Elliptic integrals and elliptic functions

Steven Wepster

Departement Wiskunde
Universiteit Utrecht

28 april 2019

About assignments

from the website:

Students work in pairs on three assignments.

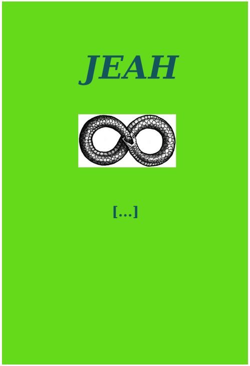
Two assignments each lecture; select three

We encourage you to explore the historical and social aspects of mathematics as well as to understand the mathematics at a more intuitive level than you have become accustomed to in regular mathematics courses.

Results are to be handed in in the form of one article-style paper

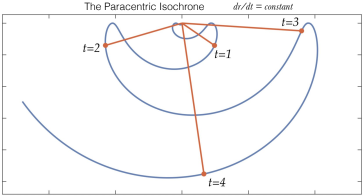
Clarification

Submit your article for publication in:
the **Journal of Elliptic Adventures in History (JEAH)**

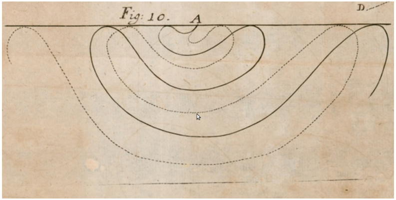


- edited by dr. V. Blåsjø and dr. S. Wepster.
- The journal is aimed at advanced undergraduates.
- The editors accept nontrivial, original papers that present interesting viewpoints and are well written.
- You want to have your paper accepted for publication but of course you are time constrained.

Leibniz: paracentric isochrone

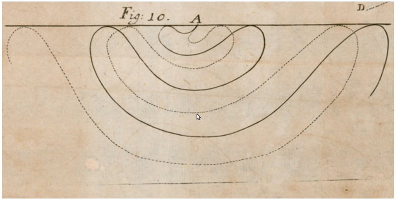


reduces to integrating $1/\sqrt{1-x^4}$



Find the curve under which a point moves under gravity (no friction) in such a way that the distance from the initial point increases uniformly with time.

differential eqn or integral



differential eqn:

$$\frac{dr}{\sqrt{ar}} = \frac{a\,dz}{\sqrt{ax(a^2 - xz^2)}}$$

this is the 18th century way of saying:

$$\int_0^r \frac{dt}{\sqrt{at}} = \int_0^z \frac{a\,dt}{\sqrt{t(a^2 - t^2)}}$$

How study such problems?

Today, we associate an integral like $\int_0^x f(t)\,dt$ with area (quadrature, $\square$)
But length is a simpler notion, geometrically.
So when geometrical construction is of concern then you prefer to express your integral as an arc length of some curve.

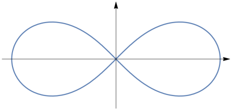
OK then which curve?

Jakob Bernouli takes $x = \sqrt{az + z^2}$ and $y = \sqrt{az - z^2}$.
Then he gets for the arclength element

$$ds = \sqrt{dx^2 + dy^2} = \frac{a\,dz}{\sqrt{z(a^2 - z^2)}}$$

while x and y satisfy:

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$$



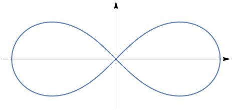
OK then which curve?

Jakob Bernouli takes $x = \sqrt{az + z^2}$ and $y = \sqrt{az - z^2}$. Then he gets for the arclength element

$$ds = \sqrt{dx^2 + dy^2} = \frac{a\,dz}{\sqrt{z(a^2 - z^2)}}$$

while x and y satisfy:

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$$

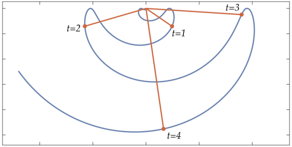


Johann: much nicer with $t^2 = az$ because the arc length turns into

$$ds = 2\frac{dt}{\sqrt{a^4 - t^4}}.$$

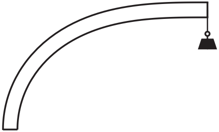
Construction by rectification

To construct



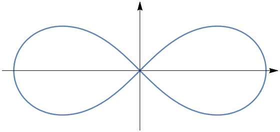
paracentric isoschrone
dr/dt = constant

rectify



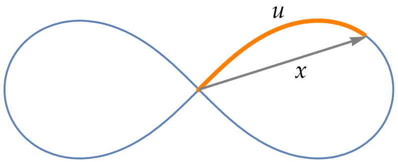
shape of elastic beam

or rectify



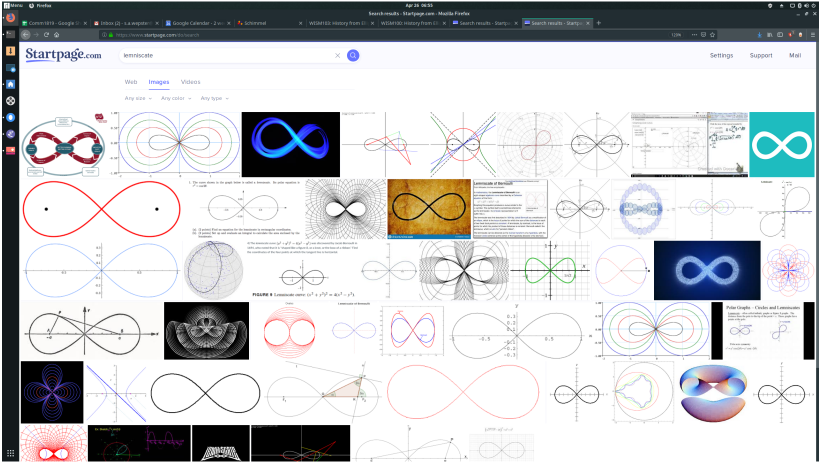
lemniscate

Lemniscate



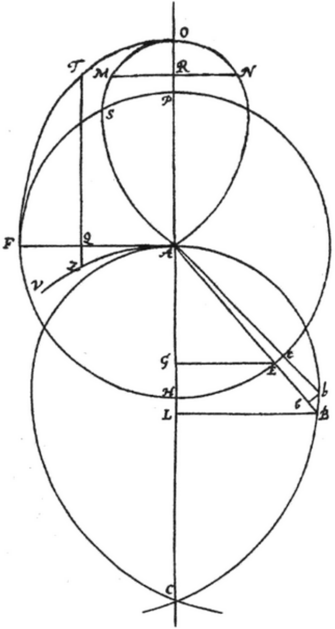
$$u(x) = \int_0^x \frac{dt}{\sqrt{1-t^4}}$$

More lemniscates



Jakob and Johann

Paracentric isochrone by rectification



Elliptic integrals

$\int \frac{dt}{\sqrt{1-t^4}}$ is a very nice example of what we now call an *elliptic integral*.

Elliptic integrals

$\int \frac{dt}{\sqrt{1-t^4}}$ is a very nice example of what we now call an *elliptic integral*.

Terminology: *any* integral of the form

$$\int R\big(x, \sqrt{P(x)}\big)\, dx$$

where $R(x, y)$ is a rational function and $P(x)$ a polynomial of degree 3 or 4.

Elliptic integrals

$\int \frac{dt}{\sqrt{1-t^4}}$ is a very nice example of what we now call an *elliptic integral*.

Terminology: *any* integral of the form

$$\int R(x, \sqrt{P(x)})\, dx$$

where $R(x, y)$ is a rational function and $P(x)$ a polynomial of degree 3 or 4.

In general these cannot be expressed in the elementary functions.

Arguably, they are the most obvious candidates for the label "next-level elementary".

Elliptic integrals

$\int \frac{dt}{\sqrt{1-t^4}}$ is a very nice example of what we now call an *elliptic integral*.

Terminology: *any* integral of the form

$$\int R\big(x, \sqrt{P(x)}\big)\, dx$$

where $R(x, y)$ is a rational function and $P(x)$ a polynomial of degree 3 or 4.

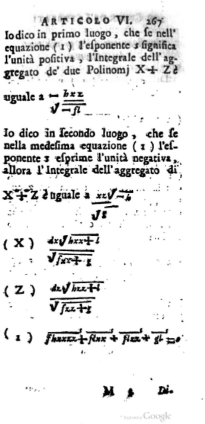
In general these cannot be expressed in the elementary functions.

Arguably, they are the most obvious candidates for the label "next-level elementary".

Other example: ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has arc length

$$\int_0^x \frac{\sqrt{a^4 - (a^2 - b^2)x^2}}{a\sqrt{a^2 - x^2}}\, dx$$

Fagnano


ARTICOLO VI. 267

Io dico in primo luogo, che se nell' equazione (1) l'esponente s significa l'unità positiva, l'Integrale dell' aggregato de' due Polinomj $X + Z$ è uguale a $-\frac{bxz}{\sqrt{-fl}}$

Io dico in secondo luogo, che se nella medesima equazione (1) l'esponente s esprime l'unità negativa, allora l'Integrale dell'aggregato di $X + Z$ è uguale a $\frac{xz\sqrt{-l}}{\sqrt{g}}$

(X) $\frac{dx\sqrt{hxx+l}}{\sqrt{fxx+g}}$

(Z) $\frac{dz\sqrt{hzz+l}}{\sqrt{fzz+g}}$

(1) $fhxxzz + flxx + flzz + gl = 0$

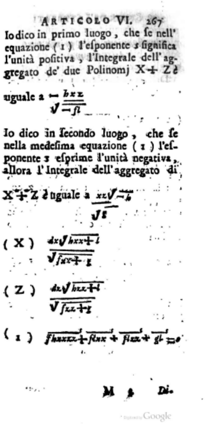
M Di-

Doubling the arc of the lemniscate:

$$2\int_0^x \frac{dt}{\sqrt{1-t^4}} = \int_0^y \frac{dt}{\sqrt{1-t^4}}$$

when $y = \dfrac{2x\sqrt{1-x^4}}{1+x^4}$.

Fagnano


ARTICOLO VI. 267

Io dico in primo luogo, che se nell' equazione (1) l'esponente s significa l'unità positiva, l'Integrale dell' aggregato de' due Polinomj X+Z è uguale a

Io dico in secondo luogo, che se nella medesima equazione (1) l'esponente s esprime l'unità negativa, allora l'Integrale dell' aggregato di X+Z è uguale a

(X)

(Z)

(1)

M Di.

Doubling the arc of the lemniscate:

$$2\int_0^x \frac{dt}{\sqrt{1-t^4}} = \int_0^y \frac{dt}{\sqrt{1-t^4}}$$

when $y = \dfrac{2x\sqrt{1-x^4}}{1+x^4}$.

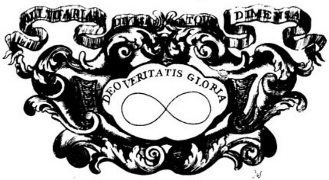
In other words:
if $u(x)$ is the arclength of radius x,
then $2u(x) = u(y)$.

Fagnano's works of 1750

PRODUZIONI
MATEMATICHE
DEL CONTE GIULIO CARLO
DI FAGNANO,
MARCHESE DE' TOSCHI,
E DI SANT' ONORIO
NOBILE ROMANO, E PATRIZIO SENOGAGLIESE
ALLA SANTITA' DI N. S.
BENEDETTO XIV.
PONTEFICE MASSIMO.
TOMO PRIMO.

IN PESARO

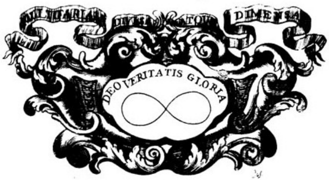
L' ANNO DEL GIUBBILEO M. DCC. L.
NELLA STAMPERIA GAVELLIANA
CON LICENZA DE' SUPERIORI.



Published in 1718, went unnoticed.

Fagnano's works of 1750


PRODUZIONI
MATEMATICHE
DEL CONTE GIULIO CARLO
DI FAGNANO,
MARCHESE DE' TOSCHI,
E DI SANT' ONORIO
NOBILE ROMANO, E PATRIZIO SENOGAGLIESE
ALLA SANTITA' DI N. S.
BENEDETTO XIV.
PONTEFICE MASSIMO.
TOMO PRIMO.
IN PESARO
L' ANNO DEL GIUBBILEO M. DCC. L.
NELLA STAMPERIA GAVELLIANA
CON LICENZA DE' SUPERIORI.



Published in 1718, went unnoticed.
Published again in 1750 and given to Euler: beginning of a new era.

Euler

Generalises the results of Fagnano, he finds

$$\int_0^{x_1} \frac{dt}{\sqrt{1-t^4}} + \int_0^{x_2} \frac{dt}{\sqrt{1-t^4}} = \int_0^{y} \frac{dt}{\sqrt{1-t^4}}$$

when $y = \dfrac{x_1\sqrt{1-x_2^4} + x_2\sqrt{1-x_1^4}}{1+x_1^2x_2^2}$

etc etc... Euler produces over 400 pages of this kind of stuff.

Euler

Generalises the results of Fagnano, he finds

$$\int_0^{x_1} \frac{dt}{\sqrt{1-t^4}} + \int_0^{x_2} \frac{dt}{\sqrt{1-t^4}} = \int_0^{y} \frac{dt}{\sqrt{1-t^4}}$$

when $y = \dfrac{x_1\sqrt{1-x_2^4} + x_2\sqrt{1-x_1^4}}{1+x_1^2x_2^2}$

etc etc... Euler produces over 400 pages of this kind of stuff.
Why?

Looks weird, but...

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.

Looks weird, but. . .

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.
Remembering that $u(x) = \arcsin x$,
we see that $u = u(x)$ has a very nice inverse $x(u) = \sin u$.

Looks weird, but...

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.
Remembering that $u(x) = \arcsin x$,
we see that $u = u(x)$ has a very nice inverse $x(u) = \sin u$.

Now, let's call $u_1 + u_2 = u_3$. Addition formula:

$$\sin(u_3) \sin(u_1 + u_2) = \sin u_1 \cos u_2 + \sin u_2 \cos u_1$$

Looks weird, but...

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.
Remembering that $u(x) = \arcsin x$,
we see that $u = u(x)$ has a very nice inverse $x(u) = \sin u$.

Now, let's call $u_1 + u_2 = u_3$. Addition formula:

$$\sin(u_3)\sin(u_1 + u_2) = \sin u_1 \cos u_2 + \sin u_2 \cos u_1$$

or with $x_i = \sin u_i$:

$$x_3 = x_1\sqrt{1 - x_2^2} + x_2\sqrt{1 - x_1^2}$$

Looks weird, but. . .

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.
Remembering that $u(x) = \arcsin x$,
we see that $u = u(x)$ has a very nice inverse $x(u) = \sin u$.

Now, let's call $u_1 + u_2 = u_3$. Addition formula:

$$\sin(u_3)\sin(u_1 + u_2) = \sin u_1 \cos u_2 + \sin u_2 \cos u_1$$

or with $x_i = \sin u_i$:

$$x_3 = x_1\sqrt{1 - x_2^2} + x_2\sqrt{1 - x_1^2}$$

Hence the sine addition formula induces an integal addition formula

$$\int_0^{x_3} \frac{dt}{\sqrt{1-t^2}} = \int_0^{x_1} \frac{dt}{\sqrt{1-t^2}} + \int_0^{x_2} \frac{dt}{\sqrt{1-t^2}}$$

Looks weird, but. . .

Let's take this simpler case: define $u(x) := \int_0^x \frac{dt}{\sqrt{1-t^2}}$.
Remembering that $u(x) = \arcsin x$,
we see that $u = u(x)$ has a very nice inverse $x(u) = \sin u$.

Now, let's call $u_1 + u_2 = u_3$. Addition formula:

$$\sin(u_3) \sin(u_1 + u_2) = \sin u_1 \cos u_2 + \sin u_2 \cos u_1$$

or with $x_i = \sin u_i$:

$$x_3 = x_1\sqrt{1 - x_2^2} + x_2\sqrt{1 - x_1^2}$$

Hence the sine addition formula induces an integal addition formula

$$\int_0^{x_3} \frac{dt}{\sqrt{1-t^2}} = \int_0^{x_1} \frac{dt}{\sqrt{1-t^2}} + \int_0^{x_2} \frac{dt}{\sqrt{1-t^2}}$$

Elliptic additon formulas are just the next level of this.

Euler on Elliptic integrals

Main Page

Search archive by:

Subject

Date

Publication

Index Number

Historical Information:

18th Century Europe

The Life of Euler

Contemporaries

Important Locations

Archive Features:

Translations

Correspondence

How Euler Did It

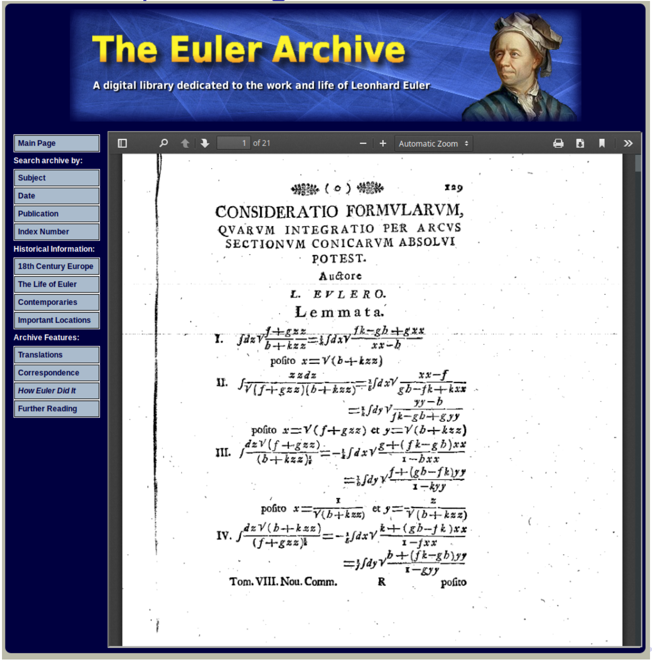
Further Reading

Elliptic Integrals

Original Titles | English Titles

28	Specimen de constructione aequationum differentialium sine indeterminatarum separatione
52	Solutio problematum rectificationem ellipsis requirentium
154	Animadversiones in rectificationem ellipsis
211	Problema, ad cuius solutionem geometrae invitantur; theorema, ad cuius demonstrationem geometrae invitantur
251	De integratione aequationis differentialis $(m\,dx)/\sqrt{(1-x^4)} = (n\,dy)/\sqrt{(1-y^4)}$
252	Observationes de comparatione arcuum curvarum irrectificibilium
261	Specimen alterum methodi novae quantitates transcendentes inter se comparandi; de comparatione arcuum ellipsis
263	Specimen novae methodi curvarum quadraturas et rectificationes aliasque quantitates transcendentes inter se comparandi
264	Demonstratio theorematis et solutio problematis in actis erud. Lipsiensibus propositorum
273	Consideratio formularum, quarum integratio per arcus sectionum conicarum absolvi potest
295	De reductione formularum integralium ad rectificationem ellipsis ac hyperbolae
345	Integratio aequationis $dx/\sqrt{(A+Bx+Cx^2+Dx^3+Ex^4)} = dy/\sqrt{(A+By+Cy^2+Dy^3+Ey^4)}$
347	Evolutio generalior formularum comparationi curvarum inservientium
448	Nova series infinita maxime convergens perimetrum ellipsis exprimens
506	Dilucidationes super methodo elegantissima, qua illustris de la Grange usus est in integranda aequatione differentiali $dx/\sqrt{X} = dy/\sqrt{Y}$
581	Plenior explicatio circa comparationem quantitatum in formula integrali $\int (Z\,dz)/\sqrt{(1+mzz+nz^4)}$ contentarum denotante Z functionem quamcunque rationalem ipsius zz
582	Uberior evolutio comparationis, quam inter arcus sectionum conicarum instituere licet
590	Theoremata quaedam analytica, quorum demonstratio adhuc desideratur
605	De miris proprietatibus curvae elasticae sub aequatione $y = \int (xx\,dx)/\sqrt{(1-x^4)}$
624	De superficie coni scaleni, ubi imprimis intentes difficultates, quae in hac investigatione occurrunt, perpenduntur
633	De binis curvis algebraicis inveniendis, quarum arcus indefinite inter se sint aequales
638	De innumeris curvis algebraicis, quarum longitudinem per arcus parabolicos metiri licet

Euler on Elliptic integrals



(o) 129

CONSIDERATIO FORMVLARVM, QVARVM INTEGRATIO PER ARCVS SECTIONVM CONICARVM ABSOLVI POTEST.

Auctore

L. EVLERO.

Lemmata.

I. $\int dz \sqrt{\frac{f+gzz}{b+kzz}} = \frac{1}{k}\int dx \sqrt{\frac{fk-gb+gxx}{xx-b}}$

posito $x = \sqrt{(b+kzz)}$

II. $\int \frac{zzdz}{\sqrt{(f+gzz)(b+kzz)}} = \frac{1}{g}\int dx \sqrt{\frac{xx-f}{gb-fk+kxx}}$

$= \frac{1}{k}\int dy \sqrt{\frac{yy-b}{fk-gb+gyy}}$

posito $x = \sqrt{(f+gzz)}$ et $y = \sqrt{(b+kzz)}$

III. $\int \frac{dz\sqrt{(f+gzz)}}{(b+kzz)^{\frac{3}{2}}} = -\frac{1}{k}\int dx \sqrt{\frac{g+(fk-gb)xx}{1-bxx}}$

$= \frac{1}{b}\int dy \sqrt{\frac{f+(gb-fk)yy}{1-kyy}}$

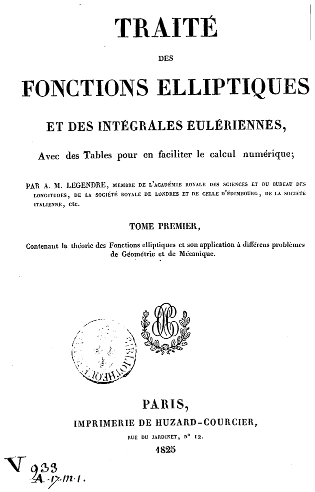
posito $x = \frac{1}{\sqrt{(b+kzz)}}$ et $y = \frac{z}{\sqrt{(b+kzz)}}$

IV. $\int \frac{dz\sqrt{(b+kzz)}}{(f+gzz)^{\frac{3}{2}}} = -\frac{1}{g}\int dx \sqrt{\frac{k+(gb-fk)xx}{1-fxx}}$

$= \frac{1}{f}\int dy \sqrt{\frac{b+(fk-gb)yy}{1-gyy}}$

Tom. VIII. Nou. Comm. R posito

Legendre



TRAITÉ
DES
FONCTIONS ELLIPTIQUES
ET DES INTÉGRALES EULÉRIENNES,
Avec des Tables pour en faciliter le calcul numérique;

PAR A. M. LEGENDRE, MEMBRE DE L'ACADÉMIE ROYALE DES SCIENCES ET DU BUREAU DES LONGITUDES, DE LA SOCIÉTÉ ROYALE DE LONDRES ET DE CELLE D'ÉDIMBOURG, DE LA SOCIÉTÉ ITALIENNE, etc.

TOME PREMIER,
Contenant la théorie des Fonctions elliptiques et son application à différens problèmes de Géométrie et de Mécanique.

PARIS,
IMPRIMERIE DE HUZARD-COURCIER,
RUE DU JARDINET, N° 12.
1825

Comprehensive theory of
elliptic integrals,
reduced to three basic types,
600 pages
obsolete within few years. . .

Looking at the wrong functions

Instead of $u = \int^x \frac{dt}{\sqrt{1-t^2}}$, we should be looking at $x = \sin u$;
Instead of $u = \int^x \frac{dt}{\sqrt{1-t^4}}$, we should be looking at its inverse:
that is what Abel called an *elliptic function*

- Gauss: sinus lemniscatus $x = sl(u)$
- Abel: $x = \phi(u)$
- Jacobi: sn, cn, dn

Gauss: an interesting string of connections

- division of an arc into n equal pieces

PAUCA SED
MATURA

Gauss: an interesting string of connections

PAUCA SED MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon

Gauss: an interesting string of connections

PAUCA SED MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon
- next level: division of lemniscate

Gauss: an interesting string of connections

PAUCA SED MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon
- next level: division of lemniscate
- seemingly unrelated: agM = **arithmetic-geometric mean**: start with numbers a_0, b_0 put $a_1 = \frac{a_0+b_0}{2}$, $b_1 = \sqrt{a_0 b_0}$, repeat for a_1, b_1 etc. Converges!

Gauss: an interesting string of connections

PAUCA SED MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon
- next level: division of lemniscate
- seemingly unrelated: agM = **arithmetic-geometric mean**:
  start with numbers a_0, b_0
  put $a_1 = \frac{a_0+b_0}{2}$, $b_1 = \sqrt{a_0 b_0}$,
  repeat for a_1, b_1 etc. Converges!
- The $agM(1, \sqrt{2})$ equals $\dfrac{\pi}{\omega}$ where
  ω =length of lemniscate

Gauss: an interesting string of connections

PAUCA SED MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon
- next level: division of lemniscate
- seemingly unrelated: agM = **arithmetic-geometric mean**: start with numbers a_0, b_0 put $a_1 = \frac{a_0+b_0}{2}$, $b_1 = \sqrt{a_0 b_0}$, repeat for a_1, b_1 etc. Converges!
- The $agM(1, \sqrt{2})$ equals $\frac{\pi}{\omega}$ where ω =length of lemniscate
- ?!?! ... this cannot be a coincidence, thinks Gauss

Gauss: an interesting string of connections

PAUCA SED

MATURA

- division of an arc into n equal pieces
- discovers constructability of 17-gon
- next level: division of lemniscate
- seemingly unrelated: agM = **arithmetic-geometric mean**:
  start with numbers a_0, b_0
  put $a_1 = \frac{a_0+b_0}{2}$, $b_1 = \sqrt{a_0 b_0}$,
  repeat for a_1, b_1 etc. Converges!
- The $agM(1, \sqrt{2})$ equals $\dfrac{\pi}{\omega}$ where ω =length of lemniscate
- ?!?! ...this cannot be a coincidence, thinks Gauss
- elliptic functions are double periodic

Double periodicity

The arc length of the leminscate equals 2ω, hence

$$sl(u + 2\omega) = sl(u),$$

i.e., 2ω is a period of sl.

Double periodicity

The arc length of the leminscate equals 2ω, hence

$$sl(u + 2\omega) = sl(u),$$

i.e., 2ω is a period of sl.
Now if

$$u = \int^{x} \frac{dt}{\sqrt{1 - t^4}}$$

then

$$iu = \int^{x} \frac{d(it)}{\sqrt{1 - (it)^4}} = \int^{ix} \frac{dt}{\sqrt{1 - t^4}}$$

hence $sl(iu) = i\, sl(u)$, i.e., $2i\omega$ is also a period of sl.

Double periodicity

The arc length of the leminscate equals 2ω, hence

$$sl(u + 2\omega) = sl(u),$$

i.e., 2ω is a period of sl.
Now if

$$u = \int^x \frac{dt}{\sqrt{1 - t^4}}$$

then

$$iu = \int^x \frac{d(it)}{\sqrt{1 - (it)^4}} = \int^{ix} \frac{dt}{\sqrt{1 - t^4}}$$

hence $sl(iu) = i\, sl(u)$, i.e., $2i\omega$ is also a period of sl.
Double periodicity is a genuine feature of elliptic functions, first recognised by Gauss in his sinus lemniscatus.

Abel

next week. . .

Ecole Polytechnique

- Founded by Monge and Carnot, 1794
- produced enigeers for the French army
- model for similar schools across Europe (TH Delft)
- Many (most?) famous French mathematicians worked here
- Before this time, Universities were **not** the place to be for advanced research!

Reform of German universities after 1815

- Inspired by Wilhelm von Humboldt
- Education based on neo-humanistic ideals: academic freedom, pure research, Bildung, etc. Anti-French.
- professors combine research and teaching. Research often shared in lectures to advanced students ("Seminarium")
- Students who finished at universities could get jobs at Gymnasia and continue with scientific research
- This system made Germany the center of the mathematical world until 1933.


Aristippus Philosophus Socraticus, naufragio cum ejectus ad Rhodiensium
litus animadvertisset Geometrica schemata descripta, exclamavisse ad
comites ita dicitur: Bene speremus, Hominum enim vestigia video.
Vitruv. Architect. lib. 6. Praef.


Aristippus Philosophus Socraticus, naufragio cum ejectus ad Rhodiensium litus animadvertisset Geometrica schemata descripta, exclamavisse ad comites ita dicitur:
Bene speremus, Hominum enim vestigia video.
Vitruv. Architect. lib. 6 Praef.


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.


Bene speremus, Hominum enim vestigia video.


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.


Bene speremus, Hominum enim vestigia video.


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.


JOHANNES BERNOULLI
MATHEMAT·PROF·


Bene speremus, Hominum enim vestigia video.


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.


JOHANNES BERNOULLI


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


GODEFROI GUILLAUME
LEIBNITZ,


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


GODEFROI GUILLAUME
LEIBNITZ,
Né le 3 Juillet 1646 mort le 14 Novembre 1716.


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre



GODEFROI GUILLAUME
LEIBNITZ,



GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre

1777 †1855
DEUTSCHE
BUNDESPOST
10



GODEFROI GUILLAUME
LEIBNITZ,



GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre
Legendre

BUNDESPOST
10

NORGE
NIELS HENRIK ABEL
550
1802-1829


GODEFROI GUILLAUME
LEIBNITZ,


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.


*1777 †1855
DEUTSCHE
BUNDESPOST
10


NORGE
NIELS HENRIK ABEL
550
1802-1829


GODEFROI GUILLAUME
LEIBNITZ,


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre


*1777 †1855
BUNDESPOST
10


NORGE
NIELS HENRIK ABEL
550
1802-1829



GODEFROI GUILLAUME
LEIBNITZ,



GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre

*1777 †1855
DEUTSCHE
BUNDESPOST
10
H.EIDENBENZ
L.SCHNELL

NORGE
NIELS HENRIK ABEL
550
1802-1829



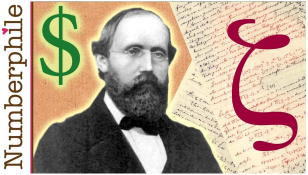
GODEFROI GUILLAUME
LEIBNITZ,


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre


∗1777 †1855
BUNDESPOST
10


NORGE
NIELS HENRIK ABEL
550
1802-1829


Numberphile

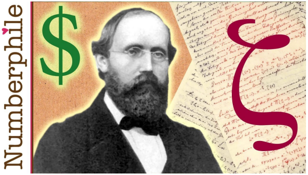

GODEFROI GUILLAUME
LEIBNITZ,


GIULIO
FAGNANI
MATEMATICO, FILOSOFO E POETA
Il ricordo
della Scuola Fagnani
a 250 anni dalla morte


Legendre.
Legendre


*1777 †1855
DEUTSCHE
BUNDESPOST
10


NORGE
NIELS HENRIK ABEL
550
1802-1829


Numberphile
$

. . .

Who pays these men, for what?

Here is a possible assignment for you: give examples for each of the following forms of financing of mathematical research.

- own/family fortune
- specific budget to solve a specific problem
- general budget to do just any math
- talent unrestrained by *lack* of budget

How do mathematical inventions happen?

Here is another possible assignment:

- by real coincidence: probably we wouldn't know this result potherwise
- by necessary coincidence: this result might have turned up in a different form, but something like this was bound to happen
- by necessity: this is the obvious and exact next step to take

This requires that you delve deeper in the mathematical and circumstantial context of the time.