Exercise 6.1

Consider the space-fractional heat equation

$$\begin{cases} u_t = D_x^{3/2} u, & x \in [0,1] \\ u(x,0) = e^{-100(x-\frac{1}{2})^2} & t \in [0,T] \end{cases}$$

with periodic boundary conditions!

and the following ODE systems:

a) $$\begin{cases} \dot{\vec{u}}(t) = -\sqrt{D_3}\,\vec{u}(t) \\ \vec{u}(0) = \vec{u}_0 \end{cases}$$

b) $$\begin{cases} \dot{\vec{u}}(t) = -\sqrt{-D_3}\,\vec{u}(t) \\ \vec{u}(0) = \vec{u}_0 \end{cases}$$

c) $$\begin{cases} \dot{\vec{u}}(t) = -\frac{1}{\sqrt{2}}\left(\sqrt{D_3} + \sqrt{-D_3}\right)\vec{u}(t) \\ \vec{u}(0) = \vec{u}_0 . \end{cases}$$

* Apply in all three cases:

1. expm: $\vec{u}(t) = e^{tM}\vec{u}_0$ with the matrix M from a), b), c).
2. Euler-Forward: choose Δt appropriately.
3. Euler-Backward: choose the Δt from 2 and "1".

* Compare the solutions with $\begin{cases} \dot{\vec{u}}(t) = D_2^{per}\,\vec{u}(t) \\ \vec{u}(0) = \vec{u}_0 \end{cases}$ ← periodic BCs!!

* Choose $\Delta x = 0.01$, $T = 0.03$ and make use of sqrtm.m for $\sqrt{\pm D_3}$.