$\boxed{\text{ex. 5.1}}$

Consider the interval $[0,1]$, the uniform grid $\{x_i = (i-1)\Delta x$, $i = 1,2,\dots,I$ and stepsize $\Delta x = \dfrac{1}{I-1}$.

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a) Construct the matrices that define the following approximations with periodic boundary conditions:

i) matrix D_{1+} ~ "upwind" discretisation: $\dfrac{u_{i+1}-u_i}{\Delta x} \approx \dfrac{\partial u}{\partial x}$ at $x = x_i$

ii) matrix D_{1-} ~ "downwind" discretisation: $\dfrac{u_i-u_{i-1}}{\Delta x} \approx \dfrac{\partial u}{\partial x}$ at $x = x_i$

iii) matrix D_{1c} ~ "central" discretisation: $\dfrac{u_{i+1}-u_{i-1}}{2\Delta x} \approx \dfrac{\partial u}{\partial x}$ at $x = x_i$

(check that $D_{1c} = \dfrac{D_{1+}+D_{1-}}{2}$)

iv) matrix D_2 ~ $\dfrac{u_{i+1}-2u_i+u_{i-1}}{(\Delta x)^2} \approx \dfrac{\partial^2 u}{\partial x^2}$ at $x = x_i$

(check that $D_2 = D_{1+} D_{1-}$)

find !!

v) matrix D_3 ~ $\dfrac{\text{- - - - - - - -}}{(\Delta x)^3} \approx \dfrac{\partial^3 u}{\partial x^3}$ at $x = x_i$

($D_3 = D_2 D_{1c}$)

find !!

vi) matrix D_4 ~ $\dfrac{\text{- - - - - - - -}}{(\Delta x)^4} \approx \dfrac{\partial^4 u}{\partial x^4}$ at $x = x_i$

($D_4 = D_2^2$)

b) In Matlab: write a function file

$\boxed{\text{FDmatrices.m}}$ with input value I

that defines the matrices $D_{1+}, D_{1-}, \ldots, D_4$ from a).

↦ First, "set $\Delta x = 1$" and choose $I = 10$.*

Check on your screen the structure of the matrices (symmetric?, skew-symmetric?, tri-diagonal?, etc)

c) Make use of the Matlab function "eig" to find the eigenvalues of the matrices from a).

Plot the eigenvalues of $D_{1+}, D_{1-}, \ldots, D_4$ in the x-y plane, where (x,y) are the real**- and imaginary parts of the eigenvalues.

Use for each matrix different colours and "*" to indicate their position in the complex plane.

↦ In c) you can use: $\Delta x = \frac{1}{I-1}$ and different values for I: $I = 10, 50, 100, \ldots$

* although Δx and I are related via the formula $\Delta x = \frac{1}{I-1}$, in part b), we ignore this, just to examine the matrices.

** use real(--) and imag(--) in Matlab.