# Introduction Scientific Computing

## Assignment 2b ("heat equation")

Consider the following differential equation (DE), which represents the stationary heat equation in one space dimension with constant source term:

$$\begin{cases} -u''(x) = 1, & x \in (0,1), \\ u(0) = 0, \quad u(1) = 0. \end{cases} \tag{1}$$

(a) Determine the <u>analytic</u> solution of DE (1).

(b) We approximate the solution of DE (1) by the following linear system:

$$-D_2 \vec{u} = \vec{b}, \tag{2}$$

where $D_2 = ...$, $\vec{u} = ...$, $\vec{b} = ...$ ($\Rightarrow$ this will be discussed in Lecture 2!).

(c) Choose $\Delta x$ and approximate the solutions of the linear system (2) by applying the Jacobi method. Check numerically, the dependence on $\Delta x$ (accuracy, convergence?). Does the convergence depend on the choice of $\Delta x$? How does $\Delta x$ influence the total number of iterations? (calculate, with the help of Matlab, the eigenvalues of the $D_2$-matrix and check how this effects the convergence criterion)

(d) The time-dependent version of the heat equation reads:

$$\begin{cases} u_t = u_{xx} + 1, & x \in (0,1), \\ u(0,t) = 0, \quad u(1,t) = 0, \quad u(x,0) = \sin(\pi x), \end{cases} \tag{3}$$

with $u(x,t)$ and $t \in [0,T]$. $T$ is the final time (not specified here).

We re-write the partial differential equation as a system of ordinary DEs (check Lecture 2 for more details):

$$\begin{cases} \dot{\vec{u}}(t) = D_2 \vec{u}(t) + \vec{f}, & t \in [0,T), \\ \vec{u}(0) = \vec{u}_0, \end{cases} \tag{4}$$

Apply the Backward-Euler method to DE-system (4). Next work out the linear system that should be solved to obtain a solution on time-level $t_{n+1}$, given the solution on time-level $t_n$. Specify the matrix and the righthand-side vector in this linear system (you do not need to implement this in Matlab).