Matrices, Least Squares and Artificial Intelligence

A-exercises

Exercise A1: Matrix operations

Given:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},$$

and:

$$B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix},$$

calculate:

1. $A + B$;
2. AB;
3. BA.

Explain whether matrix multiplication is commutative.

Exercise A2: Linear system

Solve:

$$\begin{cases} 2x + y = 5, \\ x - y = 1. \end{cases}$$

Write the problem in matrix form:

$$A\mathbf{x} = \mathbf{b}.$$

Exercise A3: Least squares

The following data are measured:

x	y
1	2.1
2	3.9
3	6.2
4	8.1

Assume:

$$y = ax + b.$$

Construct the matrix:

$$X$$

and determine the least squares parameters.

Exercise A4: Beer–Lambert calibration

A calibration experiment gives:

$$A = mc + b.$$

Explain:

1. what the parameters m and b represent;
2. how least squares determines these values;
3. how the model can predict an unknown concentration.

Exercise A5: Activation functions

Consider:

$$z = \begin{pmatrix} -2 \\ 0.5 \\ 3 \end{pmatrix}.$$

Calculate the output of the ReLU function:

$$\phi(z) = \max(0, z).$$

Explain why ReLU introduces nonlinearity.

Exercise A6: AI model interpretation

A neural network gives:

$$\hat{y} = W_2\phi(W_1x + b_1) + b_2.$$

Explain the role of:

1. W;
2. b;
3. ϕ.

Final conclusion

The most important message of this lecture is:

Artificial intelligence is built on mathematical foundations.

Matrices describe information.
Least squares learns parameters.
Activation functions create nonlinear models.
Together these concepts form the basis of modern data analysis in science and engineering.