hogere orde afgeleiden

T3	
+	16
o	39
~	33

Vb: $f(x) = x^n$

$$f'(x) = n\,x^{n-1}$$

$$f''(x) = n(n-1)\,x^{n-2}$$

$$f'''(x) = n(n-1)(n-2)\,x^{n-3}$$

$$f^{(4)}(x) = \underline{n(n-1)(n-2)(n-3)}\,x^{n-4}$$

$$\frac{n!}{(n-4)!} = \frac{n(n-1)(n-2)(n-3)\cancel{(n-4)}\cancel{\cdots 3\cdot 2\cdot 1}}{\cancel{(n-4)(n-5)(n-6)\cdots 3\cdot 2\cdot 1}}$$

faculteit

$n! = n(n-1)!$ en $0! = 1$

$$f^{(k)}(x) = \frac{n!}{(n-k)!}\,x^{n-k} \quad \text{mits } n \geq k.$$

Lineariseren

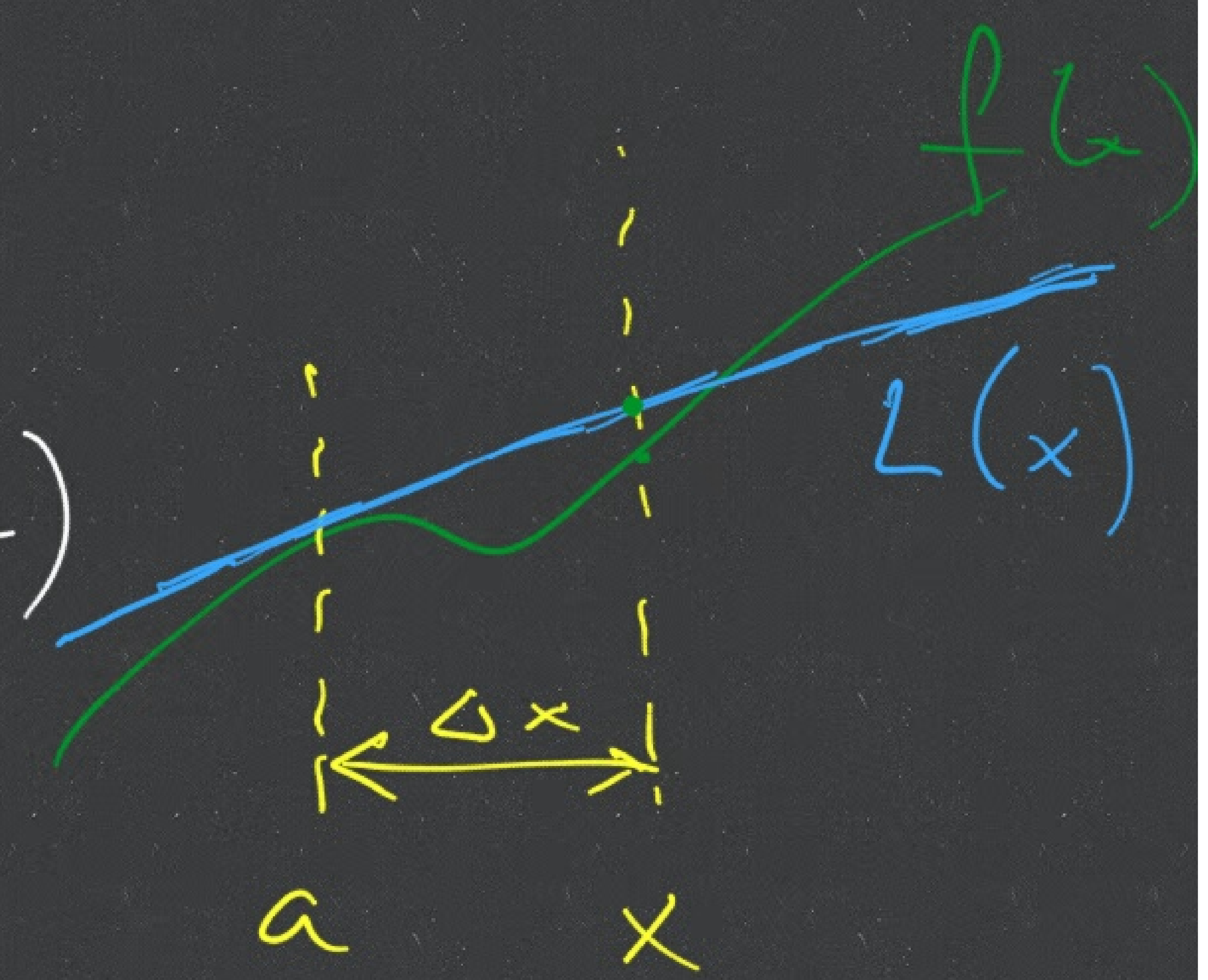
We weten al: $\dfrac{\Delta f}{\Delta x} \approx \dfrac{df}{dx}$ als Δx heel klein

maw: $\boxed{\Delta f \approx \dfrac{df}{dx}\Delta x}$

anders opschrijven met $x = a + \Delta x$

$$f(x) = f(a + \Delta x)$$

$$\Delta f = f(x) - f(a)$$



$$f(x) \approx \underbrace{f(a) + \frac{df(a)}{dx}\Delta x}$$

Definieer de linearisering van f in a is

$$L(x) = f(a) + f'(a)(x-a)$$

<u>Nut van lineariseren</u>: 1). foutschatten bij waarnemingen ($\rightarrow$ DATA).

2). f benaderen "in de buurt van" a

<u>Vb</u>: $\sqrt{5}$ benaderen mbv $f(x) = \sqrt{x}$, $a = 4$

linearisen: $f'(x) = \frac{1}{2\sqrt{x}}$, $f'(4) = \frac{1}{4}$, $f(4) = 2$

$$L(x) = f(4) + f'(4)(x-4)$$

dwz:

$$L(x) = 2 + \frac{1}{4}(x-4)$$

Bewering: $\sqrt{5} \approx 2\frac{1}{4}$

dus $L(5) = 2 + \frac{5-4}{4} = 2\frac{1}{4}$

Controleer: $\left(2\frac{1}{4}\right)^2 = \left(\frac{9}{4}\right)^2 = \frac{81}{16} = 5\frac{1}{16}$

sin(1) benaderen?

Neem $f(x) = \sin x$, steunpunt $a=0$

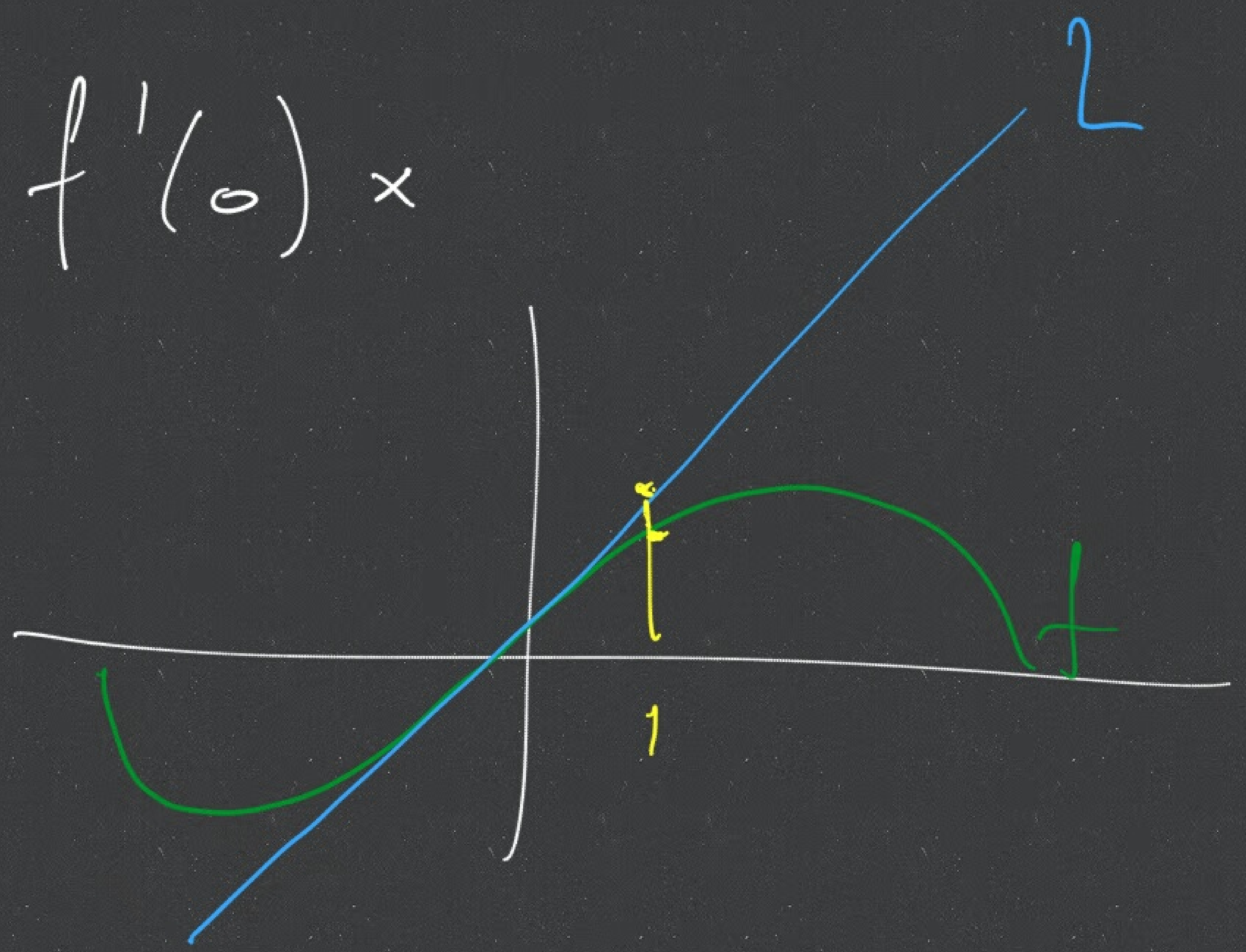
$f'(x) = \cos x$, $f'(0) = 1$, $f(0) = 0$

Linearisering: $L(x) = f(0) + f'(0)\,x$

$L(x) = x$

Benadering: $\sin 1 \approx 1$

Rekenmach: $0{,}84 \approx \sin 1$



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of: steunpt $a = \frac{\pi}{3}$, $L(x) = \frac{1}{2}\sqrt{3} + \frac{1}{2}\left(x - \frac{\pi}{3}\right)$

Betere benaderingen

Linearisering is veelterm van graad 1

$$L(x) = f(a) + f'(a)(x-a)$$

$f(a) = c_0$

$f'(a) = c_1$

Polynoom van graad 1 $P_1(x) = c_0 + c_1(x-a)$

Benaderen met hogere graads veeltermen.

Graad 2 ziet eruit als: $P_2(x) = \underbrace{c_0 + c_1(x-a)}_{P_1} + c_2(x-a)^2$

Willen: $P_2''(a) = f''(a)$

dus $2c_2 = f''(a)$

dus $c_2 = \frac{1}{2} f''(a)$

KLAD

$P_2'(x) = c_1 + 2c_2(x-a)$

$P_2''(x) = 2c_2$

Daarom kiezen we $\boxed{P_2(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2}$

Graad 3: zelfde spelletje

$$P_3(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \frac{1}{2\cdot 3} f'''(a)(x-a)^3$$

Algemeen: Graad n

!!!! $$\boxed{f(a) + f'(a)(x-a) + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n}$$

Taylorveelterm van orde n met steunpunt a van de functie f(x).

NB: 1) No life without Taylor

2) Taylor geeft de best mogelijke benadering van f bij a met gegeven orde.

3) Restterm

overslaan

<u>Voorbeeld</u> $f(x) = \sin x$, steunpunt 0

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$

$f'''(0) = -1$

$f^{(4)}(0) = f(0)$

<u>orde 1</u>: $T_1(x) = f(0) + f'(0)\,x = x$

<u>orde 2</u>: $T_2(x) = f(0) + f'(0)\,x + \frac{1}{2} f''(0)\,x^2$

$= x$

orde 3: $T_3(x) = T_2(x) + \dfrac{f'''(0)}{3!}x^3 = x - \frac{1}{6}x^3$

orde 4: zelfde

orde 5: $T_5(x) = x - \frac{1}{6}x^3 + \frac{1}{5!}x^5$

orde $2n+1$: $T_{2n+1}(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots + \dfrac{(-1)^{n+1}}{(2n+1)!}x^{2n+1}$

Voorbeeld: $\cos x$ steunpt. 0

$$T_{2n} = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots \frac{(-1)^n x^{2n}}{(2n)!}$$

Voorbeeld: e^x met steunpt. 0

$$T_n = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{4!}x^4 + \dots \frac{1}{n!}x^n$$

$$T_n(ix) = 1 + ix - \frac{1}{2}x^2 - \frac{i}{3!}x^3 + \frac{1}{4!}x^4 + \frac{i}{5!}x^5 - - -$$

$$= \underbrace{\left(1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 \dots\right)}_{\cos x} + i\underbrace{\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 \dots\right)}_{\sin x}$$

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algemeen: $T_n(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

$$= \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$