Introduction Scientific Computing

Assignment 4a (linear heat equation)

$$\begin{cases} u_t = u_{xx}, & x \in [0,1],\ t \in [0,T], \\ u(x,0) = \sin(\pi x), \\ u(0,t) = u(1,t) = 0. \end{cases}$$

Apply the first step in the Method of Lines:

$$\begin{cases} \dot{\mathbf{u}}(t) = D_2 \mathbf{u}(t), \\ \mathbf{u}(0) = \mathbf{u}_0, \end{cases} \qquad (*)$$

In the second step:

(a) **Euler Forward** for (*); choose Δt such that a (numerically) stable solution is obtained. Take $\Delta x = 10^{-2}$ and $T = 0.1$.

(b) **Euler Backward** for (*); $\Delta x = 10^{-2}$, $T = 0.1$. Compare with (a). Does the choice of Δt influence the numerical stability (and the accuracy?)?

(c) Use the MATLAB `expm` function to obtain a *semi-discrete exact solution.*

$$(\Delta x = 10^{-2},\ T = 0.1)$$

Compare with the exact solution of the heat equation.
Find the exact solution of the form $u(x,t) = g(x)h(t)$.

Assignment 4b (nonlinear Fisher PDE)

$$\begin{cases} u_t = d\,u_{xx} + \gamma\,u(1-u), & x \in [0,1],\ t \in [0,T], \\ u(x,0) = e^{-50\left(x-\frac{1}{2}\right)^2}, \\ u(0,t) = u(1,t) = 0. \end{cases}$$

Apply the first step in the Method of Lines:

$$\begin{cases} \dot{\mathbf{u}}(t) = d\,D_2\mathbf{u}(t) + \mathbf{f}(\mathbf{u}(t)), \\ \mathbf{u}(0) = \mathbf{u}_0, \end{cases}$$

where

$$\mathbf{f}(\mathbf{u}) = \gamma\,\mathbf{u} \circ (1 - \mathbf{u})$$

(elementwise product).

In the second step: **IMEX-method**

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = d\,D_2\mathbf{u}^{n+1} + \mathbf{f}(\mathbf{u}^n).$$

Choose

$$\Delta x = 10^{-2}, \quad T = 1, \quad d = 10^{-3}, \quad \gamma = 5.$$

Time stepsize $\Delta t =$? (choose a stable step size!)