Exercise $\boxed{6.2}$

Consider the space-fractional Fisher equation

$$\begin{cases} u_t = d \cdot D^{3/2} u + \gamma u(1-u) \\ u(x,0) = e^{-50(x-\frac{1}{2})^2}, & x \in [0,1] \\ & t \in [0,T] \end{cases}$$

with periodic boundary conditions.

Choose $\Delta x = 0.01$, $T = 1$, $d = 10^{-3}$, $\gamma = 5$.

Apply the IMEX-method from Day 4, with an appropriate stepsize Δt to the ODE-system:

$$\begin{cases} \dot{\vec{u}}(t) = \underbrace{-\frac{d}{\sqrt{2}}\left(\sqrt{D_3} + \sqrt{-D_3}\right)}_{\text{define } =: A} \vec{u}(t) + \vec{f}(\vec{u}(t)) \\ \vec{u}(0) = \vec{u}_0 . \end{cases}$$

$\Rightarrow (I - \Delta t\, A)\vec{u}^{\,n+1} = \vec{u}^{\,n} + \Delta t\, \vec{f}(\vec{u}^{\,n})$

Use "\\" to solve this linear system at each time-step.

Compare the solutions with the ones, if we would choose $A = D_2^{per.}$ instead.

periodic BCs !!

Use sqrtm.m to find the square root of D_3 and $-D_3$.