$\boxed{4.1}$

(linear heat equation)
$$\begin{cases} u_t = u_{xx}, & x \in [0,1],\ t \in [0,T] \\ u(x,0) = \sin(\pi x) \\ u(0,t) = u(1,t) = 0 \end{cases}$$

Apply the first step in the Method-of-Lines:

$$\begin{cases} \dot{\vec{u}}(t) = D_2\, \vec{u}(t) \\ \vec{u}(0) = \vec{u}_0 \end{cases} \qquad (*)$$

In the second step:

!!! compare with the exact solution of the heat equation

a) Euler-Forward for $(*)$; choose Δt such that a (numerically) stable solution is obtained. Take $\Delta x = 10^{-2}$ and $T = 0.1$.

b) Euler-Backward for $(*)$; $\Delta x = 10^{-2}$, $T = 0.1$. Compare with a). Does the choice of Δt influence the numerical stability? (and the accuracy?)

c) Use the matlab "expm" function to obtain a "semi-discrete" exact solution. ($\Delta x = 10^{-2}$, $T = 0.1$)

⊙ There is an exact solution for this model!