Robust Gap Removal from Binary Volumes

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Abstract

Volumetric shapes can be affected by multiple types of defects, including cracks and holes. Removing such defects is delicate, as it can also affect details of the shape, which should be preserved. We present a method for the robust detection and removal of such defects based on the shape's surface and curve skeletons. For this, we first classify gaps, or indentations, present in the input shape by their position with respect to the shape's curve skeleton, into details (which should be preserved) and defects (which should be removed). Next, we remove defects, and preserve details, by using a local reconstruction process that uses the reconstruction power of the shape's surface skeleton. We demonstrate our method by comparing it against classical morphological solutions on a wide collection of real-world shapes.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Line and curve generation

1. Introduction

Digital 3D shapes acquired by surface or volume scanning can be affected by various types of defects. Superficial defects, *e.g.* small-scale noise created by either the scanning or present in the shape itself, are easy to remove while preserving the shape's salient features (*e.g.*, edges) by classical fairing and filtering [Tau95, TWBO02]. Profound defects, *e.g.* gaps and cracks that deeply penetrate from the shape's surface to its interior, are mainly caused by serious faults in the original scanned shape, such as in the case of archaeological artifacts being scanned prior to restoration [BSK05]. Removing such defects while preserving surface detail is considerably more delicate.

Defect detection and removal is well known in 2D shape and image restoration, and done *e.g.* by inpainting methods [SC05, BCHS06]. For 3D volumes, far fewer such methods exit. Arguably the best-known such methods use morphological filters, *e.g.* closing, to find and remove (close) gaps whose size is under a user-prescribed threshold [Ser82]. While simple and fast, such methods have to be set up with great care so that they do not remove shape details to be kept (false positives), or, conversely, leave shape defects to be removed (false negatives). More advanced 3D shape restoration methods exist, *e.g.* [BVG11, BSK05, KSY09, ZGL07]. However, most such methods treat the detection-and-removal of *surface* defects, rather than *volumetric* defects. Other notable limitations include removing both defects and important details, or requiring non-trivial effort from the end user in the form of manual defect delineation or parameter setting.

We propose a method to detect and remove gap-like defects from binary volumetric shapes, with the next main advantages: (1) detection-and-removal of complex (deep, large, noisy) gaps and cracks with full surface detail preservation; (2) full automatic working; and (3) simple implementation and linear complexity *vs* shape size. For this, we use a novel combination of the input shape's curve skeleton (to detect defects) and surface skeleton (to remove gaps), described in Sec. 2. The method demonstrably obtains better results than classical morphological gap-removal on a variety of real-world complex shapes, as shown in Sec. 3. Section 4 concludes our paper.

2. Proposed method

To start with, we outline the context and requirements of our method:

1. Input shapes are represented as densely and uniformly sampled 3D binary voxel volumes $\Omega \subset \mathbb{Z}^3$;
2. Defects to be found and removed are thin-and-elongated structures that penetrate deeply into the shape Ω from its boundary $\partial\Omega$. Given the 3D nature of our shapes, such defects can be 2D structures, such as surface-like cuts and cracks. No restriction is placed on the topology or boundary geometry of these defects;
3. Small-scale shape features, such as detail on $\partial\Omega$, should be altered as little as possible by the restoration process.

Our method adapts to 3D the 2D gap-filling method presented in [SJB*14], which is briefly outlined next: Given a binary pixel shape $\Omega \subset \mathbb{Z}^2$, one first computes the medial axis, or Euclidean skeleton, $S_{\Omega_{oc}}$ of the shape Ω_{oc} obtained by morphologically opening, next closing, Ω. $DT_{\partial\Omega} : \mathbb{R}^2 \to \mathbb{R}^+$ is the Euclidean distance transform [MRH00] of Ω Using a disk structuring element of radius ρ fills all gaps in Ω whose local thickness is smaller than ρ. Next, the set of skeletal fragments $F = S_{\Omega_{oc}} \setminus \Omega$, *i.e.*, points of the skeleton of the defect-free shape Ω_{oc} which are outside of the original shape Ω , is computed. Such fragments correspond to gaps that cut *deeply* in the input shape Ω. Finally, gaps in Ω are filled by convolving the skeleton-fragment-set F with 2D disks whose radii equal the distance transform $DT_{\Omega_{co}}$ of the shape Ω_{co} obtained by first closing, then opening, Ω.

Technically, we can immediately generalize the above method to 3D shapes $\Omega \in \mathbb{Z}^3$ by using their 3D surface skeletons and distance transforms. Figure 1 (bottom path) shows the effects of this idea for a frog model cut in the middle by a single simple thick planar cut. Here, voxels $\mathbf{x} \in \Omega$ in the input shape are red, and voxels added by gap-filling are green, respectively. As visible, using the surface skeleton fills the cut present in the model quite well. Yet, many shallow gaps present on the model's surface are also filled – see *e.g.* green details between the frog's fingers and in the creases behind the hind-leg ankles. Such problems are not surprising: Indeed, the shape Ω_{oc} whose surface-skeleton $S_{\Omega_{oc}}$ we use, is an 'inflated' version of Ω_{oc}. This inflation is needed to close gaps in Ω before the skeleton computation. However, this also closes *detail* gaps, like the ones mentioned above. Hence, such gaps can generate surface-skeleton fragments which are outside Ω. Restoring the shape from such fragments produces the undesired fill-ins of detail gaps.

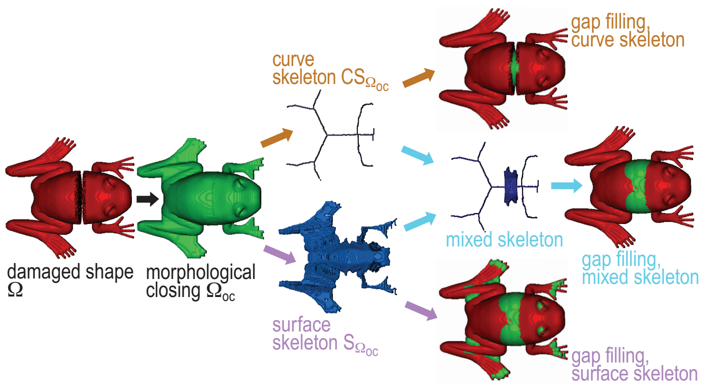

Figure 1: *3D gap restoration by surface and curve skeletons.*

The main cause of the above problem is the higher sensitivity of 3D surface skeletons to inflation-induced changes on a shape, as compared to the 2D medial axes used in [SJB*14]. To alleviate this, one could be tempted to simplify the surface skeleton $S_{\Omega_{oc}}$ prior to its use in computing the gap-set F. However, this yields an undesired locally non-smooth filling of gaps. Indeed: a gap in a 3D object is, in most cases, far from circular. So, to fill the gap in a plausible way, one needs to use the (almost) full surface skeleton, and not the simplified one. Note that this issue does not occur in 2D: The skeletal fragments present in 2D gaps are simple 1D curve segments (internal skeleton branches) which perfectly capture the simpler configuration of a 2D gap, and which are not affected by skeleton simplification. In 3D, as noted above, a gap contains a *mix* of both surface-skeleton terminal manifolds (which capture shape details) and internal surface-skeleton manifolds (which capture the coarse shape topology) [SP09].

An intuitively quick-fix to the above problem is to use the so-called curve skeleton $CS_{\Omega_{oc}}$ in the 3D detection-and-reconstruction process, instead of the surface skeleton $S_{\Omega_{oc}}$. Curve skeletons are 1D structures locally centered within their corresponding surface skeletons [CSM07]. The main advantage is that a curve skeleton has the same simple 1D structure as the classical 2D medial axis, so robustly detecting its fragments which are outside the shape, *i.e.* $CS \setminus \Omega$, is easy. However, as well known, a curve skeleton does not capture the local shape *geometry* well [CSM07, SP09]: Using such a skeleton to reconstruct Ω will fill the detected gaps by locally *tubular* structures whose thickness equals the local shape thickness (see Fig. 1, top orange path). As visible, we now avoid filling small-scale surface gaps, but we cannot fully fill the large central cut (which has a non-circular cross-section).

Summarizing the above, we conclude that, for a 3D shape

- curve skeletons are good to detect, but not to remove, gaps;
- surface skeletons are good to remove, but not to detect, gaps.

Hence, we combine the two skeleton types to solve our problem, *i.e.*:

1. Compute the curve skeleton $CS_{\Omega_{oc}}$ and use it to create the gap-set F^{CS} of all curve-skeleton voxels outside the input shape Ω;
2. Compute the set F^S of surface-skeleton voxels outside Ω;
3. From F^S, we remove all voxels not connected (within F^S) to at least a voxel in F^{CS}, by running a simple flood-fill from F^{CS} onto F^S. This removes from F^S all spurious fragments which led to filling small-surface details, yielding the final fragment-set F^{final};
4. Restore the shape Ω by convolving voxels $\mathbf{x} \in F^{final}$ by balls of radii $DT_{\partial\Omega_{co}}(\mathbf{x})$, where $DT_{\partial\Omega_{co}}$ is the distance transform of the input shape Ω after morphological closing followed by opening.

Figure 1 (middle cyan path) shows the result of our mixed curve-and-surface skeleton method on our test shape: The large central cut is restored just as nicely as when using the surface-skeleton only (Fig. 1, bottom purple path), *and* no spurious detail is filled in, just as when using the curve-skeleton only (Fig. 1, top orange path).

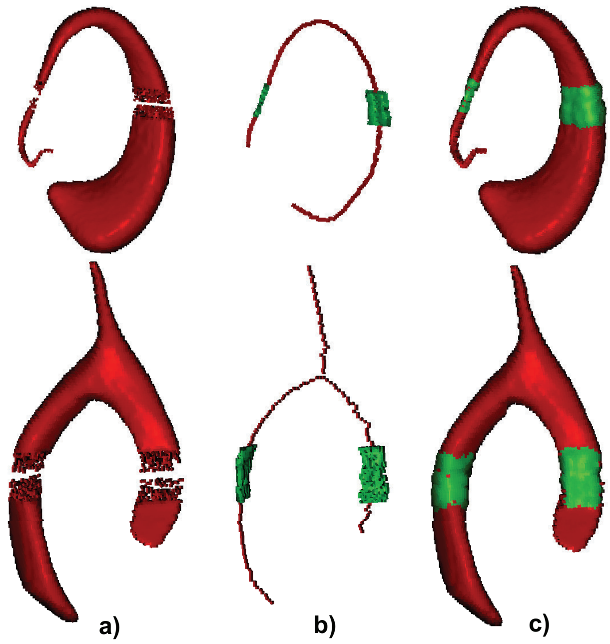

Figure 2: *Reconstruction of tubular shapes. (a) Damaged shape; (b) Curve skeleton (red) with selected portions of surface skeleton (green); (c) Reconstruction result.*

3. Results

We implemented our restoration method as follows: For morphological opening and closing, we use three well-known 3D structuring elements: cubic, ball, and cross (see Fig. 3 top). Reconstruction done by using only surface-skeletons uses the integer medial axis method (IMA) [HR08]. IMA is fast, simple to implement, and delivers high-quality (unsimplified) surface skeletons and 3D distance transforms [SJT14]. Hence, IMA is ideal if we only need surface skeletons. Reconstruction by combined curve-and-surface skeletons is done by the method in [JST16]. We chose this method for its high speed, centeredness of the delivered skeletons, ability to compute both skeleton types, and its guarantee that curve skeletons are contained in their corresponding surface skeletons. This containment is *crucial* for the success of our reconstruction: Indeed, in step 3 of our method (see Sec. 2), we perform a flood-fill from the curve skeleton onto the surface skeleton. For this to work, the curve skeleton must be embedded in the surface skeleton.

To assess both the qualitative results and scalability of our method, we ran it on about 30 shapes, voxelized from polygonal models at resolutions up to 512^3 voxels, using *binvox* [NT03]. Our method was implemented in C++ on the CPU and ran on a desktop PC Core i7 computer at 3.40 GHz with 16 GB RAM.

Figure 2 shows the reconstruction of a simple tubular shape. As visible, our combined curve-and-skeleton reconstruction method successfully detects and fills even the large jagged-edge gap shown in the figure. Figure 3 shows additional examples for shapes of a more complex geometry and topology. Here, gaps were created in the input shapes procedurally, by cutting them at various places with implicit functions modeling planes of various orientation and spheres of various

Figure 3: *Comparison of our gap-filling method (right column) with four other methods.*

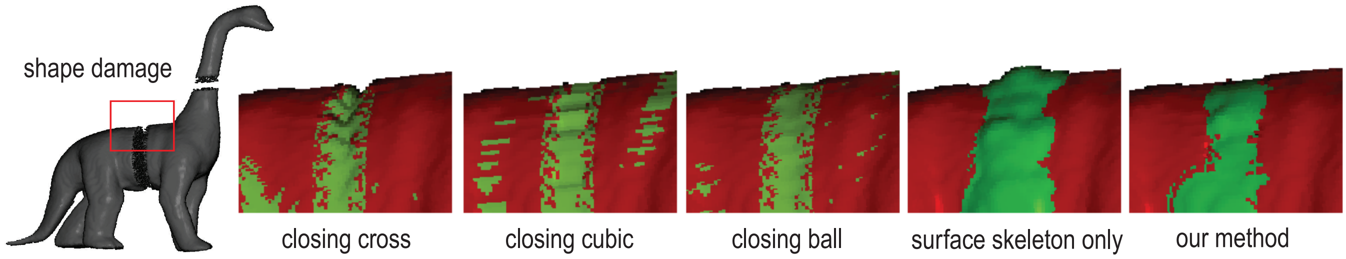

Figure 4: *Reconstruction smoothness. Our method (right) yields the locally smoothest results as compared to four other variants.*

radii, respectively. Next, noise was added on the internal cut-surfaces by randomly removing a small set of voxels from these surfaces. This creates more jagged cuts, which are arguably more challenging to restore. We compared our method (based on curve-and-surface skeletons) to four other alternatives: simple top-hat morphological gap-closing by using three structuring element types (ball, axis-aligned cube, and axis-aligned cross); and the surface-skeleton 3D extension of [SJB*14]. In Fig. 3, red shows the original points in Ω which are also present in the reconstruction, and green shows points added to Ω by the reconstruction. As visible, our combined curve-and-surface restoration method achieves the best results in terms of smoothly filling deep gaps *and* preserving surface detail.

We next discuss several relevant aspects of our restoration method:

Smoothness: As noted in virtually all inpainting-related works, restoration-like operations should produce a 'plausible' reconstruction of damaged areas [BCHS06]. While a formal definition of plausibility is hard to give, the vast majority of 2D inpainting literature mentions *smoothness* of the reconstructed signal over, and along the boundaries of, the reconstructed area to be a key desirable. Hence, we adopt this desirable for our 3D context too. As visible in Figs. 3 and 4, the simpler morphological-closing produce noisier results, while our skeleton-based reconstruction produces smoother results. Within the last category, we also see that the combined curve-and-surface reconstruction produces the smoothest results, in the sense of a reconstructed surface (green) which closely follows the curvature of the surrounding original surface (red).

Locality: As also outlined in [SJB*14] (for the equivalent 2D case), a gap-filling reconstruction should detect and remove only deep gaps that significantly cut the shape, but leave shallow gaps (detail indentations) of the input surface untouched. As for the test example in Fig. 1, Fig. 3 shows that the morphological closing methods cannot, in general, make this difference – they indiscriminately fill all gaps whose size is smaller than the structuring-element size. This is best visible by considering the amount of green in the reconstructed images which does not correspond to cut-locations visible in the shapes in the leftmost column, *e.g.*, filling the small gaps between the details of the dragon surface (top row), spikes of the trident (neptune model, fourth row from top), or frog's fingers (third row from bottom). The combined curve-and-surface method suffers far less from such issues.

Simplicity and scalability: Our method uses trivial morphological opening and closing; computing 3D curve and surface skeletons; a flood fill operation on voxel volumes; and reconstructing a 3D voxel shape from a selected set of skeleton points by the shape's medial axis transform (MAT). All these operations can be computed in linear time in the voxel size $\|\Omega\|$ of the input shape [JST16]. This makes our method work in seconds on volumes up to 512^3 voxels on the commodity PC configuration listed earlier in this section.

4. Conclusions

In this paper, we have presented a new method for automatic detection and restoration of gaps and cracks present in 3D volumetric shapes. For this, we extend the 2D gap detection and removal method in [SJB*14] to 3D, by using a novel combination of curve skeletons (for gap detection) and surface skeletons (for gap repairing). Such skeletons can be readily and efficiently provided by the 3D skeletonization method in [JST16]. Our proposed 3D restoration method was shown to produce better results, in terms of detail preservation and gap removal, as compared to classical morphological closing methods, while having an identical cost – linear in terms of the voxel count of the input shape.

Acknowledgments

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