

# Inleiding Scientific Computing

## Exercise 1c (“logistic DE”)

Consider the logistic DE from population dynamics:

$$\begin{cases} y' = y(1 - y), & t \in (0, T], \\ y(0) = y_0. \end{cases}$$

- (a) Determine the analytic solution of this DE.
- (b) Plot the solution  $y(t)$  for several initial conditions  $y_0$ . Consider

$$y_0 > 1, \quad 0 < y_0 < 1, \quad \text{and} \quad y_0 < 0.$$

- (c) Approximate this DE with the methods CT (central in time), CTnl (central in time, nonlocal), FT (forward in time), FTnl (forward in time, nonlocal) and Runge–Kutta 2. The terms “nonlocal” and these methods are explained in the extra document on the webpage.
- (d) Find the exact expressions for  $y_n$  in terms of  $y_0$ ,  $\Delta t$  and the index  $n$ , for the methods CTnl and FTnl.
- (e) Perform numerical experiments with Matlab. In particular, choose:
- for CT:  $y_0 = 0.5$ ,  $\Delta t = 0.1$  and  $T = 200$ ;
  - for CTnl:  $y_0 = 0.25$  (choose  $\Delta t$  and  $T$  yourself);
  - for FT:  $y_0 = 0.5$  with

$$\Delta t = 0.01 \ (T = 10), \quad \Delta t = 1.5 \ (T = 45), \quad \Delta t = 2.5 \ (T = 50);$$

- for FTnl:  $y_0 = 0.5$  with

$$\Delta t = 0.01 \ (T = 10), \quad \Delta t = 1.5 \ (T = 45)$$

$$\Delta t = 2.5 \ (T = 50), \quad \Delta t = 3 \ (T = 60).$$

What kind of numerical effects do you observe? Can you explain them? Do you find chaotic solutions?