Ex. $\boxed{6.1}$

1) Consider the space-fractional heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = D^{3/2} u \,, & x \in (0,1) \,, \ t \in [0,T], \\ u(x,0) = e^{-100(x-\frac{1}{2})^2} \end{cases}$$

with $\boxed{\text{periodic boundary conditions}}$,

2) and the following ODE-systems:

ⓐ $\begin{cases} \dot{\vec{u}}(t) = -\sqrt{D_3}\, \vec{u}(t), \\ \vec{u}(0) = \vec{u}_0 \,, \end{cases}$

ⓑ $\begin{cases} \dot{\vec{u}}(t) = -\sqrt{-D_3}\, \vec{u}(t), \\ \vec{u}(0) = \vec{u}_0 \,, \end{cases}$

ⓒ $\begin{cases} \dot{\vec{u}}(t) = -\dfrac{1}{\sqrt{2}}\left(\sqrt{D_3} + \sqrt{-D_3}\right) \vec{u}(t), \\ \vec{u}(0) = \vec{u}_0 \end{cases}$

ⓓ $\begin{cases} \dot{\vec{u}}(t) = D_2\, \vec{u}(t), \\ \vec{u}(0) = \vec{u}_0 \end{cases}$

ⓔ $\begin{cases} \dot{\vec{u}}(t) = -\sqrt{\sqrt{(-D_2)^3}}\, \vec{u}(t), \\ \vec{u}(0) = \vec{u}_0 \,. \end{cases}$

3) Apply in all five cases: 1) expm : $\vec{u}(t) = e^{tM} \vec{u}_0$ with M the respective matrix from ⓐ,..,ⓔ.

2) Euler-Forward; choose Δt appropriately

3) Euler-Backward; choose the Δt from 2) and use "\" in Matlab

4) Compare the solutions from ⓐ,..,ⓔ.

5) Choose $\Delta x = 0.01$, $T = 0.03$ and make use of sqrtm.m .