Dynamic Grid Adaptation for Computational Magnetohydrodynamics

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Abstract. In many plasma physical and astrophysical problems, both linear and nonlinear effects can lead to *global* dynamics that induce, or occur simultaneously with, *local* phenomena. For example, a magnetically confined plasma column can potentially posses global magnetohydrodynamic (MHD) eigenmodes with an oscillation frequency that matches a local eigenfrequency at some specific internal radius. The corresponding linear eigenfunctions then demonstrate large-scale perturbations together with fine-scale resonant behaviour. A well-known nonlinear effect is the steepening of waves into shocks where the discontinuities that then develop can be viewed as extreme cases of 'short wavelength' features. Numerical simulations of these types of physics problems can benefit greatly from *dynamically controlled grid adaptation schemes.*
Here, we present a progress report on two different approaches that we envisage to evaluate against each other and use in multi-dimensional hydro- and magnetohydrodynamic computations. In *r-refinement*, the number of grid points stays fixed, but the grid 'moves' in response to persistent or developing steep gradients. First results on 1D and 2D MHD model problems are presented. In *h-refinement*, the resolution is raised locally without moving individual mesh points. We show 2D hydrodynamic 'shock tube' evolutions where hierarchically nested patches of subsequently finer grid spacing are created and destroyed when needed. This *adaptive mesh refinement* technique will be further implemented in the Versatile Advection Code, so that its functionality carries over to any set of near conservation laws in one, two, or three space dimensions.

1 Introduction

Computational magnetohydrodynamics is rapidly developing into a standard tool for investigating the behaviour of a plasma (a charge-neutral 'soup' of ions and electrons). The numerical algorithms used in state-of-the-art software packages for multi-dimensional MHD studies heavily borrow on well-established techniques employed in computational fluid dynamics. However, significant complications arise due to the presence of a magnetic field, together with its dynamical

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influence. E.g., Riemann solver based methods must allow for the presence of three basic wave modes in the plasma: a fast magnetosonic, an Alfvén, and a slow magnetosonic signal travel outwards to communicate localized, isolated perturbations to further out regions. In addition, the magnetic field itself satisfies a basic law that constrains possible solutions of MHD problems: the absence of magnetic charges or monopoles cause the field to be solenoidal $\nabla \cdot \mathbf{B} = 0$.

In spite of these complications, various methods have been developed and applied successfully for simulating magnetically controlled fluid dynamics. The resulting richness in physics phenomena often involve both long and very short lengthscales. To name but a few recently investigated topics:

– **Sunspot eigenoscillations:** the natural vibration modes of sunspots, when modeled as magnetic flux tubes embedded in unmagnetized surroundings, include so-called *leaky, resonantly damped modes* [11]. These modes correspond to global oscillations of the sunspot that affect the entire surroundings through outwards travelling sound waves, but also have internal narrow boundary layers where energy is dissipated Ohmically.
– **Secondary, induced plasma instabilities:** resistive MHD studies of velocity shear layers, susceptible to the Kelvin-Helmholtz instability (known from wind-induced ripples on a pond), demonstrated how small-scale reconnection events can occur by secondary tearing instabilities [8].
– **Complex interacting bow shock patterns:** numerical simulations of idealized plasma flow problems around perfectly conducting, rigid objects, revealed how under certain inflow conditions, the resulting bow shock consists of several small-scale and large-scale features with interconnecting weak and strong discontinuities [3].

It should be clear that numerical simulations of these plasma physical processes need to employ a sufficiently high resolution to capture both the fine-scale structure and the overall dynamics. For steady problems, a priori knowledge of the regions where a high spatial resolution is needed can be incorporated by using a static, stretched grid. However, for unsteady problems where typically long-term interactions are of interest, a dynamically adjusted grid resolution is needed. Therefore, we are currently assessing the use of two different grid adaptation schemes for multi-dimensional hydro- and magnetohydrodynamic problems. We demonstrate the workings of a moving grid method on some MHD problems in Sect. 2, and explain in some more detail the patch-based adaptive mesh refinement (AMR) scheme [1] in Sect. 3. We are implementing the latter approach in the Versatile Advection Code [12] [VAC, see `http://www.phys.uu.nl/~toth`]. The VAC software has already demonstrated its capacities for doing multi-dimensional magneto-fluid-dynamical simulations in a wealth of astrophysical and fundamental plasma physical applications. The lack of adaptivity in the mesh geometry has so far been compensated by the fact that we can run on massively parallel platforms [7,14]. Still, to make further quantitative parametric studies into fully nonlinear regimes, an efficient grid adaptivity is desirable.

2 Adaptive Method Of Lines

In the Method Of Lines (MOL) approach, the grid points reposition themselves dynamically in accord with the local resolution requirements [17]. This means that the new grid point positions must also be calculated simultaneously with the physical variables (like density, momenta, energy, etc.) at these new locations. The governing physics equations are therefore first transformed to new coordinates, i.e. $\xi \equiv \xi(\mathbf{x}, t)$ and $\theta = t$ where $\mathbf{x}$ and t denote the original cartesian coordinates and time, respectively. In this coordinate transformation, the $\xi(\mathbf{x}, t)$ itself obeys a suitably constructed partial differential equation that controls the mesh movements. To obtain an efficient and gradually adjusting adaptive grid, a so-called equidistribution principle is being used, enhanced with smoothing procedures in the spatial and the time direction. This adaptive grid PDE, together with the transformed governing PDE model, is then semi-discretized and one obtains a large system of ordinary differential equations. This system can be solved with an appropriate stiff time-integrator.

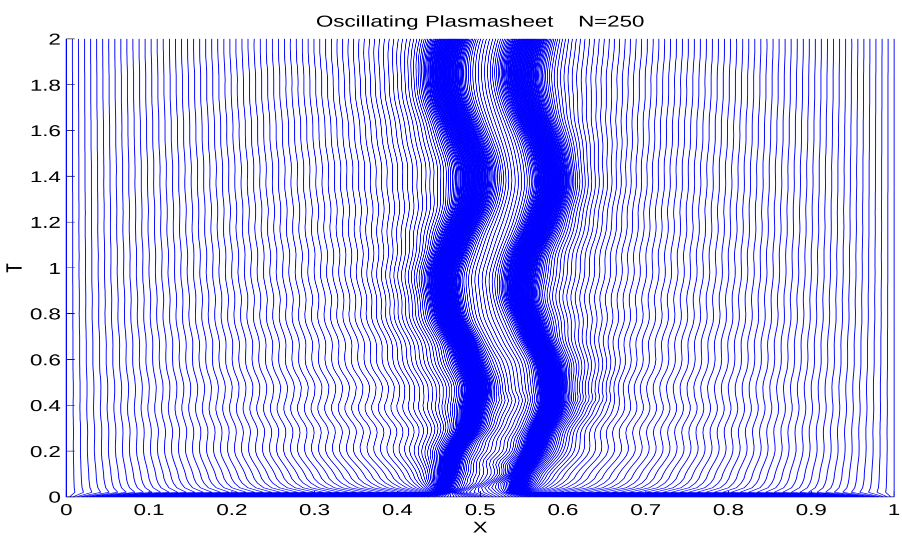

Fig. 1. The grid history in a 1D MHD simulation of an oscillating plasma sheet embedded in a vacuum. Starting with an equidistant grid of 250 grid points, the sheet boundaries are automatically recognized as regions where grid points need to be clustered. After this rapid initial adjustment (prior to times $T < 0.05$), the mesh clearly follows the oscillation.

We have implemented and applied this MOL-approach to various MHD model problems [18]. The earliest study by Dorfi and Drury [4] already demonstrated this method on one dimensional Euler shock tubes. We have successfully tested the method to similar MHD shock problems and are evaluating different means to generalize the method to two and three space dimensions. The difficulty in generalizing MOL methods to more than one space coordinate is that the corners of 2D or 3D cells should be prevented from folding over onto each other. Also, in multidimensional MHD applications, we must pay particular attention to the solenoidal condition on the magnetic field.

Two results are shown here: Fig. 1 shows the time history of the grid point locations for a 1.5D MHD problem first introduced by Tóth, Keppens, and Botchev [15]. A high density plasma sheet, embedded in a 'vacuum' bounded by rigid walls, is set into motion by an initial imbalance in the magnetic pressure between the left and right vacuum region. The resulting dynamics is a linear oscillation of the plasma sheet which retains its identity. The oscillation is governed by alternating compressions and expansions of the magnetic field trapped in the vacuum regions. In the grid line history, the originally uniformly spaced mesh rapidly concentrates around the discontinuities that form the plasma sheet edges, and are seen to follow the slow waving motion of the sheet.

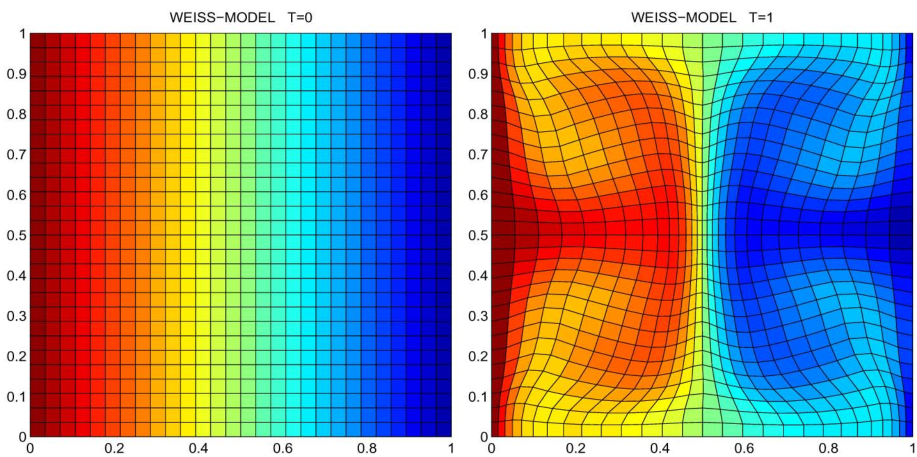

Fig. 2. A 2D kinematic flux expulsion. The left panel shows the initial cartesian mesh and the shading corresponds to the magnetic vector potential. Right panel: an imposed four-cell convection pattern causes the initially straight, uniform field to distort, which is recognized and followed by the 2D grid cell movements.

In Fig. 2, a 2D kinematic flux expulsion is simulated. As in the original work by Weiss [16], a prescribed convection pattern [velocity distribution $\mathbf{V}(\mathbf{x})$] is used in the induction equation for the magnetic field $\mathbf{B}$, namely

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \tag{1}$$

where η indicates a magnetic diffusivity. Starting with an initially uniform field and mesh, the field lines deform in response to perpendicular flow, while parallel flow simply follows the field lines. The diffusion becomes important in regions of strong gradients only. In the figure, the shading corresponds to the magnetic field potential: field lines would be isolines in this plot. One can see that at later times, some field lines are curled up and the grid is distorted to capture the localized strong variations. In a forthcoming publication [18], we plan to discuss these and other problems in detail and compare them with high resolution solutions on static grids, obtained with the Versatile Advection Code [12]. To get similarly accurate solutions on a non-adaptive grid, many more grid points must be used in each space direction.

3 Adaptive Mesh Refinement

One of the best known Adaptive Mesh Refinement (AMR) methods is the one originally developed by Berger [1]. The AMR philosophy is to allow for a user-defined number of grid levels (indexed by l), that have fixed refinement ratios r_l between their spatial step sizes Δx_l (time steps Δt_l), so that

$$r_l \equiv \frac{\Delta x_{l-1}}{\Delta x_l} \equiv \frac{\Delta t_{l-1}}{\Delta t_l}. \tag{2}$$

In a patch-based approach, a refinement criterium applied to all grid patches on level l yields a collection of (scattered) points where a higher resolution is needed. Such a refinement criterium can be based on physical quantities – like a flow divergence or a current – exceeding user specified threshold values. For efficiency, these quantities can be estimated from a low order solution on the grids while the solution method used in the actual time integration can be of higher order. Another refinement criterium often used in AMR implementations is a point-to-point comparison between the conservative variables (e.g. density) obtained by a normal, 'fine' step on grid patch $G_{i,l}$ of resolution $[\Delta x_l, \Delta t_l]$ and by a 'coarse' step of resolution $[2\Delta x_l, 2\Delta t_l]$. By saving previous time steps of the solution on patch $G_{i,l}$, this only involves one coarse and one fine time step advance, which again can be of low order.

In all cases, the points thus flagged for refinement are clustered in groups and surrounded by clouds of 'buffer' points to anticipate the expected spreading of the dynamics over a larger area. All the resulting points are then grouped in rectangles (in 2D), which by subsequent bisections form the most efficient next candidate grid patches on level $l+1$. Extra measures can be taken to ensure a proper nesting of grids: each level $l+1$ grid must be entirely contained in level l grids with at least one grid cell of a level l grid neighbouring its sides. Exceptions are possible near the computational domain edge.

The time integration must proceed in a well-defined order, such that all grids at all levels agree in the physical solution after each time step $t_1^n \rightarrow t_1^n + \Delta t_1$ on the coarsest grid(s) present. A hypothetical sequence of three subsequent timesteps is schematically represented in Fig. 3, showing the possibility for grid

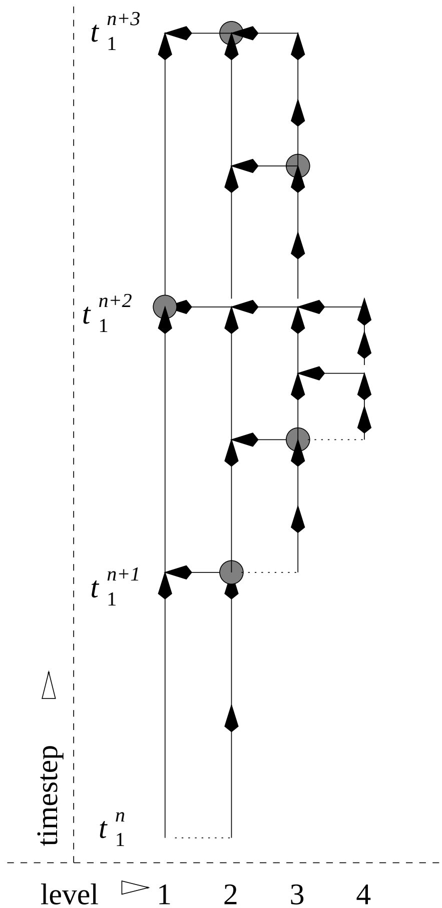

Fig. 3. A hypothetical sequence of three time steps, in an AMR simulation with a maximal allowed nesting level of 4. Vertical 'advance' arrows time-integrate all grids at that particular level; horizontal 'update' arrows pass the more accurate solutions down the level tree; and 'refine' actions (grey circles) may lead to higher level creation or destruction.

level creation and destruction. Starting from time t_1^n, the n-th time step as judged from the level 1 grids, the scheme is traversed from left to right, bottom to top, and with horizontal 'update' arrows preceeding vertical time 'advance' steps. In the first time step shown, the 'advance' of level 1 is followed by two 'advance' steps on level 2, which are the only levels present at time t_1^n. When level 2 has caught up in time with level 1 (both arrived at time t_1^{n+1}), the coarse solution is 'updated' – indicated by a horizontal arrow – with the finer level 2 solution, where available. This process continues in the second and third time step. However, the sequence is complicated by allowing for newly created (or destroyed) grids on levels $l + 1$ up to a maximally allowed level (taken as 4 in Fig. 3). This happens at the locations marked by the grey circles: the grids at that level are unchanged, but all higher level grids can suddenly appear (after the first and halfway in the second time step), disappear (after the second timestep), or simply get rearranged or be left unchanged (halfway in the third time step). The criterion for when a specific 'refine' action (grey circle) takes place is simple: when k timesteps are taken on a certain level, it is evaluated for refinement ($k = 2$ in Fig. 3). However, the maximally allowed finest level 4 is never refined, and a downward cycle of update steps should not lead to duplicate refinements.

We have a Fortran 90 implementation of an AMR scheme, usable for the Euler equations in two space dimensions. The integrator is a finite volume, conservative Flux Corrected Transport [2] algorithm. It should be clear that the update steps mentioned above also involve 'fix' operations at boundaries between level l and level $l + 1$ grids: to ensure global conservation, the fluxes as obtained by the addition of the fine cell fluxes that make up one coarse level l cell replace the fluxes obtained from the level l cells that are covered by a finer mesh.

As an example calculation, we show in Fig. 4 a two dimensional generalization of Harten's [6] shock tube problem, where the bottom right hand corner of a rectangular domain has different constant state variables than those in the rest of the domain. The simulation allows for four grid levels, which are automatically created at time $t = 0$ and nicely follow the discontinuities present. At a slightly later time, the discontinuities in each direction develop locally in combinations of rarefactions, shocks and contact discontinuities. Note in particular how the hierarchically nested grid structure rearranges itself to capture the evolving flow features. Thereby, grids can merge, disappear, shrink or grow in size as imposed by the physics.

We will further translate the Fortran 90 code into LASY syntax [13], so that both 1D, 2D, and 3D applications can be run with the same source code. The coupling with the Versatile Advection Code will open up the possibility to apply the AMR technique to any set of (near) conservation laws, like the (resistive) MHD equations.

4 Conclusions and Outlook

This progress report summarizes our continuing efforts to evaluate and exploit grid adaptation schemes in challenging magnetohydrodynamic computations. We demonstrated the workings of two different approaches, r-refinement and h-refinement, for some idealized model problems. The application of MOL-techniques to MHD problems, in particular in 2D and 3D versions, is a novel research area which should be pursued further along the lines indicated in this manuscript. The more established AMR technique has been applied in MHD problems recently, e.g. see [10,9,5], but a dimension independent implementation in combination with a choice in the actual set of conservation laws to solve, will become feasible for the first time when we finish our efforts.

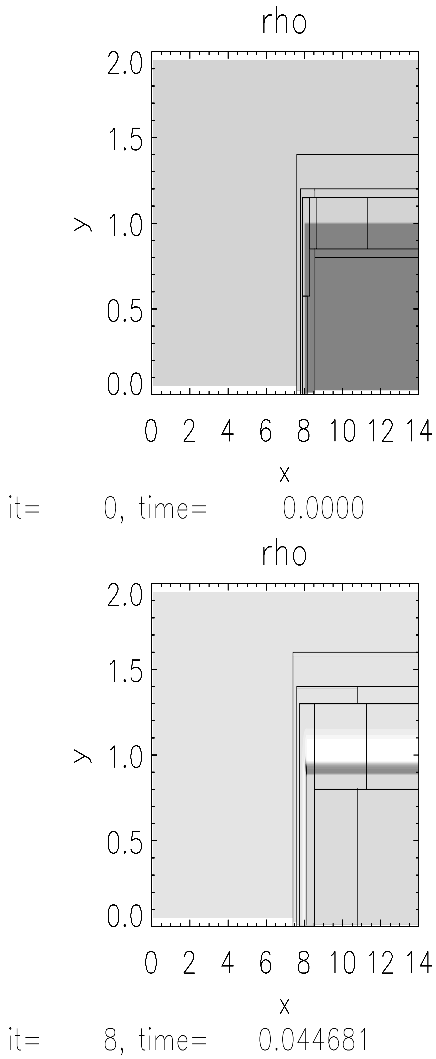

Fig. 4. A 2D hydrodynamic shock tube problem. We show density ρ (left) and y-velocity (right) at times $t = 0.0$ (top) and eight CFL-limited timesteps later (bottom). Four refinement levels, with $r_l = 2, l = 2, 3, 4$, automatically form a nested structure that follows the initial discontinuity. Level 1 is the full square, and the thin lines are the boundaries of the grid patches, which are nested into that. As time evolves, the grids adjust dynamically: note how at $t = 0.0$, five grids on level 4 were formed, which have merged and broadened into three level 4 grids at the last time shown. The underlying level 2 and 3 grids also changed, always ensuring a proper nesting.

The number of applications that become amenable to large-scale numerical simulations promise to keep us and other physicists alike busy for the years to come.

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