Ex. $\boxed{6.2}$

Consider the space-fractional Fisher equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = -d\left(-\dfrac{\partial^2}{\partial x^2}\right)^{\alpha/2} u + \gamma u(1-u), \ x \in (0,1), t \in (0,T], \\ u(x,0) = 0.1\, e^{-50(x-\frac{1}{2})^2} \end{cases}$$

with $\boxed{\text{periodic boundary conditions}}$ and $d = 10^{-3}, \gamma = 5, T = 1$.

For $\Delta x = 0.01$, apply the IMEX-method from lecture 4, and an appropriate stepsize Δt, to the ODE-systems:

"$\alpha = 3/2$": 1) $\begin{cases} \dot{\vec{u}}(t) = \dfrac{-d}{\sqrt{2}}\left(\sqrt{D_3} + \sqrt{-D_3}\right)\vec{u}(t) + \vec{f}(\vec{u}(t)), \\ \vec{u}(0) = \vec{u}_0, \end{cases}$

2) $\begin{cases} \dot{\vec{u}}(t) = -d\sqrt{\sqrt{(-D_2)^3}}\,\vec{u}(t) + \vec{f}(\vec{u}(t)), \\ \vec{u}(0) = \vec{u}_0. \end{cases}$

* Use "\" to solve the linear system in each timestep $t^n \to t^{n+1}$.
* Compare the solutions in 1) and 2).
* Use sqrtm in Matlab for $\sqrt{D_3}$, $\sqrt{-D_3}$, $\sqrt{\sqrt{(-D_2)^3}}$.
* Do not forget to include the periodic boundary conditions in the matrices in 1) and 2) !
* Finally, compare the solutions with the ODE-system, if we use the matrix D_2 instead of $\sqrt{D_3}$, etcetera.