Oneigenlijke integralen

$$\int_a^b f(x)\,dx$$

$n = 85$
$+$ 11
$\circ$ 27
$-$ 47

- type 1: $a = -\infty$ en/of $b = +\infty$
- type 2: $f(x) \to \pm\infty$ in het interval (a, b)

Aanpak: met limiet

Vb: $f(x) = \dfrac{1}{x}$

$\displaystyle\int_1^\infty \frac{1}{x}\,dx$ type 1. Eerst: $\displaystyle\int_1^R \frac{1}{x}\,dx = \log x \Big|_1^R = \log R$

dan: $\displaystyle\int_1^\infty \frac{1}{x}\,dx = \lim_{R\to\infty} \int_1^R \frac{1}{x}\,dx = \lim_{R\to\infty} \log R = \infty$

DWZ: $\displaystyle\int_1^\infty \frac{dx}{x}$ is divergent.

<u>type 2</u>: $\int_0^1 \frac{dx}{x}$ Probleem: $\frac{1}{x} \to \infty$ als $x \to 0$.

$$\int_R^1 \frac{dx}{x} = \log x \Big|_R^1 = -\log R, \text{ dus}$$

$$\int_0^1 \frac{dx}{x} = \lim_{R\to 0^+} \int_R^1 \frac{dx}{x} = \lim_{R\to 0^+} (-\log R) = +\infty \text{ Divergent}$$

<u>Vb</u>: $f(x) = x^{-\alpha}$; met $\alpha \neq 1$

- Bekijk $\int_1^\infty x^{-\alpha}\,dx$ type 1, $\lim_{R\to\infty} \int_1^R x^{-\alpha}\,dx = \lim_{R\to\infty} \frac{x^{1-\alpha}}{1-\alpha}\Big|_{x=1}^{x=R}$

$$= \lim_{R\to\infty} \left(\frac{R^{1-\alpha}}{1-\alpha} - \frac{1}{1-\alpha}\right) = \lim_{R\to\infty} \frac{R^{1-\alpha}-1}{1-\alpha} \xrightarrow{\alpha>1} = \frac{1}{\alpha-1} \text{ CONVERGENT}$$

$$\xrightarrow{\alpha<1} = \infty \text{ DIVERGENT}$$

• Nu $\displaystyle\int_0^1 x^{-\alpha}\,dx = \lim_{R\to 0^+} \int_R^1 x^{-\alpha}\,dx$

$$= \lim_{R\to 0^+} \left.\frac{x^{1-\alpha}}{1-\alpha}\right|_{x=R}^{x=1}$$

$$= \lim_{R\to 0^+} \frac{1-R^{1-\alpha}}{1-\alpha}$$

$$= \begin{cases} \xrightarrow{\alpha>1} \; = +\infty & \text{DIV.} \\[2ex] \xrightarrow{\alpha<1} \; = \dfrac{1}{1-\alpha} & \text{CONV.} \end{cases}$$

$$\int_0^\infty \frac{1}{x}\,dx$$ type 1&2 tegelijk.

$$\boxed{KIS}$$ Keep it simple.

Kies je favoriete a met $0 < a < \infty$ en splits

$$\int_0^\infty \frac{1}{x}\,dx = \int_0^a \frac{1}{x}\,dx + \int_a^\infty \frac{1}{x}\,dx$$

$$\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$$ splits.

$$\int_{-\infty}^0 \frac{1}{1+x^2}\,dx + \int_0^\infty \frac{1}{1+x^2}\,dx = 2\int_0^\infty \frac{1}{1+x^2}\,dx$$

Dat was Stelling 2 / p. 364

Nu stelling 3 (p. 365): het maken van afschattingen waarmee je alsnog kunt zeggen of een lastige onbepaalde integraal conv. of div. is.

St 3 Als op een interval (a, b) (evt. $a = -\infty$ of $b = \infty$)

geldt dat $0 \leq f(x) \leq g(x)$

dan geldt ook $0 \leq \int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$

en bovendien:

- als $\int_a^b g(x)\,dx$ conv dan ook $\int_a^b f(x)\,dx$
- als $\int_a^b f(x)\,dx$ div. dan ook $\int_a^b g(x)\,dx$

NB

- als $\int_a^b g(x)\,dx$ div. dan weet je niks over $\int f$.
- als $\int_a^b f(x)\,dx$ conv. " " " " over $\int_a^b g\,dx$

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vb.

$f(x) = |\sin x|$

$g(x) = 2$ voor alle x

$$0 \leq f(x) \leq g(x)$$

$$0 \leq \underbrace{\int_0^\infty f(x)\,dx}_{\text{div.}} \leq \underbrace{\int_0^\infty 2\,dx}_{\text{div.}}$$