method of Jacobi

$A\vec{x} = \vec{b}$

given matrix

given vector

to be found (numerically)

calculate $\max_{\lambda_1,\lambda_2,\lambda_3} |\lambda(I - D^{-1}A)|$

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rewrite : $D^{-1}A\vec{x} = D^{-1}\vec{b}$

D = diagonal matrix ("diagonal of A")

$$0 = D^{-1}\vec{b} - D^{-1}A\vec{x}$$

$$\vec{x} = D^{-1}\vec{b} - D^{-1}A\vec{x} + \vec{x}$$

$$\vec{x} = D^{-1}\vec{b} + (I - D^{-1}A)\vec{x}$$

choose $\vec{x}_0$ ("initial guess")

define iteration process: $\vec{x}_{j+1} = D^{-1}\vec{b} + (I - D^{-1}A)\vec{x}_j$

$j = 0, 1, 2, \ldots$

Theorem : if A is "strictly diagonally dominant", then this process converges to $\vec{x}$

(no proof)

Examples : (homework exercise)

initial guess $\vec{x}_0$?

compare with $A\backslash\vec{b}$

1) $A = \begin{pmatrix} 3 & 0 & 4 \\ 7 & 4 & 2 \\ -1 & 1 & 2 \end{pmatrix}$ no convergence

2) $A = \begin{pmatrix} -3 & 3 & -6 \\ -4 & 7 & -8 \\ 5 & 7 & -9 \end{pmatrix}$ convergence

3) $A = \begin{pmatrix} 4 & 1 & 1 \\ 2 & -9 & 0 \\ 0 & -8 & -6 \end{pmatrix}$ (slow) convergence

4) $A = \begin{pmatrix} 7 & 6 & 9 \\ 4 & 5 & -4 \\ -7 & -3 & 8 \end{pmatrix}$ (slow) convergence