PhD students
- group leader Visualization and Graphics





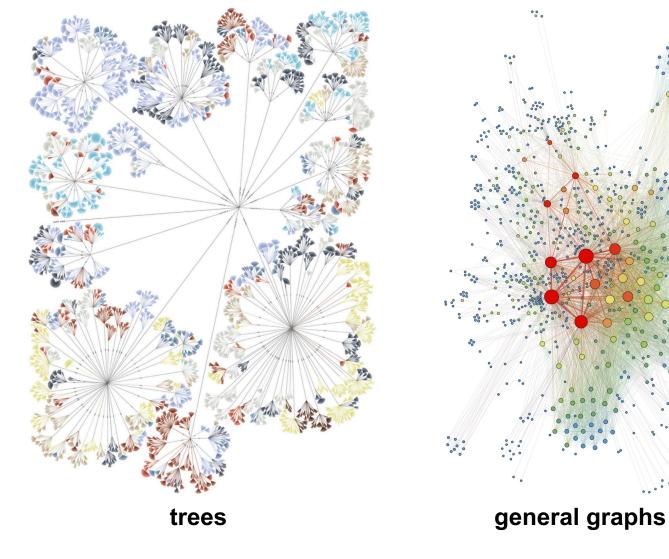
a.c.telea@uu.nl





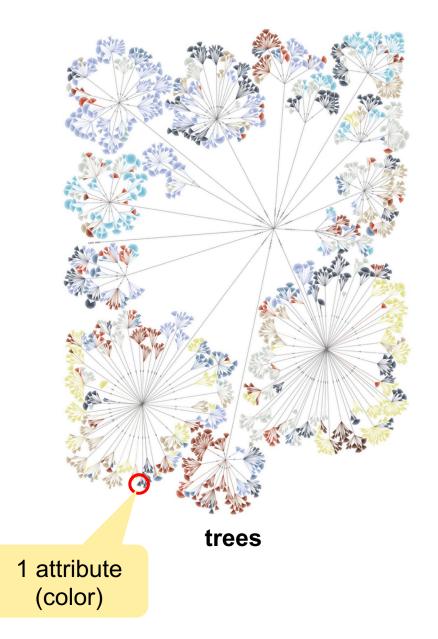


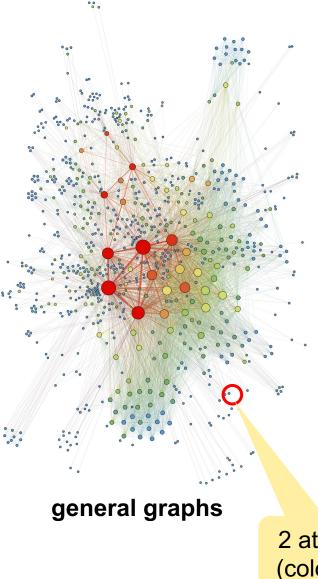
Visualizing Relational Data



You have learned how to do this

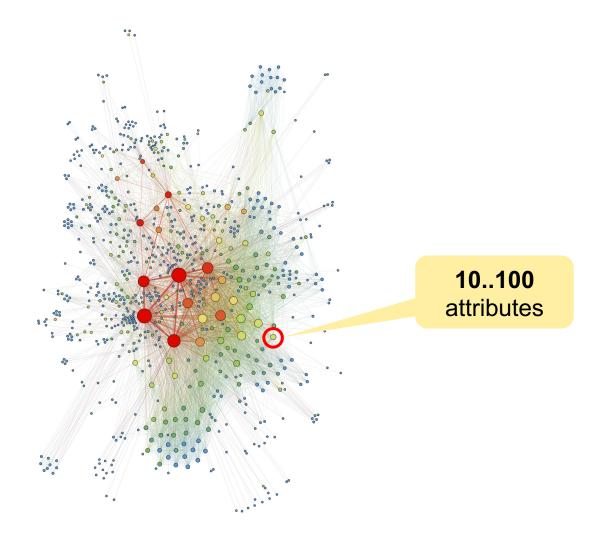
What about data attributes?





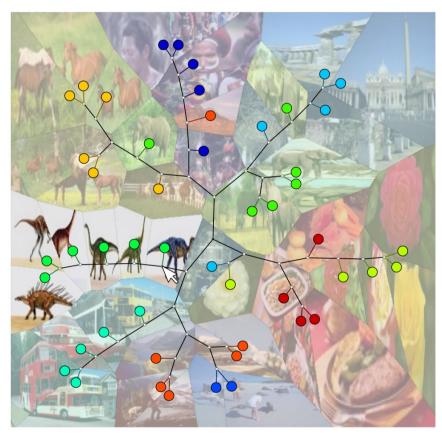
2 attributes (color, size)

What about data attributes?



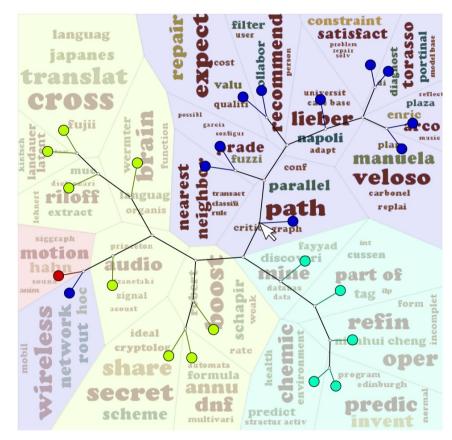
How to visualize this?

Examples of trees with data attributes



Attributes (per node)

- type (color)
- image

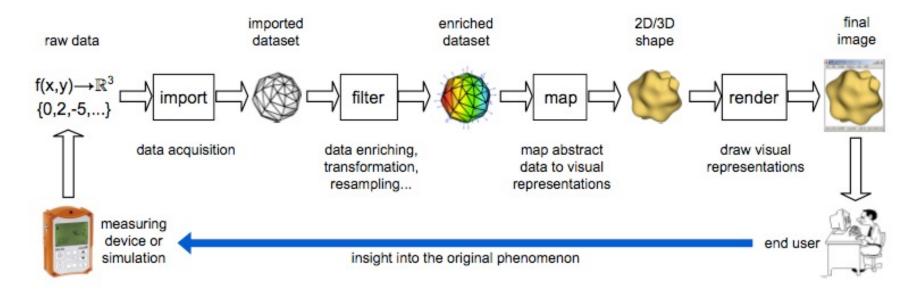


Attributes (per node)

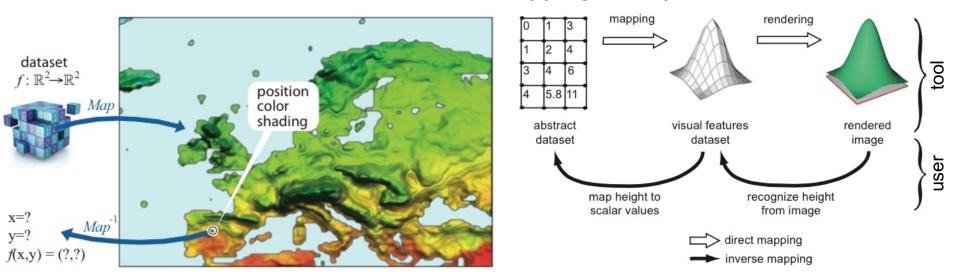
- type (color)
- multiple text tags

How to visualize many attributes?

The Visualization Pipeline

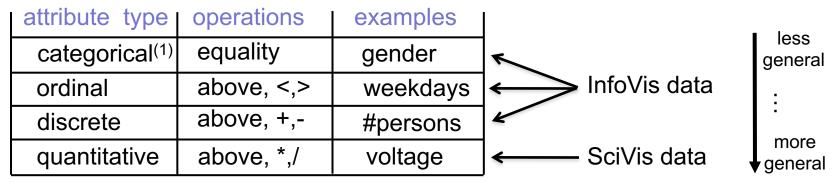


Direct vs Inverse Mapping Principles



A. Telea, Data Visualization – Principles and Practice, 2nd ed., CRC Press, 2014

What are data attributes?



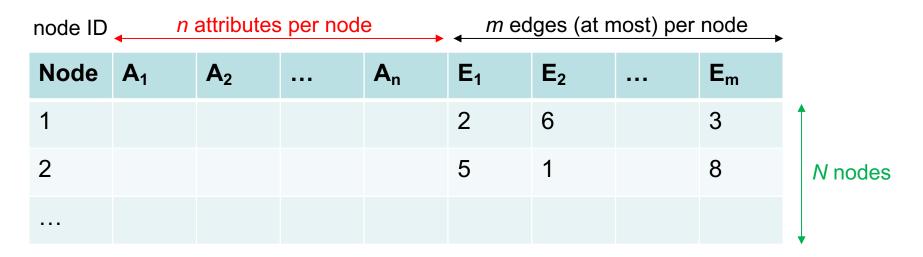
¹ relations can be seen as ordered pairs of categorical attributes

Structure of a relational+attribute dataset

node ID	◀	n attributes per node				<i>m</i> edges (at most) per node					
Node	A ₁		A ₂		\mathbf{A}_{n}		E ₁	E ₂		E _m	
1							2	6		3	
2		any kinds/values of all above four attribute					5	1		8	
			typ	Des							

What about 'big data'?

Two independent things to measure



Number of nodes N

- each node (table row) has the same type and number of attributes
- also called samples, observations, or data points

Number of attributes *n*

- each attribute (table column) is of a given type (ordinal, categorical, etc)
- also called dimensions or variables

What about 'big data'?



What is harder to visualize?

1000 data points having each 1 numerical attribute

100 data points having each 10 numerical attributes

Why?

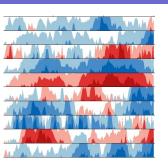
What about 'big data'?

What is harder to visualize

- N=1000 data points having n=1 single numerical attribute each
- N=100 data points having n=10 numerical attributes each

The number of values **n*N** to show is the same, but...

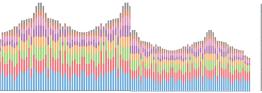
150 1.00 1D graphs/charts work pretty well ©

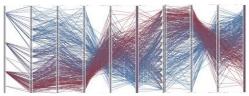


100 samples x 10 attributes



many chart kinds, many problems (not scalable, cluttered, abstract, ...)

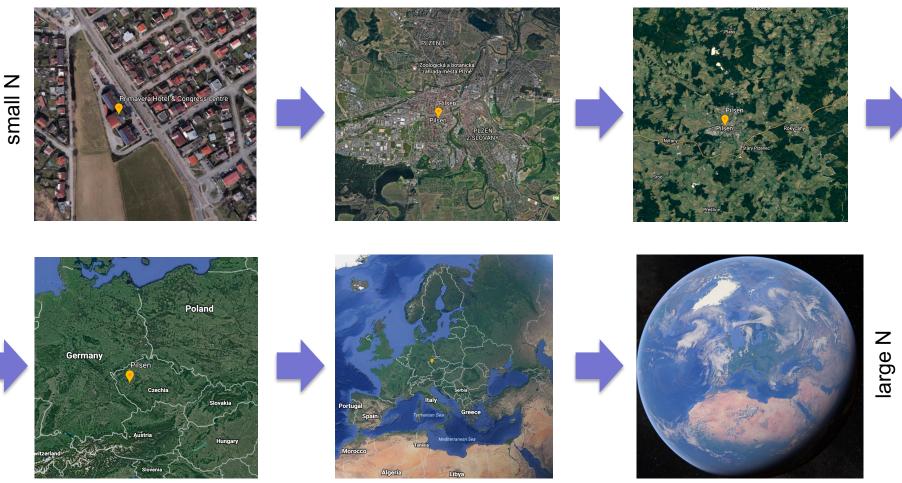




1000 samples x 1 attribute

Solution: Aggregation

Aggregating the samples (N)

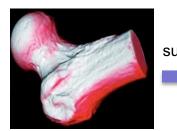


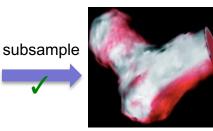
Simple idea: reduce #data points (N) by grouping related samples (e.g. averaging)

Does this always work?

Sample Aggregation Challenges

Continuous, numerical, spatial data





bone dataset, 80K points

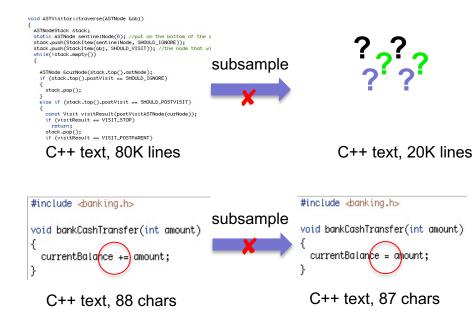
bone dataset, 20K points



bone detail, 88 polygons

- bone detail, 87 polygons
- we throw away 75% of the data
- the semantics stays the same
- interpolation: simple
- resampling: Cauchy-continuous ©

Discrete, non-numerical, non-spatial data



- we throw away one single character
- the semantics becomes fully different!

•interpolation: often not possible

• resampling: not Cauchy continuous 😕

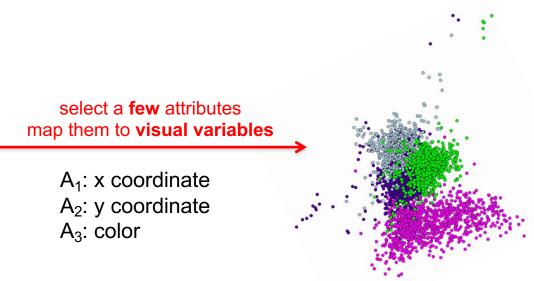
No simple solution for all datasets 🛞

Solution: Aggregation

Aggregating the dimensions (n) by selection

table

id	category	name	date	time	open	hiah	low	close
536	sif	SIF1	2004-11-29	13:00	0.800000	1.800000	0.800000	0.800000
535	sif	SIF1	2004-11-29	14:00	0.800000	0.800000	0.800000	0.800000
533	sif	SIF1	2004-11-29	16:00	0.795000	0.795000	0.795000	0.795000
530	sif	SIF1	2004-11-30	14:00	0.795000	0.795000	0.795000	0.795000
532	sif	SIF1	2004-11-30	12:00	0.800000	0.800000	0.795000	0.795000
531	siF	SIF1	2004-11-30	13:00	0.795000	0.795000	0.795000	0.795000
528	sif	SIF1	2004-11-30	16:00	0.795000	0.795000	0.795000	0.795000
29	sif	SIF1	2004-11-30	15:00	0.795000	0.795000	0.795000	0.795000
527	sif	SIF1	2005-00-02	12:00	0.785000	0.790000	0.785000	0.790000
526	sif	SIF1	2005-00-02	13:00	0.790000	0.795000	0.790000	0.795000
525	sif	SIF1	2005-00-02	14:00	0.795000	0.795000	0.795000	0.795000
524	sif	SIF1	2005-00-02	15:00	0.800000	0.800000	0.800000	0.800000
20	siF	SIF1	2005-00-03	15:00	0.795000	0.795000	0.795000	0.795000
523	sif	SIF1	2005-00-03	12:00	0.795000	0.795000	0.795000	0.795000
522	sif	SIF1	2005-00-03	13:00	0.795000	0.795000	0.795000	0.795000
521	sif	SIF1	2005-00-03	14:00	0.795000	0.795000	0.795000	0.795000
519	sif	SIF1	2005-00-03	16:00	0.795000	0.795000	0.795000	0.795000
518	sif	SIF1	2005-00-06	11:00	0.790000	0.790000	0.790000	0.790000
14	sif	SIF1	2005-00-06	15:00	0.795000	0.795000	0.795000	0.795000
517	sif	SIF1	2005-00-06	12:00	0.795000	0.795000	0.795000	0.795000
516	sif	SIF1	2005-00-06	13:00	0.795000	0.795000	0.795000	0.795000
15	sif	SIF1	2005-00-06	14:00	0.795000	0.795000	0.795000	0.795000
513	sif	SIF1	2005-00-06	16:00	0.795000	0.795000	0.795000	0.795000
09	sif	SIF1	2005-00-07	14:00	0.790000	0.795000	0.790000	0.795000
12	sif	SIF1	2005-00-07	11:00	0.795000	0.795000	0.795000	0.795000
11	sif	SIF1	2005-00-07	12:00	0.795000	0.795000	0.795000	0.795000
10	sif	SIF1	2005-00-07	13:00	0.790000	0.790000	0.790000	0.790000
808	sif	SIF1	2005-00-07	15:00	0.790000	0.790000	0.790000	0.790000
506	sif	SIF1	2005-00-08	13:00	0.795000	0.795000	0,795000	0.795000
507	sif	SIF1	2005-00-08	12:00	0.790000	0.790000	0.790000	0.790000
505	sif	SIF1	2005-00-08	14:00	0.795000	0.795000	0.795000	0.795000



visualization

Advantages

very easy to do

Problems

- which (few) dimensions to select to visualize?
- what to do with the other (tens..hundreds of) dimensions?

 \geq

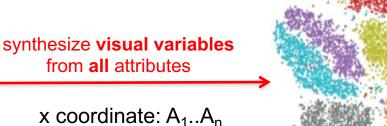
Solution: Aggregation

Aggregating the dimensions (n) by synthesis

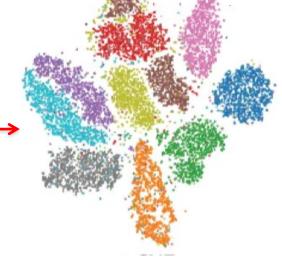
n attributes

2D projection

id	category	name	date	time	open	high	low	close
636	sif	SIF1	2004-11-29	13:00	0.800000	0.800000	0.800000	0.800000
635	sif	SIF1	2004-11-29	14:00	0.800000	0.800000	0.800000	0.800000
633	sif	SIF1	2004-11-29	16:00	0.795000	0.795000	0.795000	0.795000
630	sif	SIF1	2004-11-30	14:00	0.795000	0.795000	0.795000	0.795000
632	sif	SIF1	2004-11-30	12:00	0.800000	0.800000	0.795000	0.795000
631	sif	SIF1	2004-11-30	13:00	0.795000	0.795000	0.795000	0.795000
628	sif	SIF1	2004-11-30	16:00	0.795000	0.795000	0.795000	0.795000
629	sif	SIF1	2004-11-30	15:00	0.795000	0.795000	0.795000	0.795000
627	sif	SIF1	2005-00-02	12:00	0.785000	0.790000	0.785000	0.790000
626	sif	SIF1	2005-00-02	13:00	0.790000	0.795000	0.790000	0.795000
625	sif	SIF1	2005-00-02	14:00	0.795000	0.795000	0.795000	0.795000
624	sif	SIF1	2005-00-02	15:00	0.800000	0.800000	0.800000	0.800000
620	siF	SIF1	2005-00-03	15:00	0.795000	0.795000	0.795000	0.795000
623	sif	SIF1	2005-00-03	12:00	0.795000	0.795000	0.795000	0.795000
622	sif	SIF1	2005-00-03	13:00	0.795000	0.795000	0.795000	0.795000
621	sif	SIF1	2005-00-03	14:00	0.795000	0.795000	0.795000	0.795000
619	sif	SIF1	2005-00-03	16:00	0.795000	0.795000	0.795000	0.795000
618	sif	SIF1	2005-00-06	11:00	0.790000	0.790000	0.790000	0.790000
614	sif	SIF1	2005-00-06	15:00	0.795000	0.795000	0.795000	0.795000
617	sif	SIF1	2005-00-06	12:00	0.795000	0.795000	0.795000	0.795000
616	sif	SIF1	2005-00-06	13:00	0.795000	0.795000	0.795000	0.795000
615	sif	SIF1	2005-00-06	14:00	0.795000	0.795000	0.795000	0.795000
613	sif	SIF1	2005-00-06	16:00	0.795000	0.795000	0.795000	0.795000
609	sif	SIF1	2005-00-07	14:00	0.790000	0.795000	0.790000	0.795000
612	siF	SIF1	2005-00-07	11:00	0.795000	0.795000	0.795000	0.795000
611	sif	SIF1	2005-00-07	12:00	0.795000	0.795000	0.795000	0.795000
610	sif	SIF1	2005-00-07	13:00	0.790000	0.790000	0.790000	0.790000
608	sif	SIF1	2005-00-07	15:00	0.790000	0.790000	0.790000	0.790000
606	sif	SIF1	2005-00-08	13:00	0.795000	0.795000	0.795000	0.795000
607	sif	SIF1	2005-00-08	12:00	0.790000	0.790000	0.790000	0.790000
605	sif	SIF1	2005-00-08	14:00	0.795000	0.795000	0.795000	0.795000



 A_2 : y coordinate A_1 ... A_3 : color



Advantages

• visualization encodes all the data (samples, attributes)

Problems

- what do the visual variables mean?
- how to decode the *n* attributes from them?

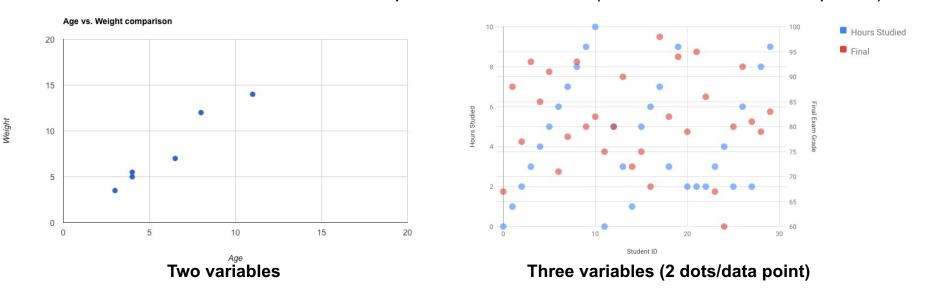
1. Scatterplots



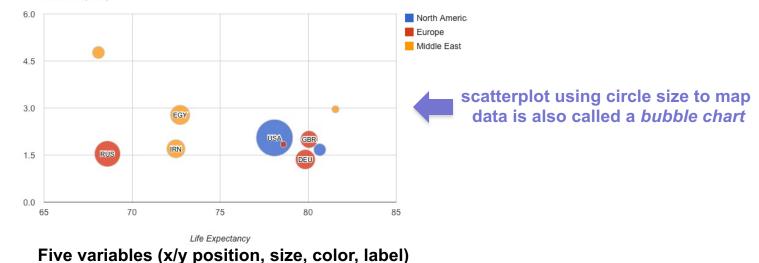
Scatterplots

Fertility Rate

• show the correlation of 2..5 ordinal/quantitative variables (measured at the same points)



Correlation between life expectancy, fertility rate and population of some world countries (2010)



Scatterplot challenges

Size coding vs occlusion

• stock data (dot = stock, *x* = traded volume, *y* = percentage change, *color* = industry sector)



disk size = market capitalization shows more data, but too much occlusion!

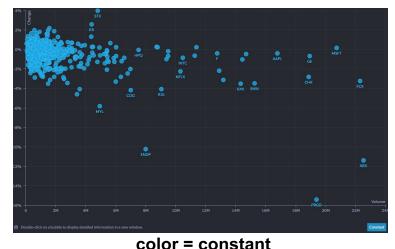


disk size = constant shows less data (3 variables), but less occlusion



color = 1-year performance (quantitative) less clutter than when mapping categorical variable *industry sector* to color

Color coding vs occlusion

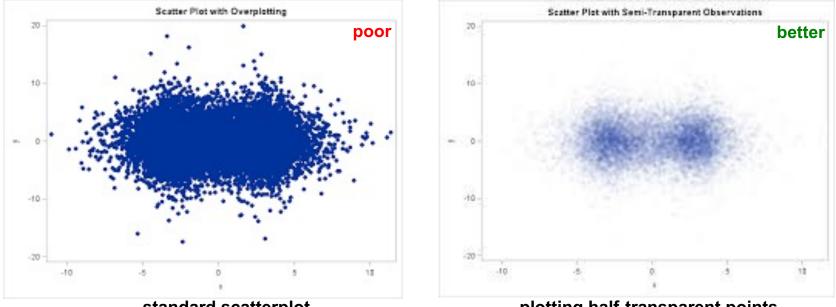


shows less data (2 variables) – classical scatterplot easiest plot to interpret from all

Encoding density

Take a very large scatterplot

- tens..hundreds of thousands of data items (points)
- how to handle overplotting (occlusion)?



standard scatterplot we have no idea what happens inside the big blob!

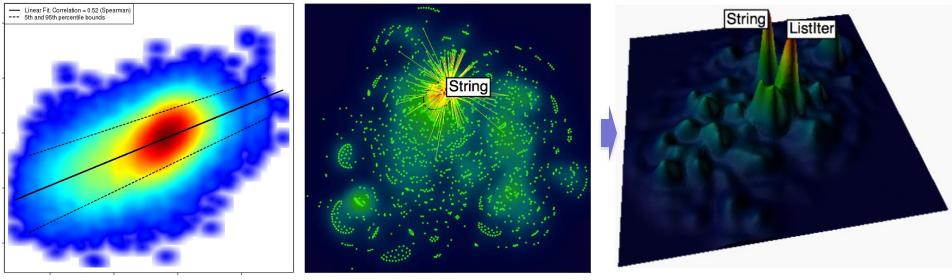
plotting half-transparent points saturation shows local point density!

Transparency coding

- very simple technique
- emphasizes data-rich regions, suppresses outliers
- in the limit: scatterplots become continuous density fields (for millions of points)

Encoding density

Going from discrete scatterplots to continuous fields



color-coded density field we see local point density well

color-coded density field we see local point density well

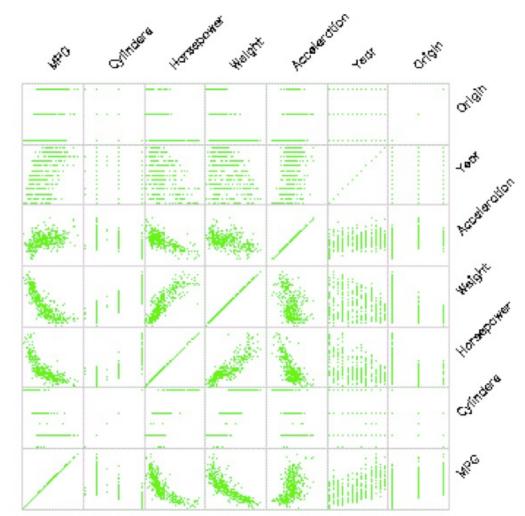
color-and-height coded density field we see local point density even better

Main idea

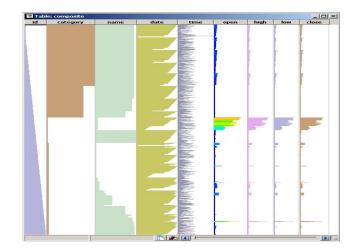
- interpret the point-density as a **continuous** 2D scalar field
- · this transforms the discrete scatterplot into a continuous field
- visualize this continuous field using classical field visualization methods
 - continuous color coding (e.g. using rainbow colormap)
 - 3D height plots, contours
- abstract from details, show overview (good for big data)

Scatterplot matrices (SPLOMs)

- consider all pairs of **columns** c_i , c_j
 - construct scatterplots P_{ij} for all pairs c_i , c_j over all rows
 - arrange P_{ij} in a (symmetric) matrix; shows correlation of any c_i with any c_j
- nice, but does not scale to tens..hundreds of columns



2. Data Tables



- one of the most ubiquitous types of (InfoVis) data
- table: set of rows (observations) and columns (dimensions)
- columns can have different types
- rows and columns are not uniquely ordered
- drawing a large table (> 10 columns or 50 rows) becomes useless...

id	category	name	date	time	open	high	low	close
636	siF	SIF1	2004-11-29	13:00	0.800000	0.800000	0.800000	0.800000
635	sif	SIF1	2004-11-29	14:00	0.800000	0.800000	0.800000	0.800000
633	sif	SIF1	2004-11-29	16:00	0.795000	0.795000	0.795000	0.795000
630	sif	SIF1	2004-11-30	14:00	0.795000	0.795000	0.795000	0.795000
632	sif	SIF1	2004-11-30	12:00	0.800000	0.800000	0.795000	0.795000
631	sif	SIF1	2004-11-30	13:00	0.795000	0.795000	0.795000	0.795000
628	sif	SIF1	2004-11-30	16:00	0.795000	0.795000	0.795000	0.795000
629	sif	SIF1	2004-11-30	15:00	0.795000	0.795000	0.795000	0.795000
627	sif	SIF1	2005-00-02	12:00	0.785000	0.790000	0.785000	0.790000
626	sif	SIF1	2005-00-02	13:00	0.790000	0.795000	0.790000	0.795000
625	sif	SIF1	2005-00-02	14:00	0.795000	0.795000	0.795000	0.795000
624	sif	SIF1	2005-00-02	15:00	0.800000	0.800000	0.800000	0.800000
620	sif	SIF1	2005-00-03	15:00	0.795000	0.795000	0.795000	0.795000
623	sif	SIF1	2005-00-03	12:00	0.795000	0.795000	0.795000	0.795000
622	sif	SIF1	2005-00-03	13:00	0.795000	0.795000	0.795000	0.795000
621	sif	SIF1	2005-00-03	14:00	0.795000	0.795000	0.795000	0.795000
619	sif	SIF1	2005-00-03	16:00	0.795000	0.795000	0.795000	0.795000
618	sif	SIF1	2005-00-06	11:00	0.790000	0.790000	0.790000	0.790000
614	sif	SIF1	2005-00-06	15:00	0.795000	0.795000	0.795000	0.795000
617	sif	SIF1	2005-00-06	12:00	0.795000	0.795000	0.795000	0.795000
616	sif	SIF1	2005-00-06	13:00	0.795000	0.795000	0.795000	0.795000
615	sif	SIF1	2005-00-06	14:00	0.795000	0.795000	0.795000	0.795000
613	sif	SIF1	2005-00-06	16:00	0.795000	0.795000	0.795000	0.795000
609	sif	SIF1	2005-00-07	14:00	0.790000	0.795000	0.790000	0.795000
612	sif	SIF1	2005-00-07	11:00	0.795000	0.795000	0.795000	0.795000
611	sif	SIF1	2005-00-07	12:00	0.795000	0.795000	0.795000	0.795000
610	sif	SIF1	2005-00-07	13:00	0.790000	0.790000	0.790000	0.790000
608	sif	SIF1	2005-00-07	15:00	0.790000	0.790000	0.790000	0.790000
606	sif	SIF1	2005-00-08	13:00	0.795000	0.795000	0.795000	0.795000
607	sif	SIF1	2005-00-08	12:00	0.790000	0.790000	0.790000	0.790000
605	sif	SIF1	2005-00-08	14:00	0.795000	0.795000	0.795000	0.795000

How to visualize big tables?

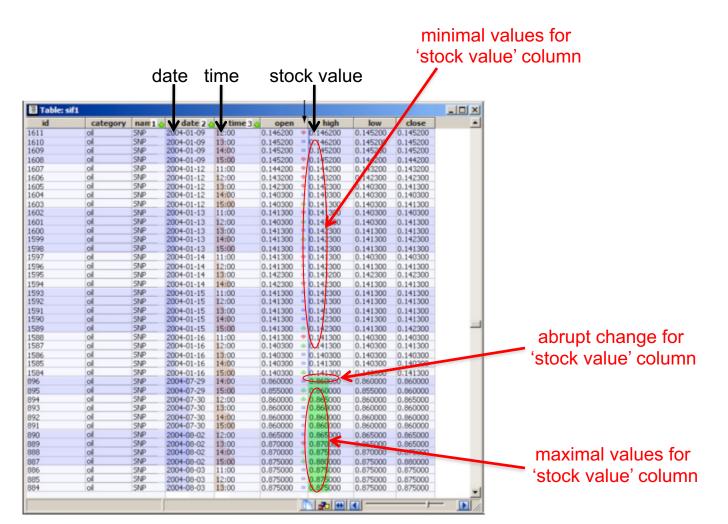
Example: stock exchange data

m transactions

n attributes of a transaction

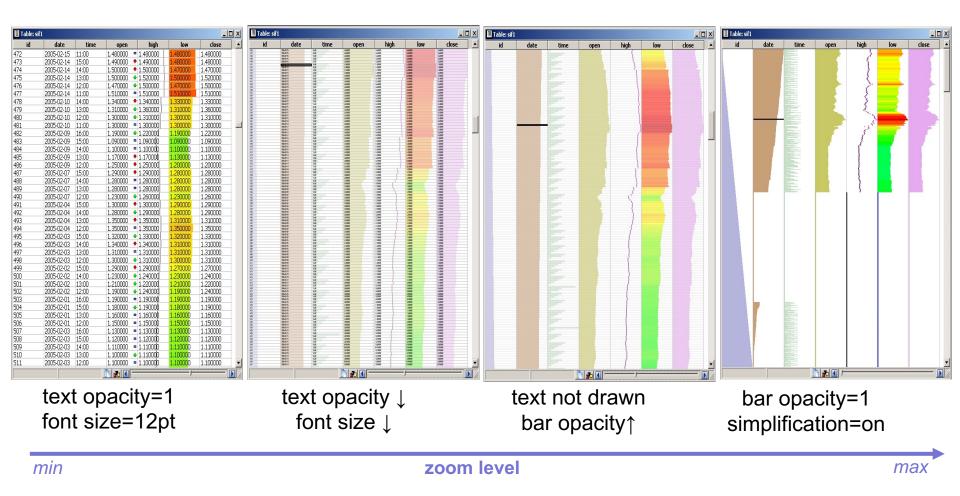
First enhancement: overlay bar charts on columns Added value

- quickly scrolling through the table pre-attentively highlights minima, maxima, and large changes
- this allows us to explore large tables easily



Second enhancement: the 'table lens' technique [Rao et al, '94]

• shows the table at any user-chosen level of detail



R. Rao, S. Card (1994) The table lens: merging graphical and symbolic representations in an interactive focus + context visualization for tabular information, Proc. ACM CHI, 318-322

Third enhancement: single-column sorting

- **sort** table on values of user-selected column (attribute)
- zoom-out mode shows distribution and correlation of column values

Stock data example

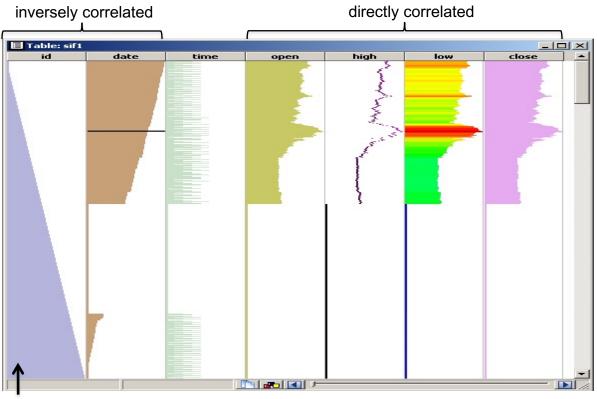
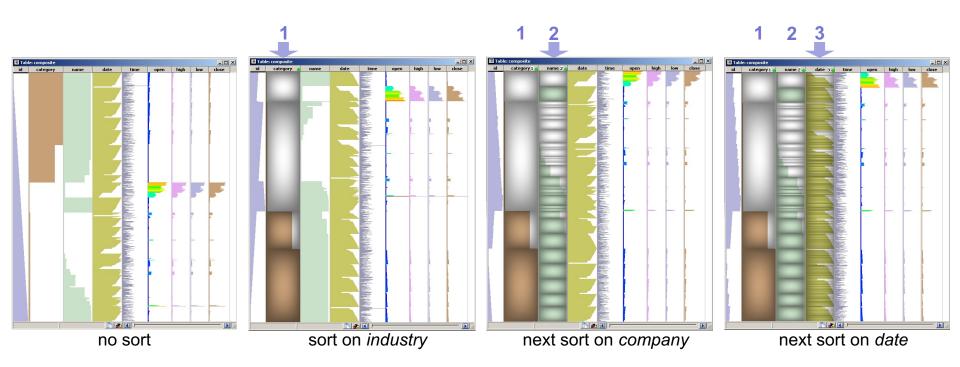


table sorted on transaction ID

Fourth enhancement: multiple-column sorting and row grouping

- **sort** table on multiple user-selected column values
- emphasize **same-value** column ranges with *cushions*¹

Show stock data grouped by industry, company, and date



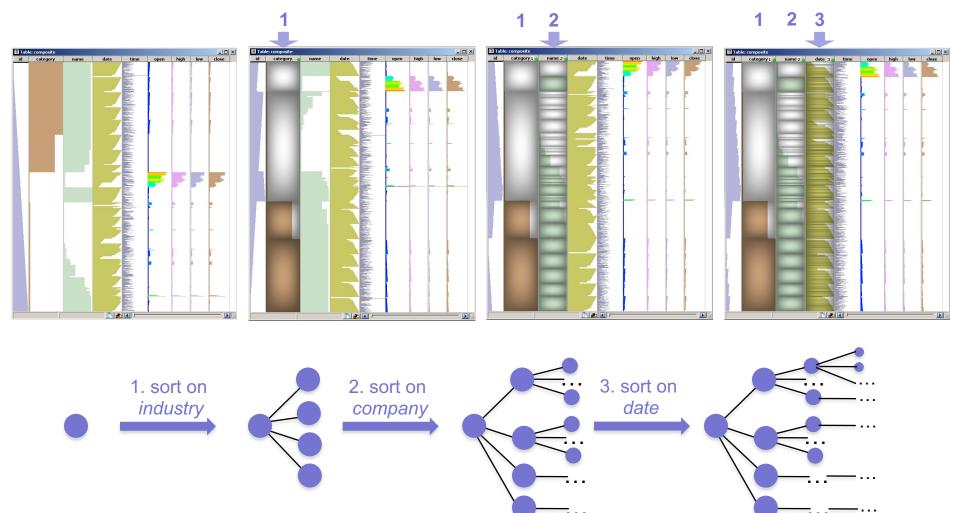
Sorting has two roles

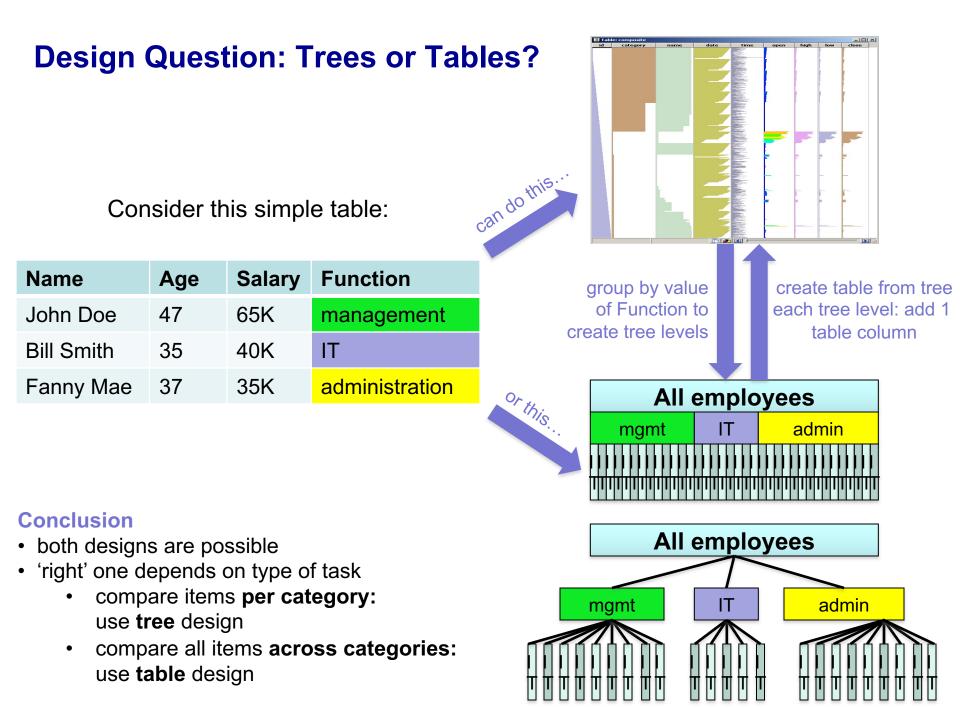
- group rows having same value in an attribute
- show how other attributes vary within a group

Multiple sorting: generates on-the-fly a hierarchy (tree) from the table

- one tree-level per sort
- one node per group of sorted rows having the same value

We'll see soon how to visualize this tree!

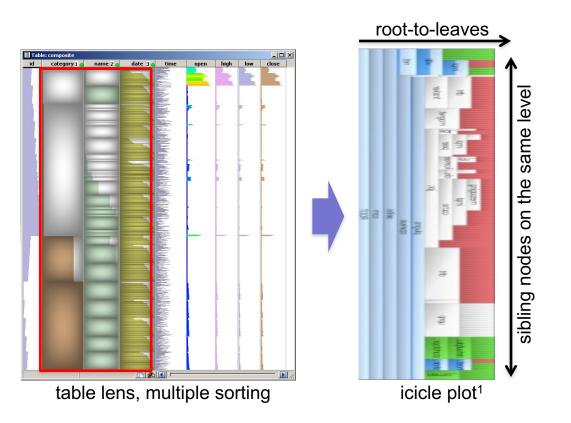




Icicle plots

Basic idea

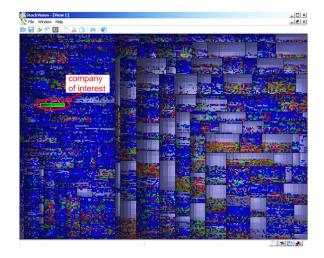
- nodes: rectangles; edges: not drawn explicitly, but shown by node positions
- one level per vertical band (root at left, leaves at right)
- siblings stacked vertically within a band
- compact display, no clutter; node size = sum of subtree importance



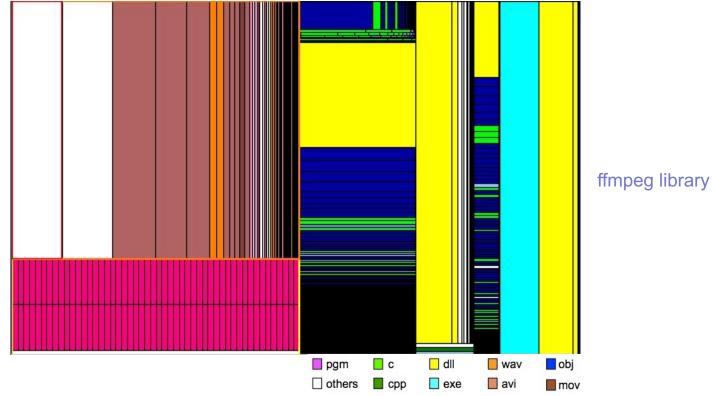
We shall see several variations of the icicle plot further!

¹J. Kruskal, J. Landwehr, Icicle plots: Better displays for hierarchical clustering, JSTOR, 1983

3. Treemaps



Treemaps [Shneiderman '92]



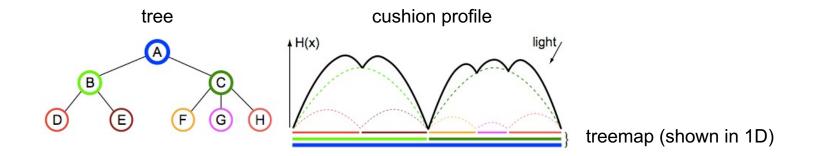
Basic idea: 'slice and dice' layout

- 1 node = 1 rectangle
- child node rectangles: *nested* in the parent node rectangle (recursive subdivision)
- leaf rectangle *size* and *color* show data attributes
- edges: not drawn explicitly!
- very compact: tens of thousands of nodes on one screen! no pixel wasted
- aspect ratios are not very good; hierarchy depth unclear

How can we improve the basic idea?

Two extensions of basic treemaps

- enforce near-square aspect ratios during rectangle subdivision¹
- use shading: cushion profiles to convey hierarchical structure²



For each rectangle r_i (except root)

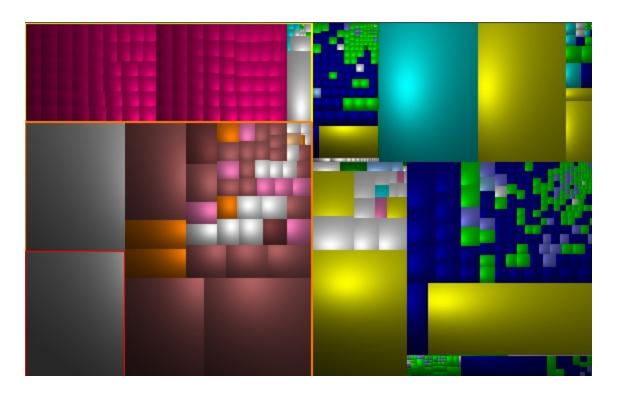
• define 2D parabola $h_i(x,y)$ with height $H_i = f^d$ (d = depth of r_i in tree)

Compute global height $h = \Sigma_i h_i$

For each image pixel I(x,y)

- I(x,y) = shading(h(x,y))
- use an oblique light source for better results

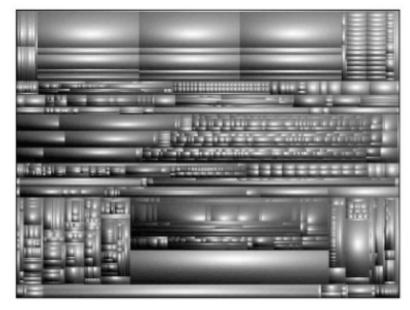
¹ M. Bruls, K. Huizing, J. J. van Wijk (1999) Squarified Treemaps, Proc. VisSym, 322-330 ² J. J. van Wijk, H. van de Wetering (1999) Cushion treemaps: Visualization of hierarchical information, Proc. InfoVis, 135-142



ffmpeg C library

- rectangle borders are not *explicitly* drawn \rightarrow gain space
- borders are *implicit* in the shading discontinuities
- discontinuity strength conveys tree depth levels
- near-square aspect ratio of cells allows easier size (area) comparison

Comparison of methods



hard disk, ~30K files

Slice and dice layout

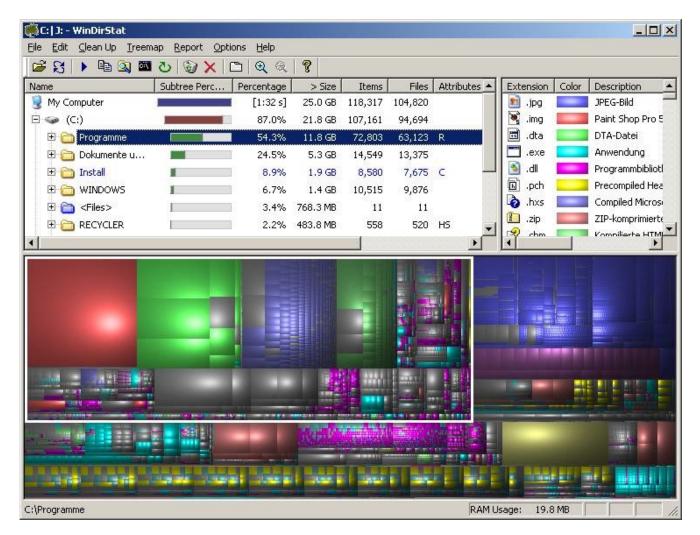
- unbalanced cell sizes (hard to compare)
- tree structure is not very clear

Squarified layout

- balanced cell sizes (easier to compare)
- tree structure is extremely salient

Example 1: WinDirStat tool¹

• visualize a file system (*e.g.* your hard disk)



¹ http://windirstat.info

Squarified Cushion Treemaps

Example 2: Map of the Market¹

Visualize stock exchange data online

- hierarchy: nesting of companies within sectors
- rectangle size: market capitalization
- color: gain (green) ... loss (red)



Squarified Cushion Treemaps

Example 3: Another Map of the Market¹

Visualize stock exchange data online

- hierarchy: nesting of companies within sectors
- rectangle size: market capitalization
- color: gain (green) ... loss (red)

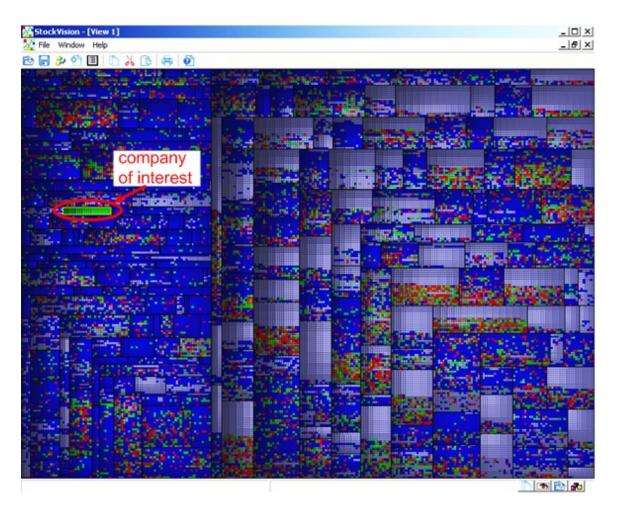


¹ http://www.finviz.com/map.ashx

Squarified Cushion Treemaps

Treemaps from tables

- recall the multiple-sorting idea for tables?
 - implicitly creates a *hierarchy* from a table (with just a few clicks to sort columns)
 - visualize hierarchy with a treemap



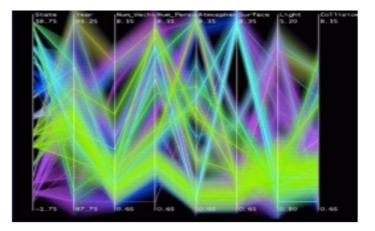
Stock exchange table

- three levels:
 - market sector
- companies
- prices / day over 1 month
- colors
 - red: daily loss
 - green: daily gain
- light blue: unavailable data

Discovered a small emerging company with steady growth!

A. Telea (2006) Combining Extended Table Lens and Treemap Techniques for Visualizing Tabular Data, Proc. EuroVis

4. Parallel coordinates



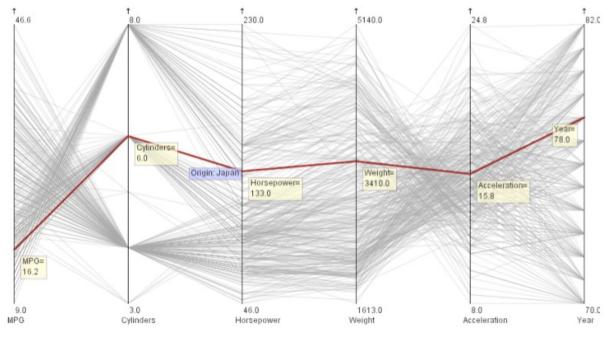
Parallel coordinates

Take again a table

- rows: car brands
- columns: car parameters (MPG, cylinders, horsepower, weight, acceleration, fabrication year)

Parallel coordinates

- table columns: different y axes
- table cells: points on their corresponding axes
- table rows: polylines connecting their points
- column correlations: 'bundles' of close lines



A. Inselberg (2009) Parallel Coordinates: Visual Multidimensional Geometry and its Applications. Springer Implementations: Mondrian (theursus.de/mondrian), Prefuse (prefuse.org), Xdat (xdat.org), Xmdv (http://davis.wpi.edu/xmdv)

Tables vs parallel coordinates

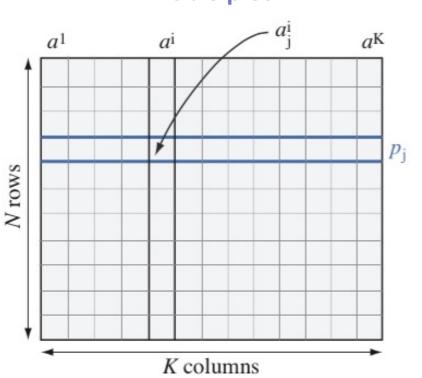
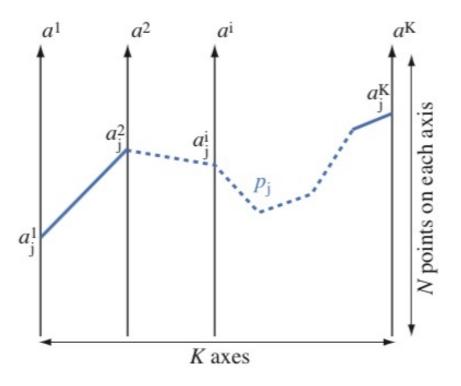


Table plot

Parallel coordinates



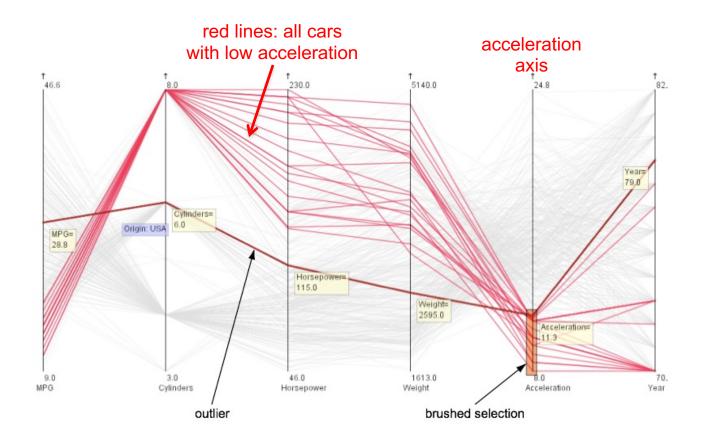
- row: horizontal line
- column: vertical line
- column-column correlation: not easy to see
- no overdraw/clutter

- row: skewed polyline
- column: vertical line (permuted values)
- column-column correlation: easy to see
- overdraw/clutter present

Parallel coordinates

Selection

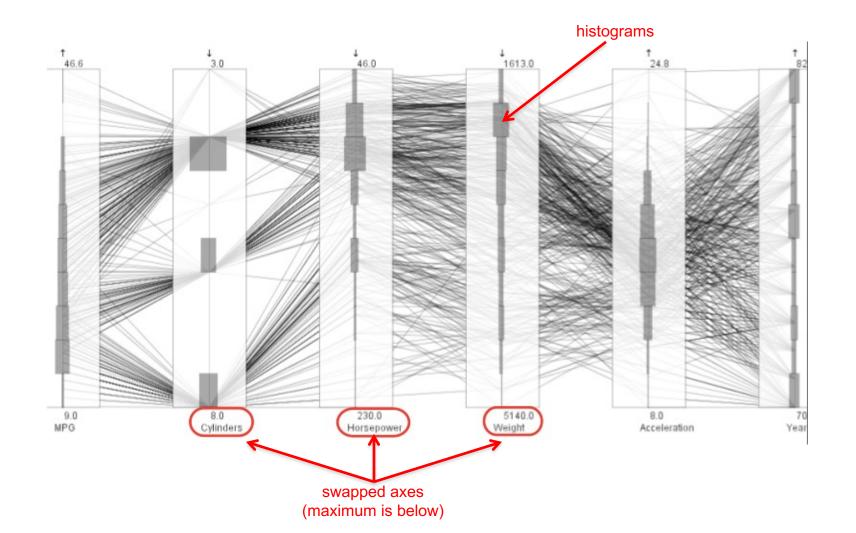
- use mouse to select attribute ranges on axes
- highlight all rows (lines) passing through selection
- supports queries such as
 - show all cars with a low acceleration
 - find what attributes (e.g. MPG, cylinders, weight, ...) low-acceleration cars have



Parallel coordinates

Enhancements

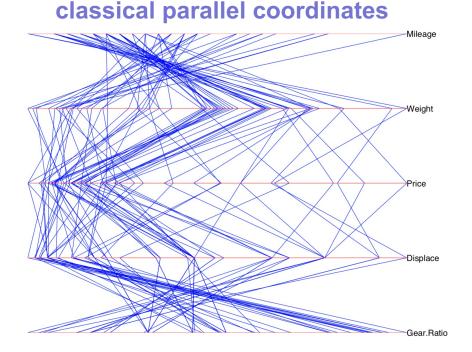
- permute axes (horizontally) and swap their direction (vertically) to minimize line crossings
- add **histograms** on axes to show #rows per unit-data value



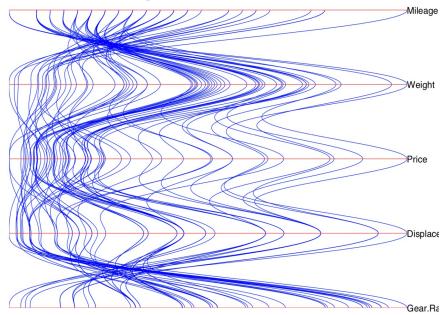
Smooth parallel coordinates

Enhancements

- use curves (splines) instead of polylines
- reduces visual clutter
- makes visually following a sample (curve) easier



smooth parallel coordinates

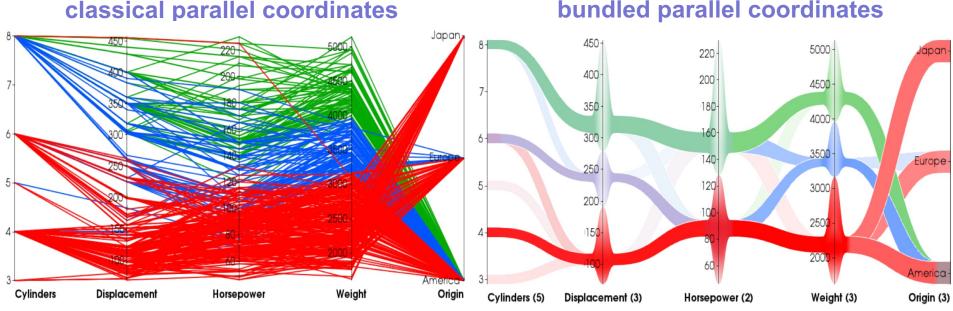


M. Rida et al (2006). Multivariate continuous data – Parallel Coordinates. Graphics of Large Datasets: Visualizing a Million. Springer

Bundled parallel coordinates

Enhancements

- use **curves** (splines) instead of polylines (as in smooth parallel coordinates)
- bundle the curves (as in graph bundling)
- massively simplifies the visualization, reduces clutter
- following groups of similar samples (close curves) is much easier

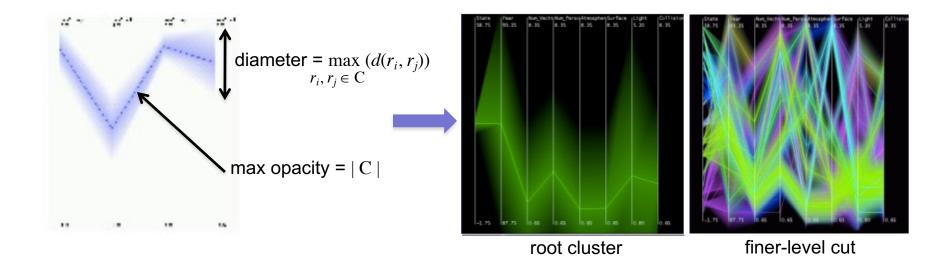


bundled parallel coordinates

G. Palmas et al (2014). An Edge-Bundling Layout for Interactive Parallel Coordinates. Proc. PacificVis

Hierarchical parallel coordinates

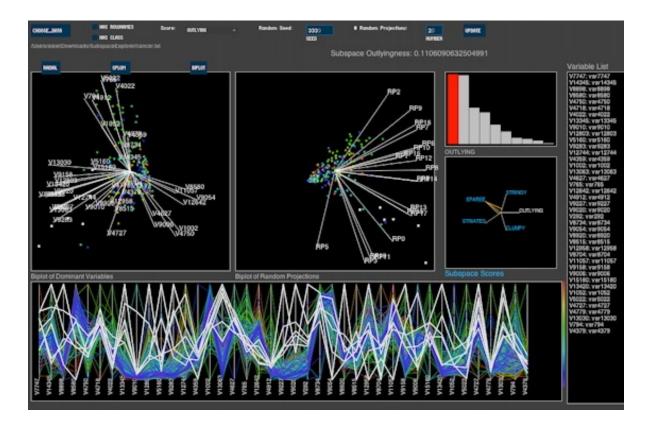
- reduce clutter for very large datasets (10⁶..10⁹ rows)
- hierarchically cluster rows r_i
 - (1) create a cluster $C_i = \{r_i\}$ for each row. Set $S = \{C_i\}$
 - (2) find two most similar clusters C_i , C_j using an Euclidean distance metric $d(r_i, r_j) = \sum_k (r_{ik} r_{jk})^2$
 - (3) build parent cluster $C = (C_i, C_j), S = S \setminus (C_i \cup C_j) \cup C$
 - (4) repeat from step 2 until $S = \{ \text{ root cluster } \}$
- select a 'cut' K in the cluster tree S at desired level-of-detail
- visualize each cluster $C \in K$ with an opacity band which encodes cluster size and diameter



Putting it all together

Low-dimensional data visualization

- easy-to-use tool: SPLOMs, parallel coordinates, and projections (next module)
- Java implementation (runs anywhere), simple text input format



Summary: Low-dimensional data visualization

For what

• datasets with many samples *N* but few (2..10) dimensions *n*

Main design idea

• allocate one visual variable for one..a few dimensions

Techniques

- scatterplots, scatterplot matrices
- table lenses
- table-tree duality
- icicle plots, treemaps
- parallel coordinates

Open challenge: What to do with many dimensions?