

# Introduction Scientific Computing

## Assignment 6.3

Consider the blow-up differential equation:

$$\begin{cases} \dot{u}(t) = [u(t)]^p, & t \in [0, T^*], & p \geq 2, \\ u(0) = u_0. \end{cases} \quad (1)$$

(a) Determine the exact solution  $u(t)$  of model (1) and the blowup time  $T^*$ .

- Plot the solutions for some values of the parameter  $p$ .

(b) For  $p = 2$  and  $u_0 = 1$ , apply the methods Euler-Forward and Euler-Backward to model (1).

- Instead, you could also use an ODE-solver from Matlab.
- Do you find accurate solutions? In particular, do the methods detect the blow-up time  $T^*$ ?

(c) Apply a Sundman-transformation (as discussed in lecture 6) for  $p = 2$ : consider three different choices for the transformation.

- Compare the numerical solutions with the solutions from parts (a) and (b).
- Comment on the accuracy and applicability of the different methods.
- Also, show a few graphs of the numerical solutions.