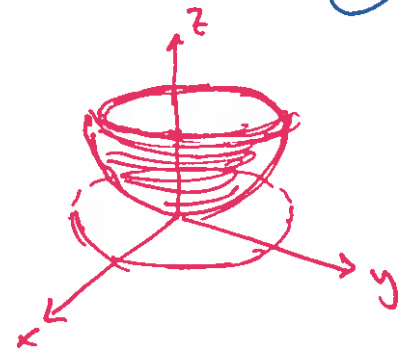


Voorbeeld 15.6: Ex 5

$$\vec{F} = 2 \frac{x\hat{i} + y\hat{j}}{x^2 + y^2} + \hat{k}$$



$$S: \vec{r} = u \cos v \hat{i} + u \sin v \hat{j} + u^2 \hat{k}$$

$$\text{op } [0, 1] \times [0, 2\pi] \text{ dwz } \begin{matrix} 0 \leq u \leq 1 \\ 0 \leq v \leq 2\pi \end{matrix}$$

Gevraagd: downward flux of \vec{F} thru S
↓
neem $-\hat{N}$

Oplossing: Normaalvec vinden via raakvectoren in $\vec{r}(u, v)$

$$\frac{\partial \vec{r}}{\partial u} = \cos v \hat{i} + \sin v \hat{j} + 2u \hat{k}$$

$$\frac{\partial \vec{r}}{\partial v} = -u \sin v \hat{i} + u \cos v \hat{j}$$

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = -2u^2 \cos v \hat{i} - 2u^2 \sin v \hat{j} + u \hat{k} = \vec{n}$$

downward: neem $d\vec{S} = -\vec{n} dS$
 $= (2u^2 \cos v, 2u^2 \sin v, -u) du dv$

Parametrisering gebruiken in \vec{F} :

$$\vec{F} = \frac{2 \cos v}{u} \hat{i} + \frac{2 \sin v}{u} \hat{j} + \hat{k}$$

Invullen:

$$\iint_S \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^{2\pi} (4u - u) du dv = 3\pi$$

NB: Geen subs, dus geen Jacobiaan!