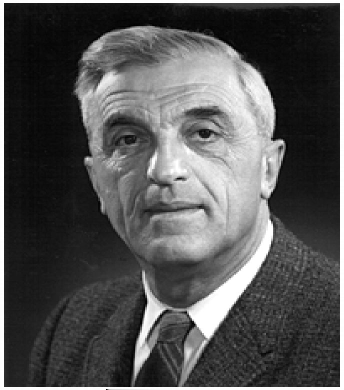
UMC Utrecht

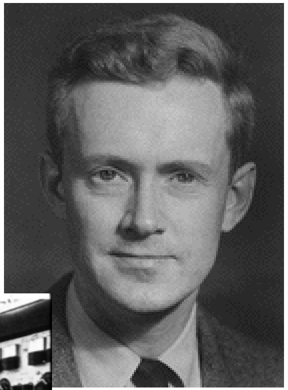
Accelerating the MRI exam

Alessandro Sbrizzi
University Medical Center, Utrecht

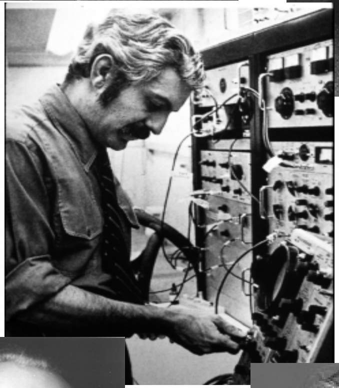
Brief history of MRI

1946 Felix Bloch and Edward Purcell independently discover the magnetic resonance phenomena (Nobel Prize in 1952)

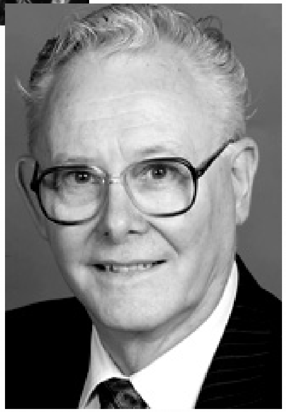




1971 Raymond Damadian: nuclear magnetic relaxation times of tissues and tumors differed→Clinical Application



1973/1974 Paul C. Lauterbur and Peter Mansfield: spatial localization through Gradient Fields and Fourier Transform→Imaging (Nobel Prize in 2003S)



Some applications

Tumors (MRI and Spectroscopy)

Multiple Sclerosis

Ischemic Stroke

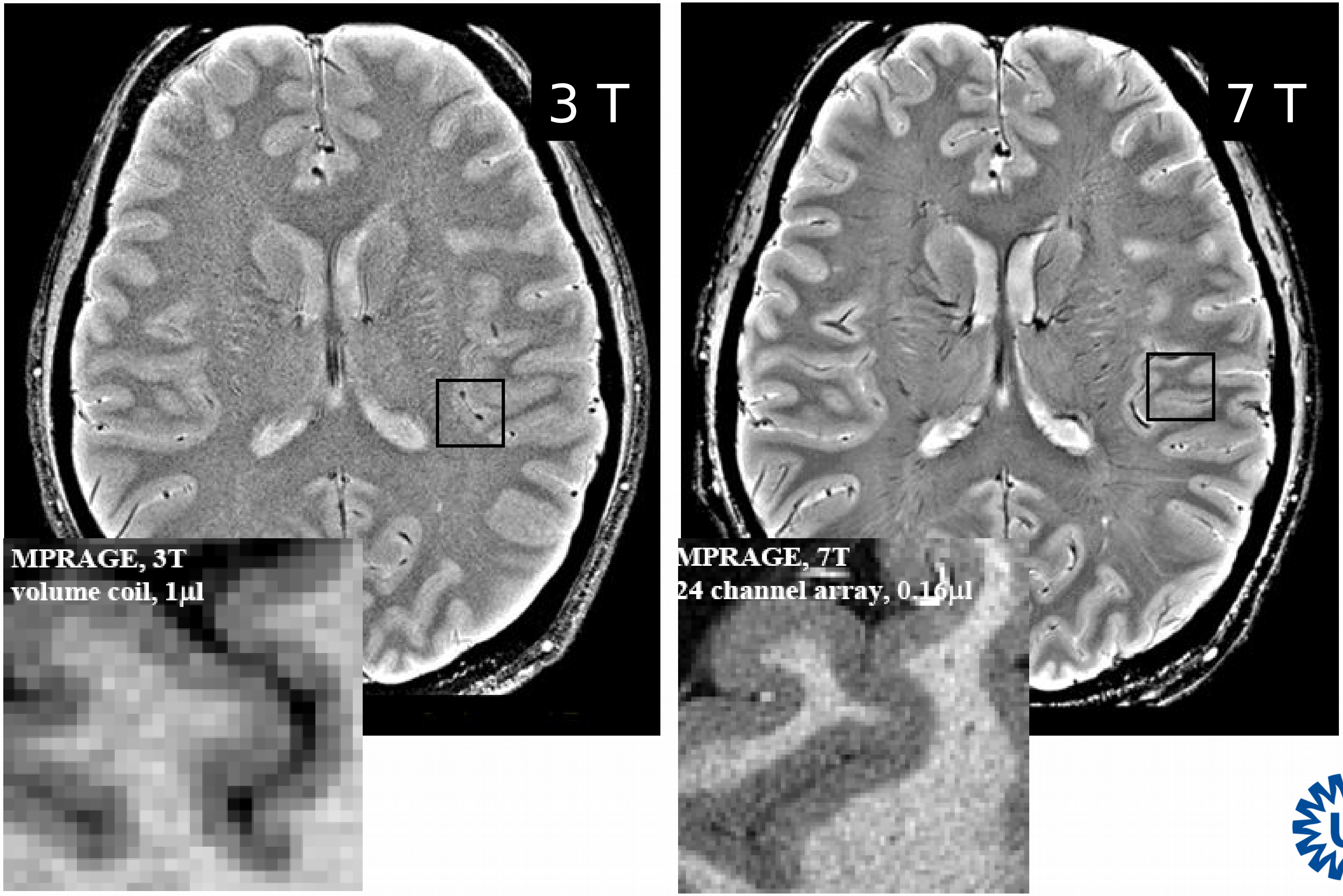
Stenosis or aneurysms (MR Angiography)

Brain Functioning (fMRI)

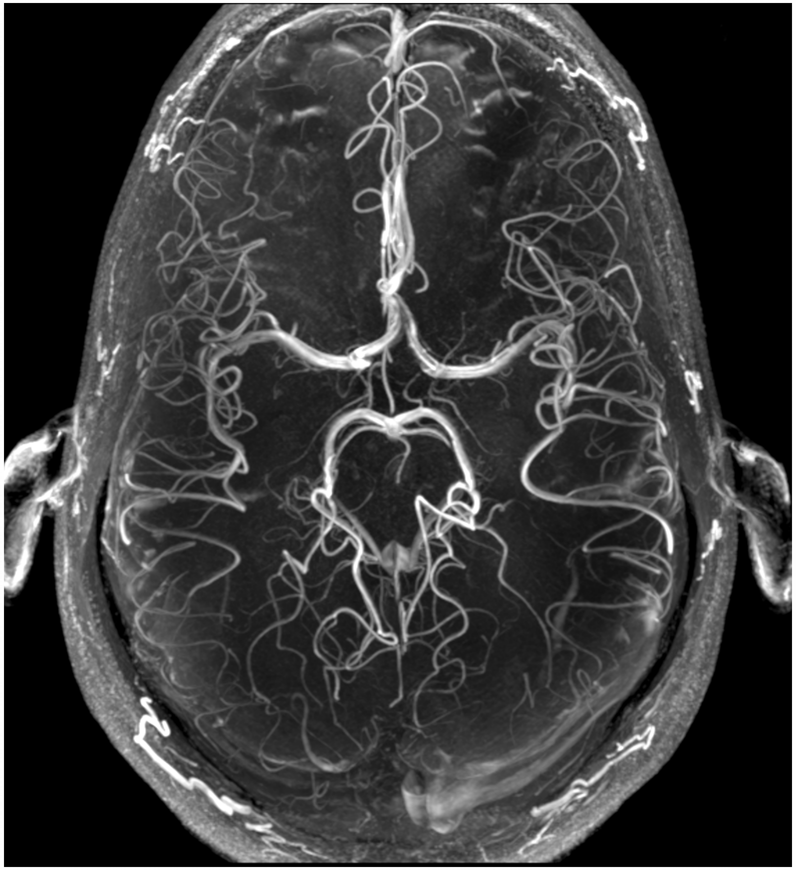
MR guided surgery (High Focused Ultrasound, MRI-Linac)

…

Towards higher fields MRI



Brain vasculature at 7T MRI



UMC Utrecht high-field MRI

About 40 researchers in total

Leader: Prof. Peter Luijten

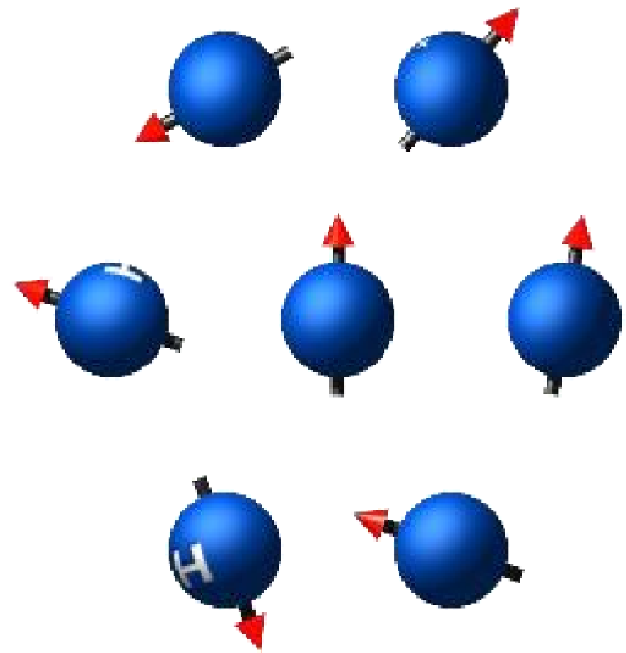
Embedded in the large Imaging Division at UMC Utrecht (about 100 MRI researchers)

Actitivity: engineering, architecture design, numerical simulations, experiment design, in-vivo applications, medical feed-back from clinics, diagnosis, treatment monitoring, image-guided-therapies

Physical Principles of MRI

Hydrogen protons (spins) behave like tiny rotating magnets

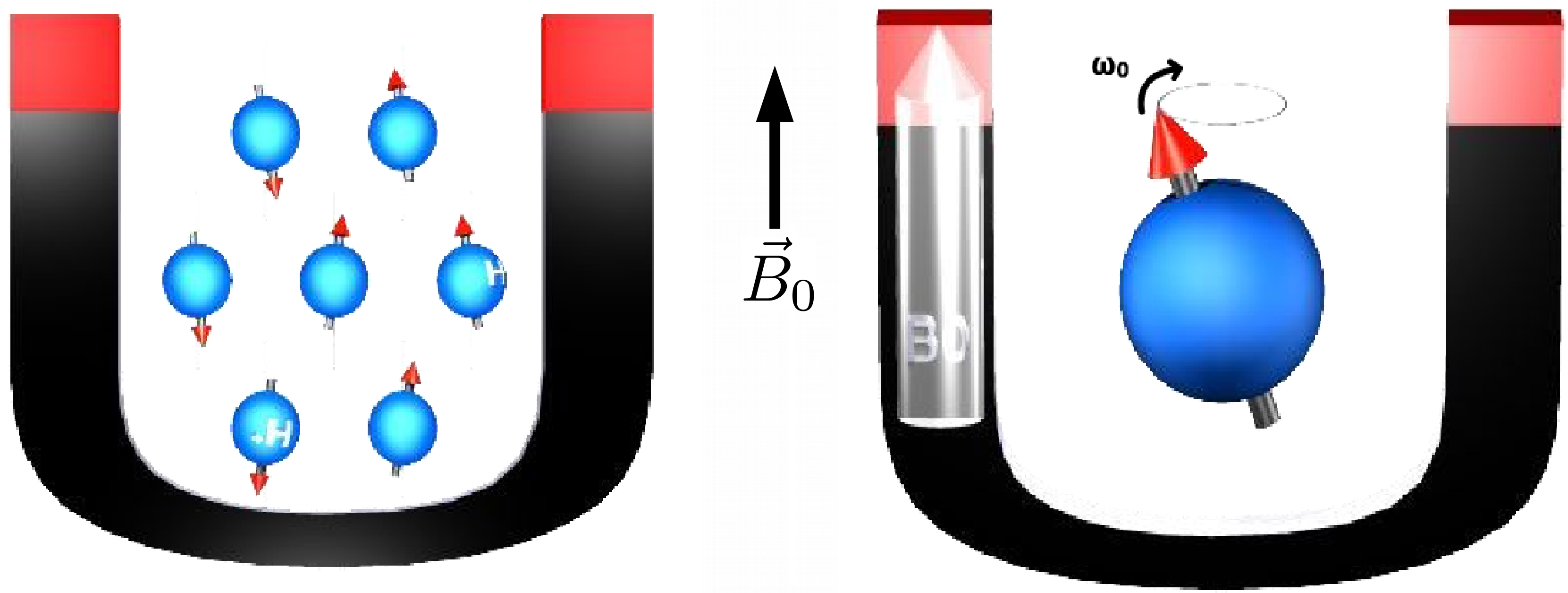
Normally they are randomly distributed



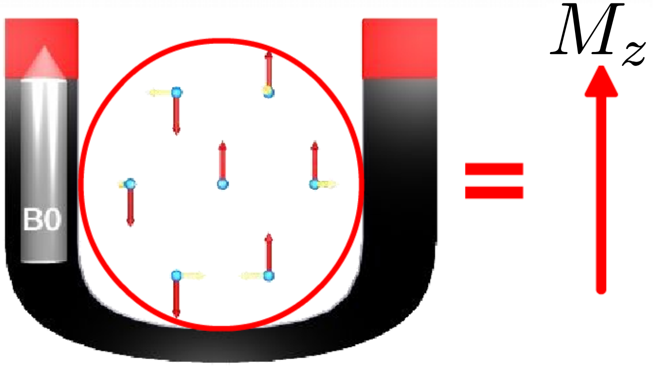
Physical Principles of MRI

Within a large external magnetic field (called B_0), nuclear spins align

Spins precess about the axis of the B_0 field at Larmor frequency

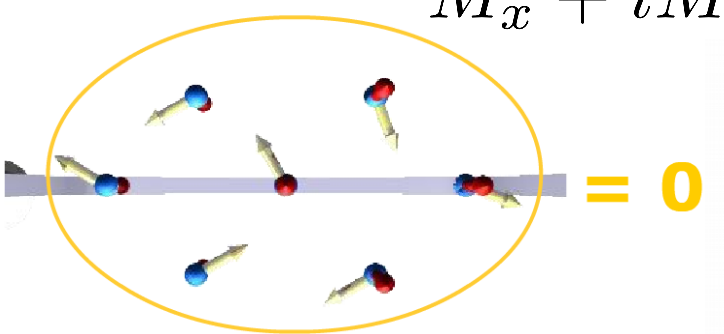


Physical Principles of MRI



Net magnetization:

$$\vec{M} \equiv \frac{1}{V} \sum \vec{\mu}$$



Magnetization: $\vec{M} = (M_x, M_y, M_z)^T$

Longitudinal magnetization: M_z

Transverse magnetization: $M_\perp \equiv M_x + iM_y$

Bloch Equation in the *rotating frame*

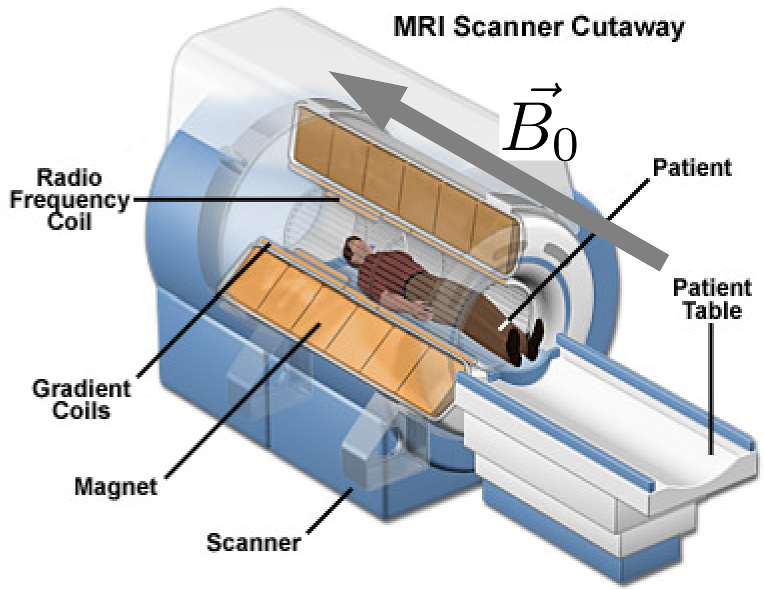
$$\frac{d\vec{M}}{dt} = \begin{pmatrix} -\frac{1}{T_2} & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1,y} \\ -\gamma\vec{G}\cdot\vec{r} & -\frac{1}{T_2} & \gamma B_{1,x} \\ \gamma B^{1,y} & -\gamma B_{1,x} & -\frac{1}{T_1} \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{pmatrix}$$

with initial condition $\vec{M}(0) = (0, 0, M_0)^T$.

$$\begin{pmatrix} 0 & \gamma\vec{G}\cdot\vec{r} & -\gamma B_{1,y} \\ -\gamma\vec{G}\cdot\vec{r} & 0 & \gamma B_{1,x} \\ \gamma B^{1,y} & -\gamma B_{1,x} & 0 \end{pmatrix} + \begin{pmatrix} -\frac{1}{T_2} & 0 & 0 \\ 0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{pmatrix}$$

rotation — Decay (relaxation)

The MRI Scanner

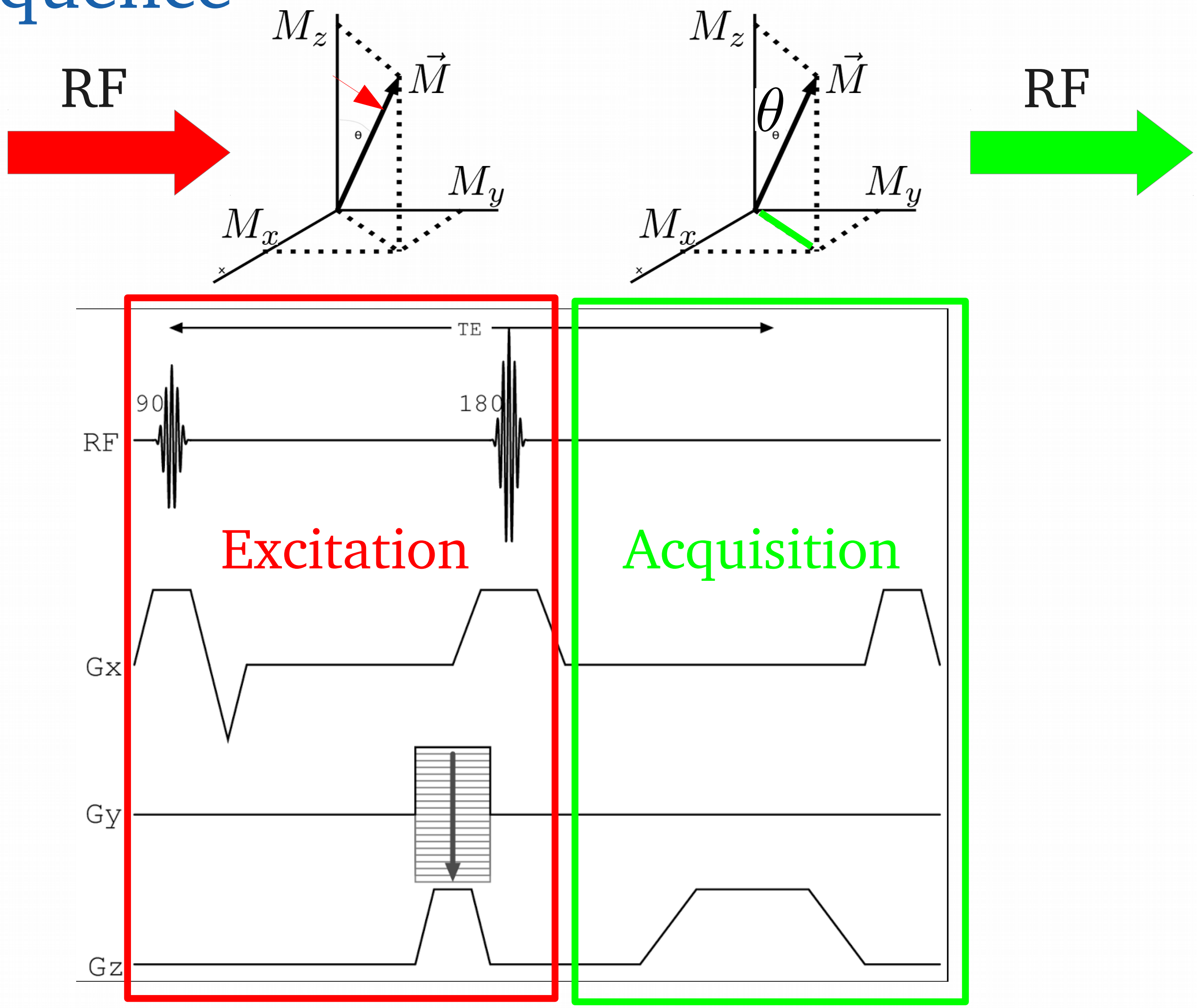


The main, static magnetic field B_0 (to align the spins)

The Radio Frequency field B_1 (to tip down the spins)

The Gradient field, G (spatial localization)

MRI sequence



The image equation

$$\sigma(t) \propto \int_{\mathbb{V}} M_{\perp}(\vec{r}) e^{-i\gamma \int_0^t \vec{G}(\tau)\cdot\vec{r}d\tau} d\vec{r}$$

- σ is the measured signal
- $M_{\perp}$ is the transverse magnetization

Set $\vec{k}(t) \equiv \frac{\gamma}{2\pi} \int_0^t \vec{G}(\tau) d\tau$, then:

$$\sigma(\vec{k}) = \int_{\mathbb{V}} M_{\perp}(\vec{r}) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$

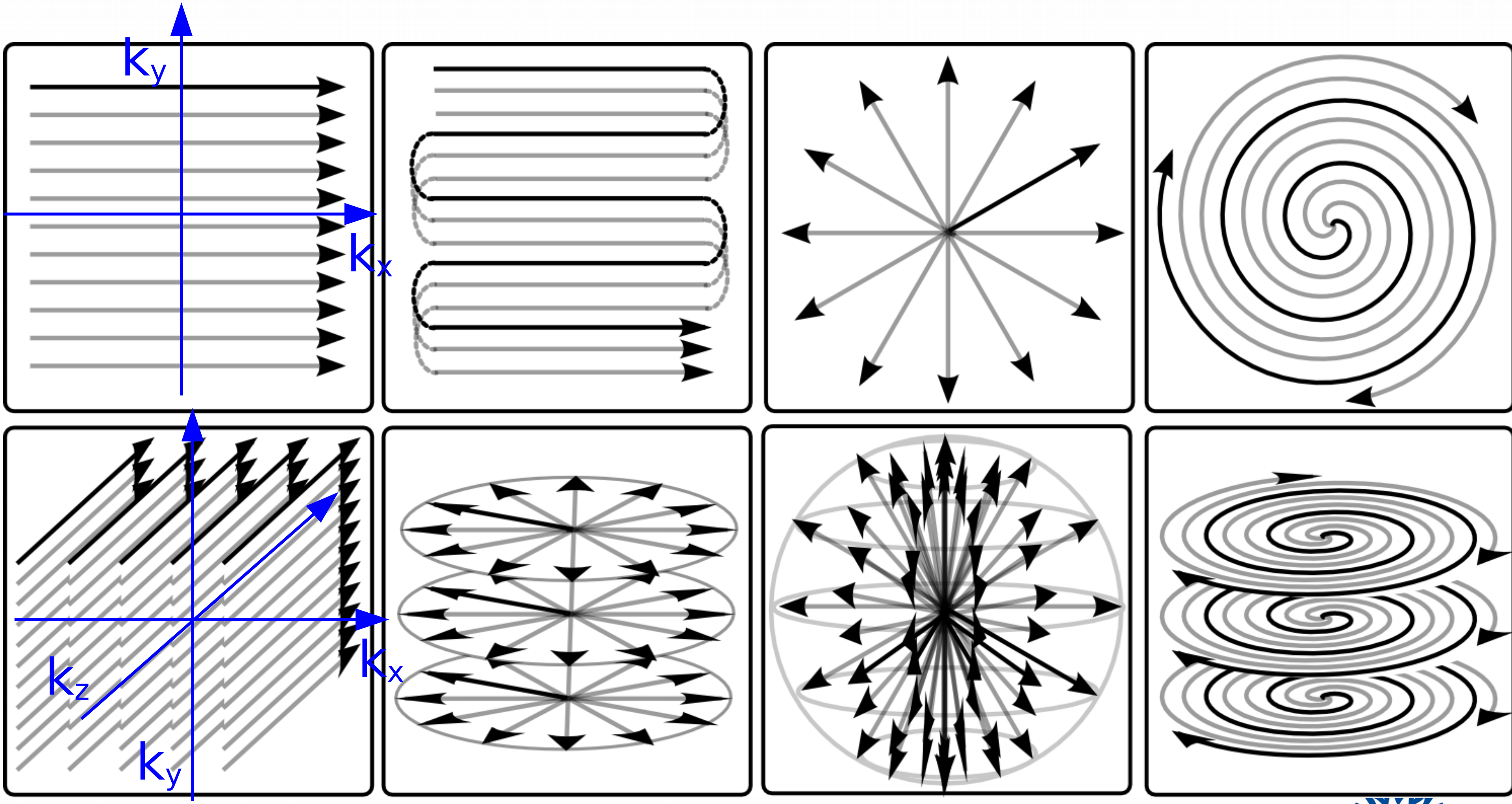
The image equation

$$\sigma(\vec{k}) = \int_{\mathbb{V}} M_{\perp}(\vec{r}) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$

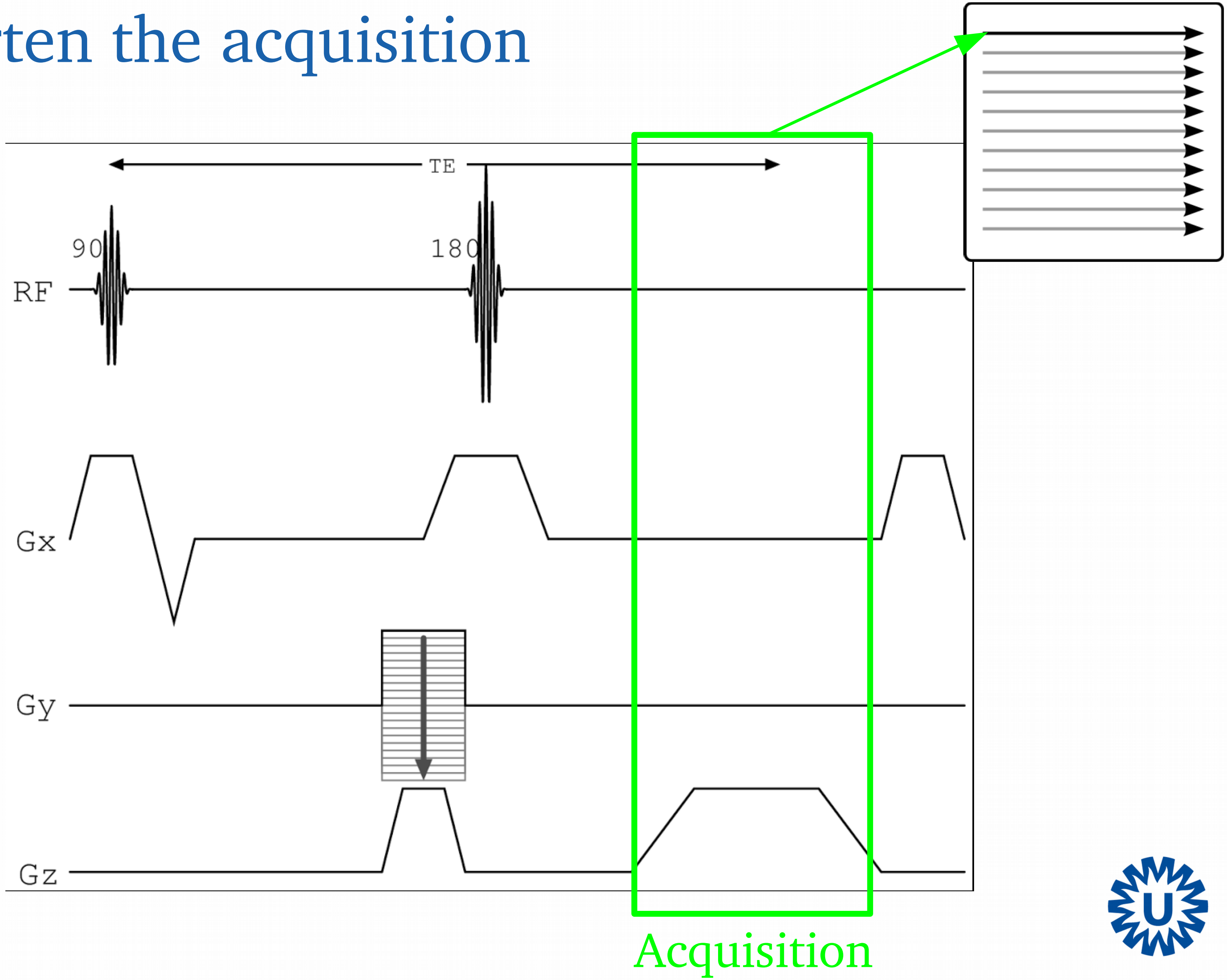
$$\Rightarrow M_{\perp}(\vec{r}) = \int_{\mathbb{K}} \sigma(\vec{k}) e^{i2\pi\vec{k}\cdot\vec{r}} d\vec{k}$$

- $\vec{k} = \frac{\gamma}{2\pi} \int_0^t \vec{G}(\tau) d\tau$ can be seen as a *spatial frequency*.
- The data is acquired along a trajectory in the k-space
- Nyquist criterion for k-space sampling

Traveling in the K-space

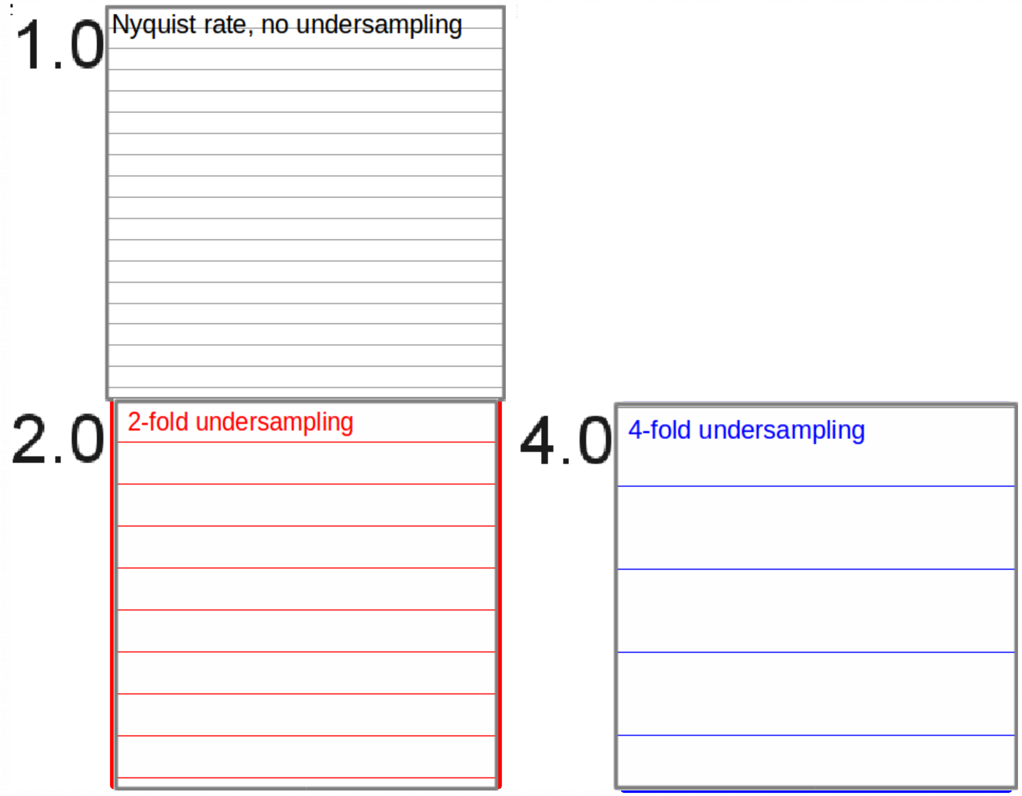


Shorten the acquisition



Fold-over effect

Undersampling the k-space in the vertical direction: shorter scan time, but:



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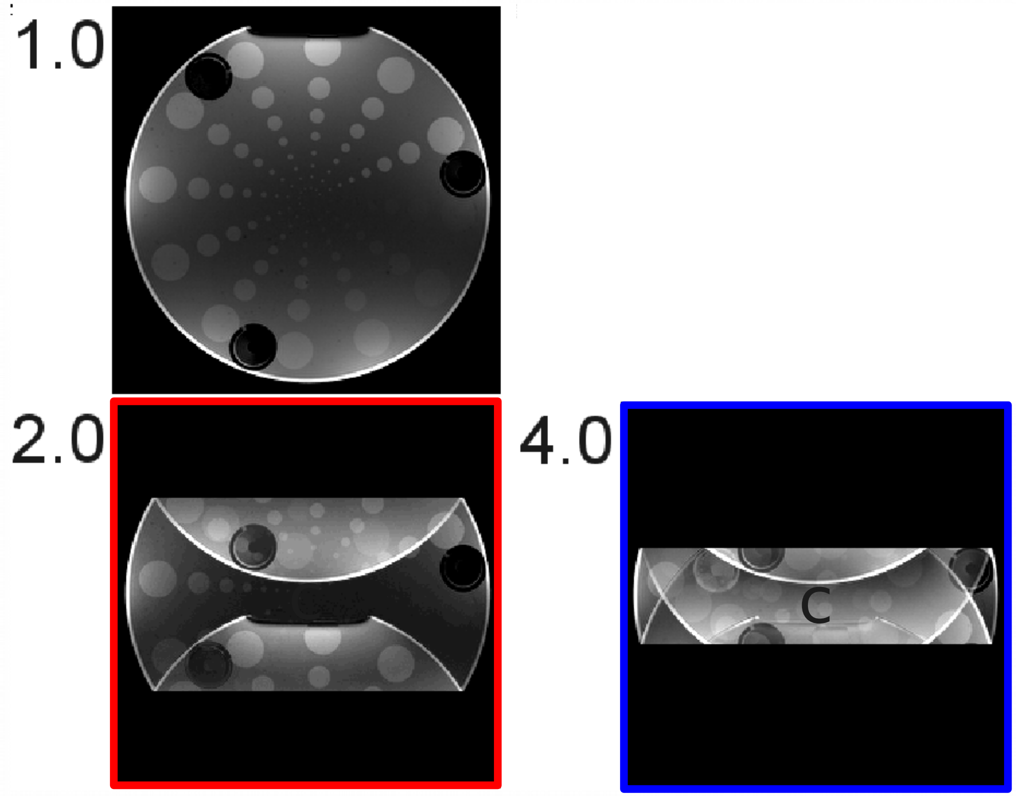
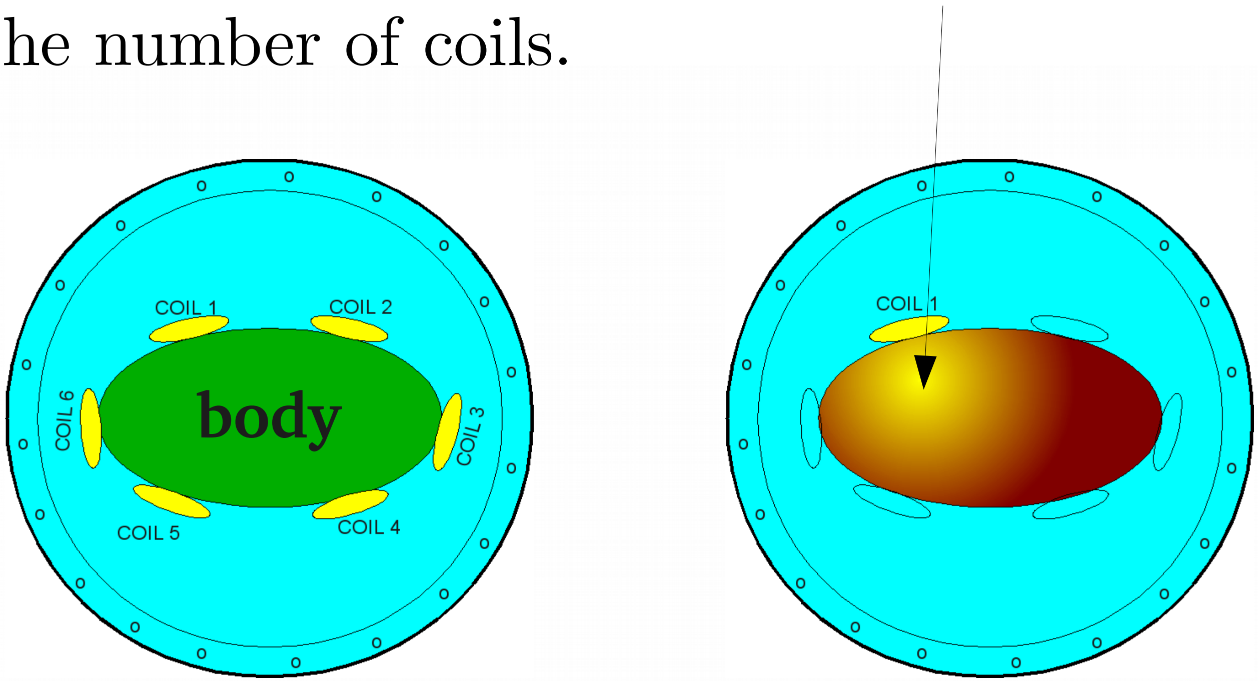
Undersampling the k-space in the vertical direction: shorter scan time, but:



For a regular R-fold undersampling:
$$z_m = \frac{1}{R} \sum_{j=0}^{R-1} \rho_{m+jN/R}$$

Receive coil arrays

Since 1999: multiple receive coils, each with own spatial dependence, the sensitivity, $S^p(\vec{r})$, $p = 1, \ldots, P$. P is the number of coils.

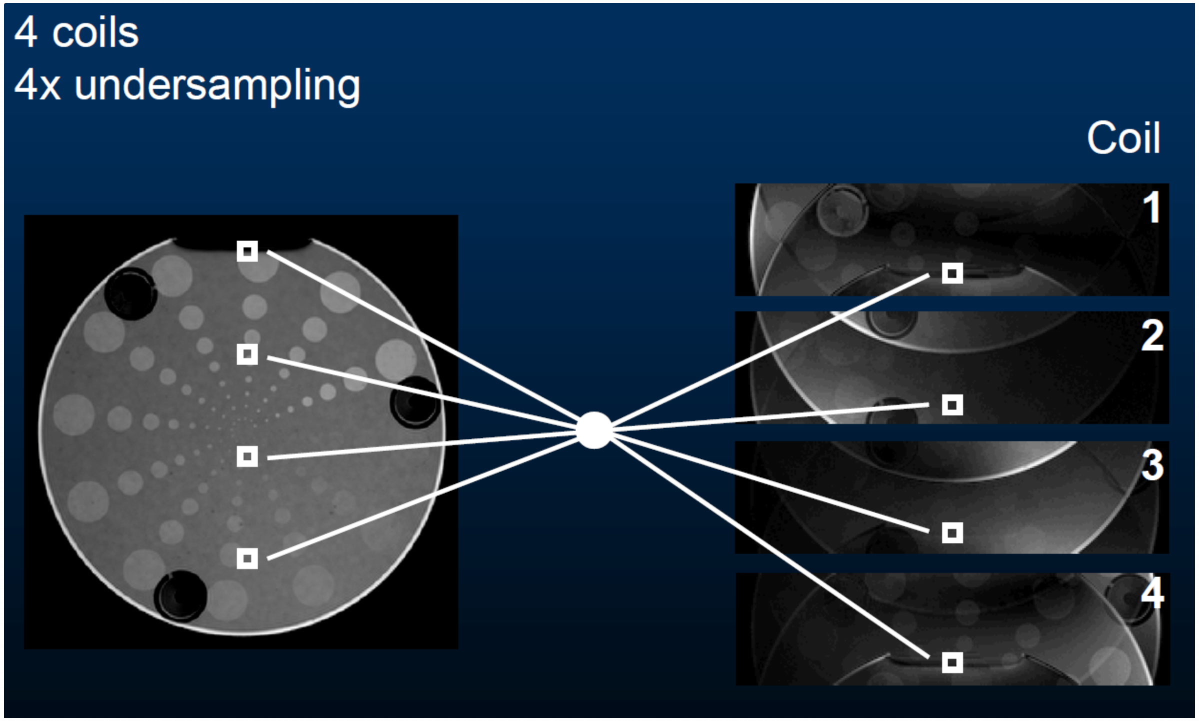


Cross section of the scanner bore

Parallel imaging

Each coil 'sees' a own version, (z^p_m), of the folded true image

$$z^p_m = \sum_{j=0}^{R-1} S^p_{m+jN/R}\,\rho_{m+jN/R}$$



Parallel imaging

Each coil 'sees' a own version, (z_m^p), of the folded true image

$$z_m^p = \sum_{j=0}^{R-1} S^p_{m+jN/R}\rho_{m+jN/R}$$

$$\begin{bmatrix} z_m^1 \\ z_m^2 \\ \vdots \\ z_m^P \end{bmatrix} = \begin{bmatrix} S_m^1 & S^1_{m+N/R} & \cdots & S^1_{m+N(R-1)/R} \\ S_m^2 & S^2_{m+N/R} & \cdots & S^2_{m+N(R-1)/R} \\ \vdots & \vdots & \ddots & \vdots \\ S_m^P & S^P_{m+N/R} & \cdots & S^P_{m+N(R-1)/R} \end{bmatrix} \begin{bmatrix} \rho_m \\ \rho_{m+N/R} \\ \vdots \\ \rho_{m+N(R-1)/R} \end{bmatrix}$$

Pruessmann et al, Magn Reson Med 1999

Cartesian SENSE (1999)

1. simultaneously collect the under-sampled data for each coil
2. apply FFT^{-1} and obtain the folded images
3. for each voxel in the reduced Field of View solve the system (unfolding).

Pruessmann et al, Magn Reson Med 1999

Generalized SENSE

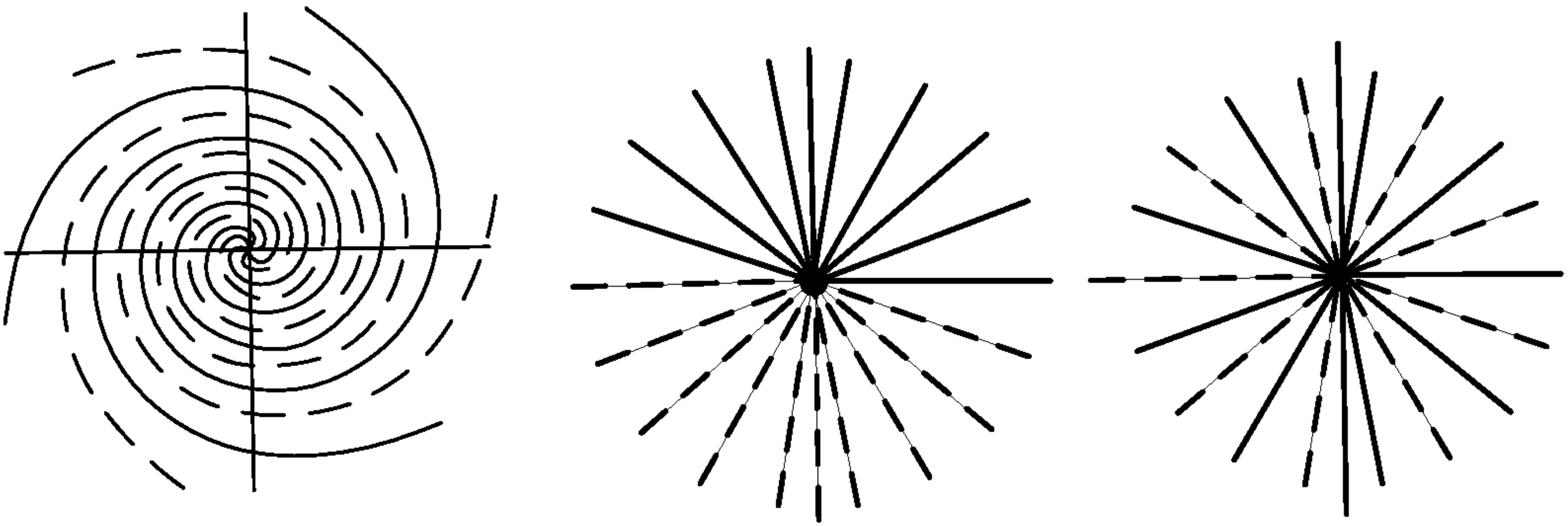
The image equation for the p-th coil:

$$\sigma_p(\vec{k}) = \int_{\mathbb{V}} S_p(\vec{r}) M_{\perp}(\vec{r}) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$

$\sigma_p(\vec{k})$ is the signal from p-th coil, $S_p(\vec{r})$ the sensitivity function.

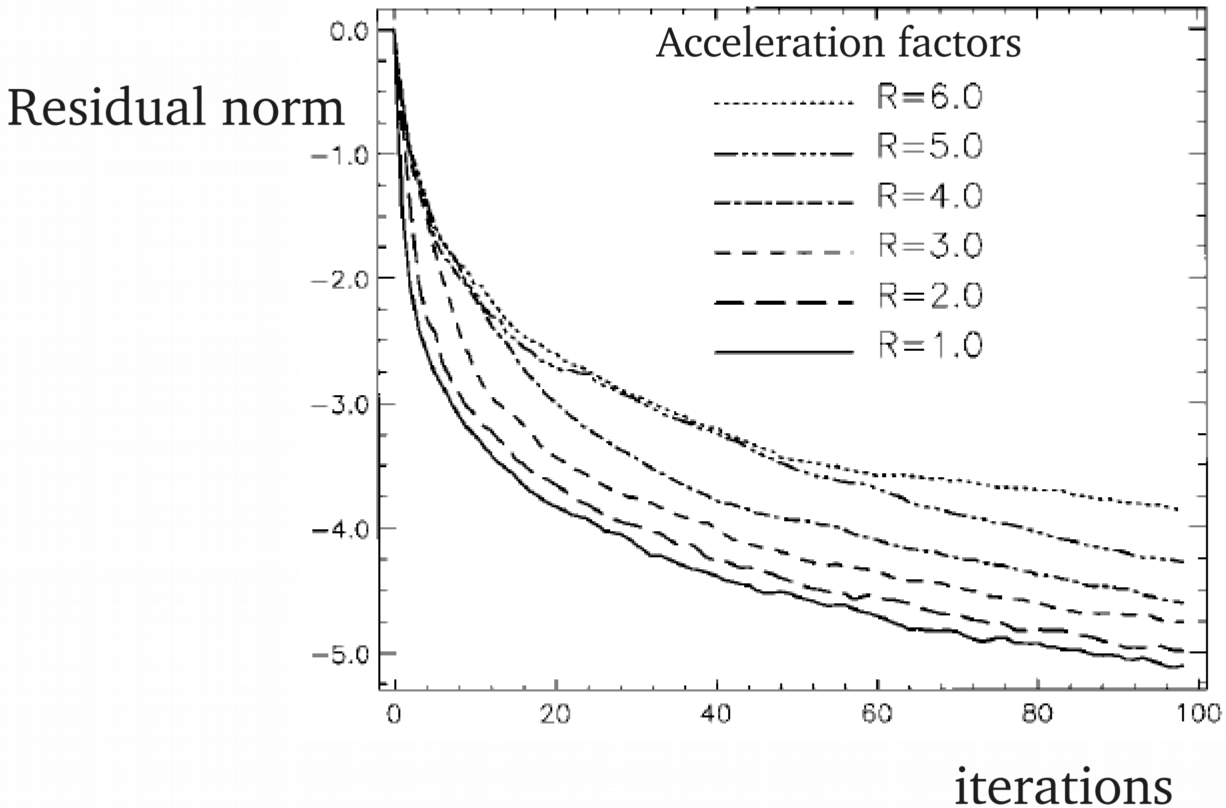
Solve the integral equation at once.

Generalized SENSE



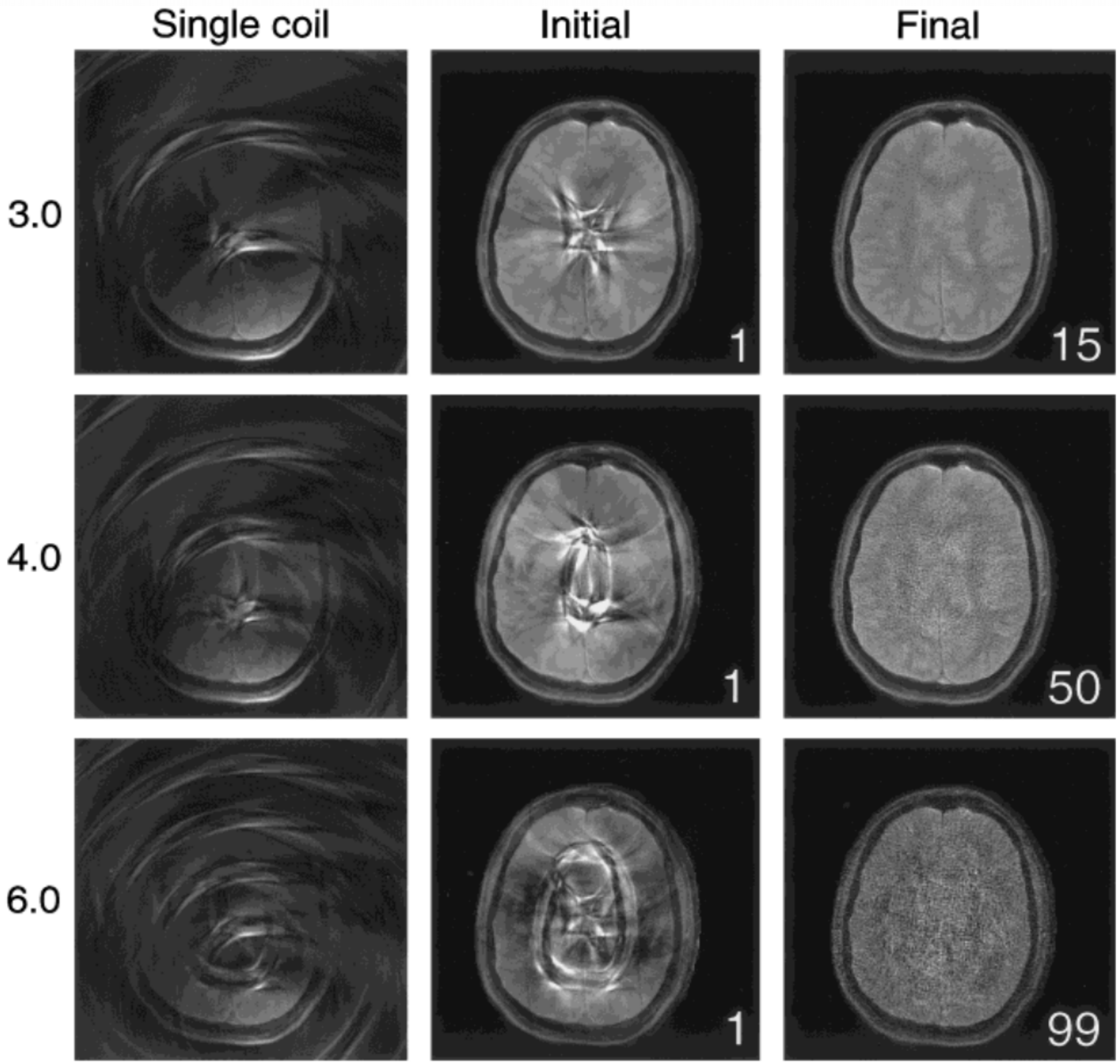
- Acquisition scheme does no longer need to be cartesian.
- Efficient k-space trajectory and undersampling-scheme.
- Non-uniform FFT
- Iterative recon methods.
- Preconditioning.

Generalized SENSE. Spiral k-space sampling



Pruessmann et al, Magn Reson Med 2001

Generalized SENSE



Pruessmann et al, Magn Reson Med 2001

Compressed Sensing MRI (since 2007)

- We can reconstruct a $k-$*sparse* signal $\vec{x} \in \mathbb{R}^N$ from $M = \mathcal{O}(k \log(N))$ measurements $\vec{b} = \Phi\vec{x}$.
- The $M \times N$ matrix Φ has to satisfy certain properties (RIP).
- A random Gaussian matrix works.

Compressed Sensing MRI (since 2007)

- If the signal $\vec{x}$ allows a sparse representation $\vec{x} = \Psi^T \vec{z}$, we reconstruct from measurements $\vec{b} = \Phi\Psi^T \vec{z}$.
- RIP is hard to verify for a given matrix
- At least, Φ and Ψ must be *incoherent*, that is,

$$\mu(\Phi, \Psi) \equiv \max_{i,j} |(\Phi\Psi^T)_{ij}|$$

must be small.

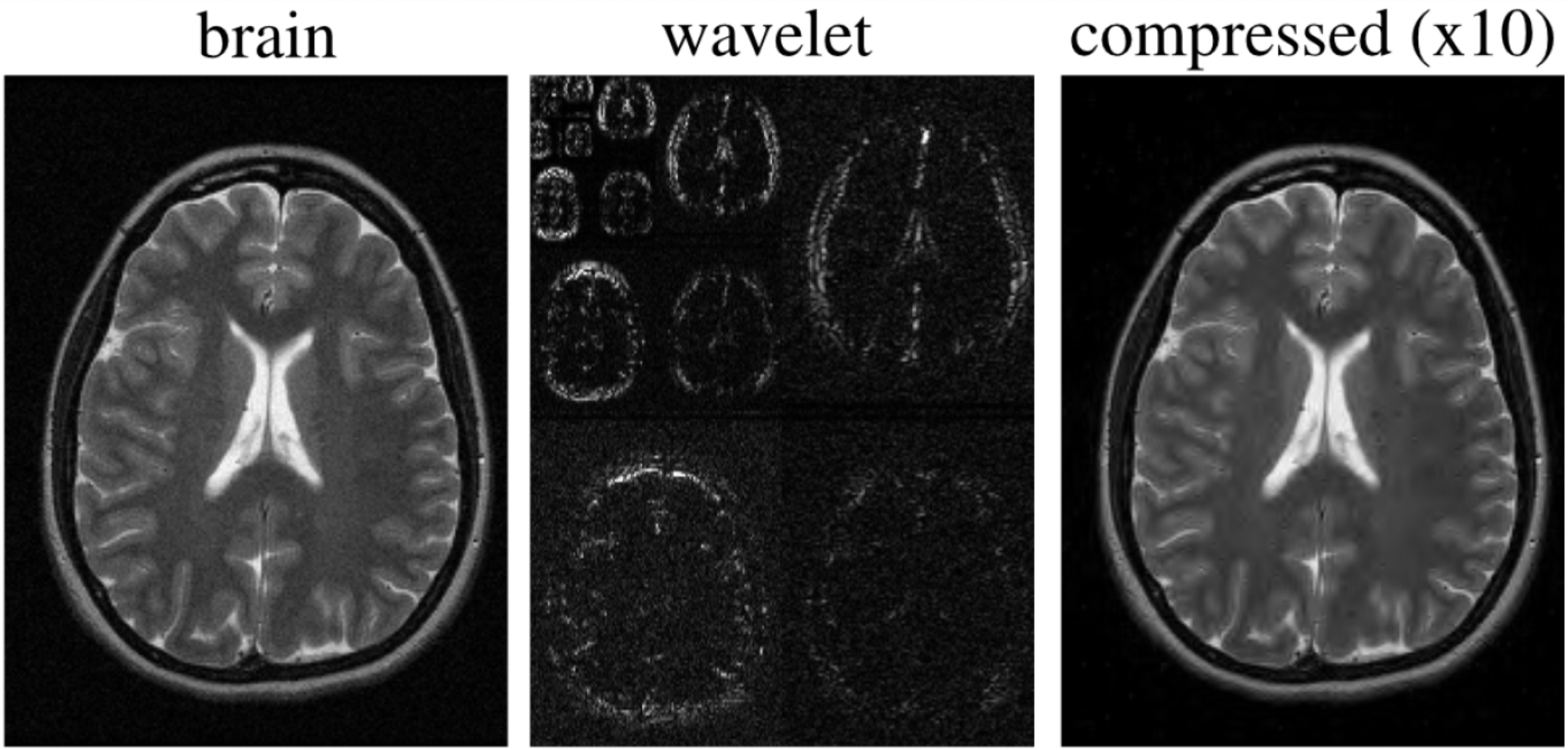
Compressed Sensing MRI (since 2007)

We can retrieve the solution by solving a BPDN problem

$$\min_{\vec{z}} \|\vec{z}\|_1 \qquad \text{s.t.} \qquad \|\Phi\Psi^T\vec{z} - \vec{b}\| \leq \epsilon,$$

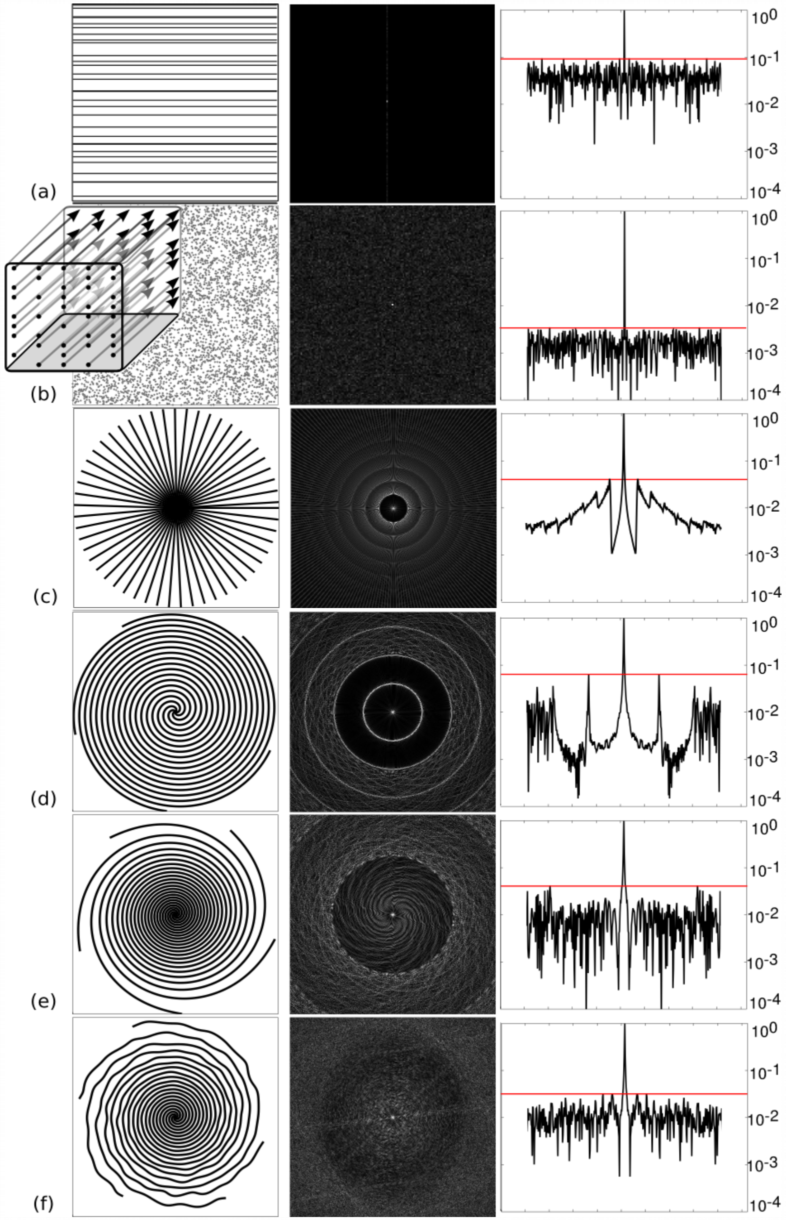
where ϵ is the noise level.

Sparsity in MRI



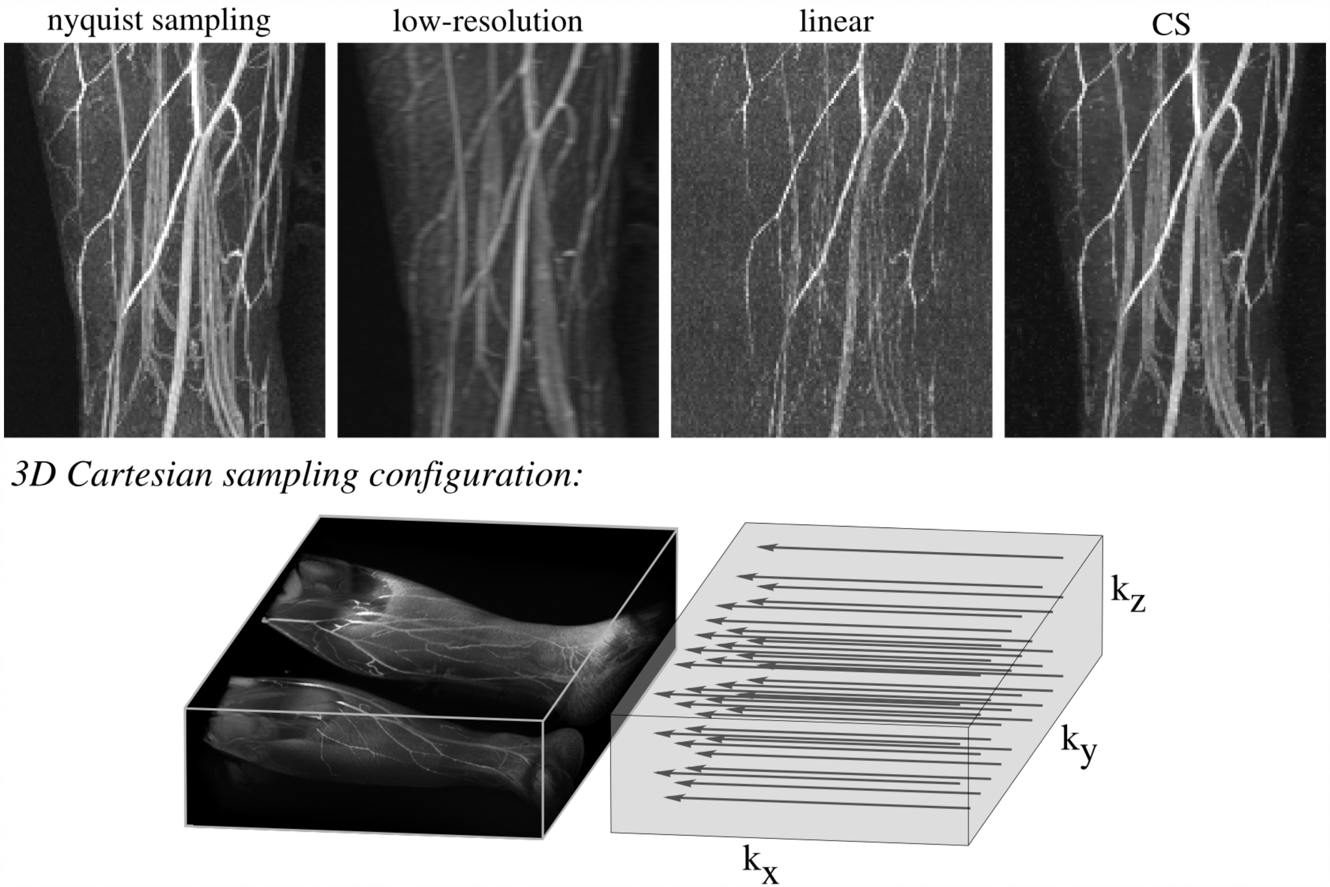
Lustig et al, Magn Reson Med 2007

Sampling



Lustig et al, Magn Reson Med 2007

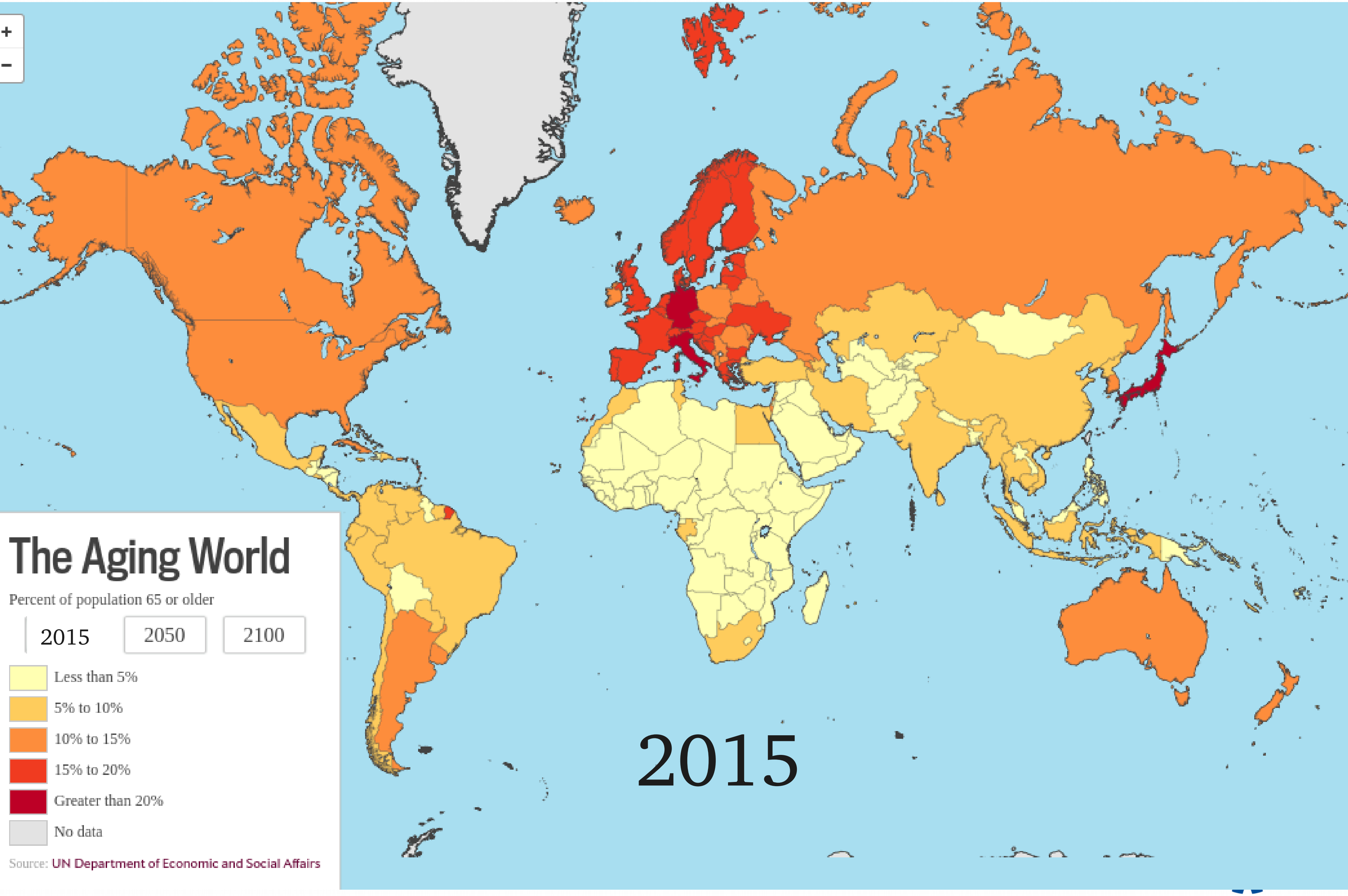
Example CS-MRI



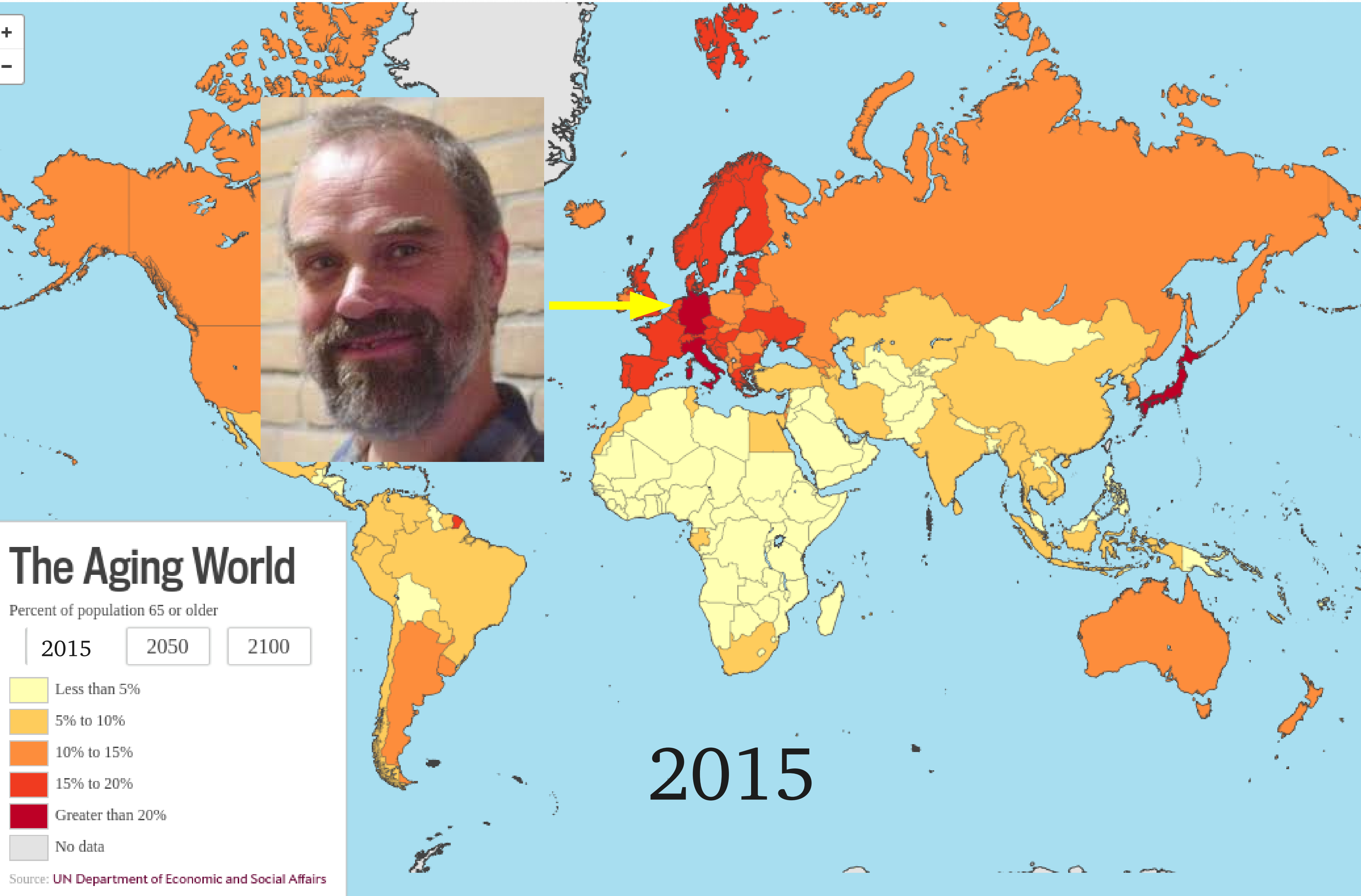
Lustig et al, Magn Reson Med 2007

Which future for MRI?

Collaboration with Dr. Nico van den Berg



+
−
The Aging World
Percent of population 65 or older
2015
2050
2100
Less than 5%
5% to 10%
10% to 15%
15% to 20%
Greater than 20%
No data
Source: UN Department of Economic and Social Affairs
2015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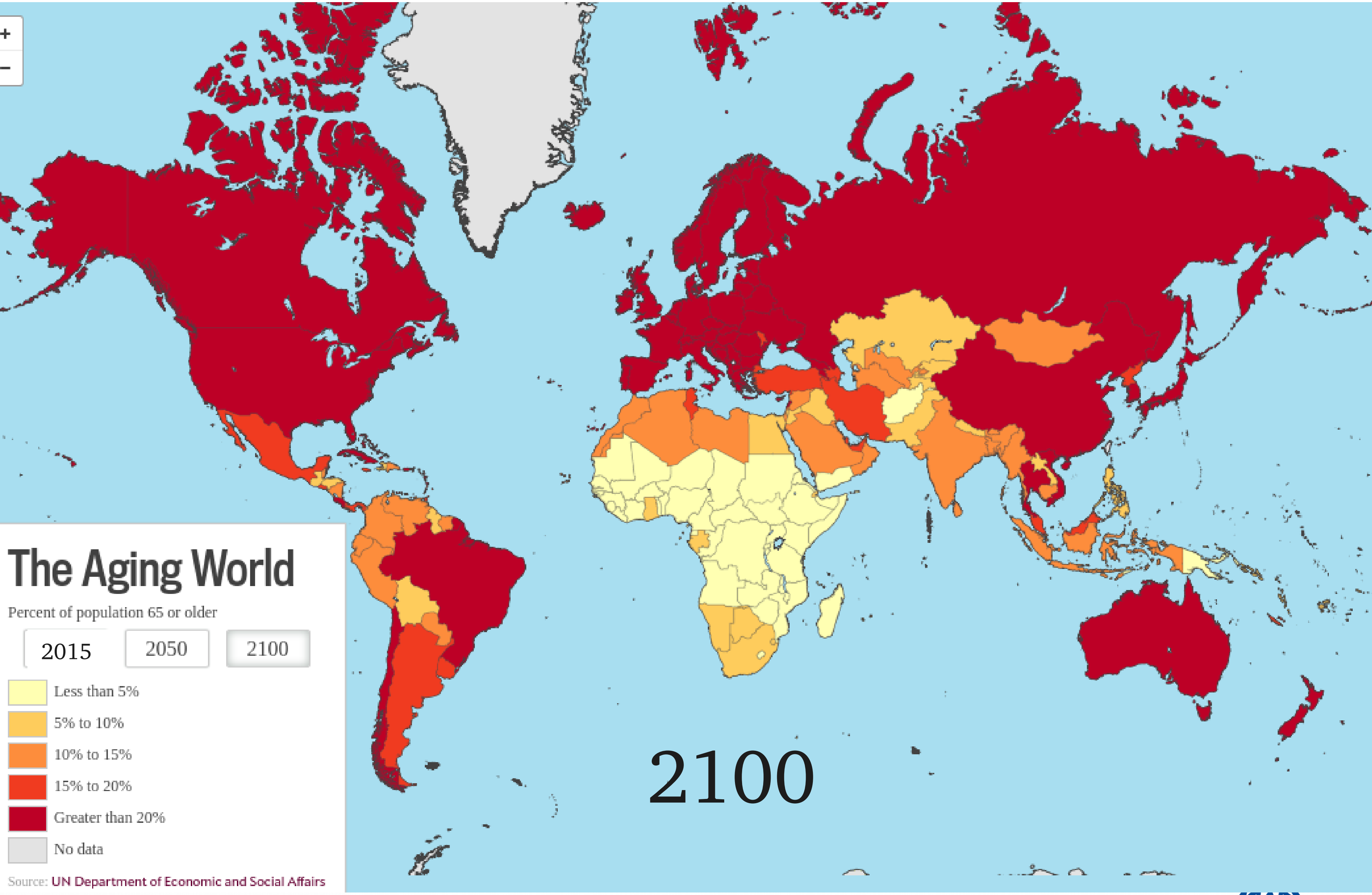
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Source: UN Department of Economic and Social Affairs
2100

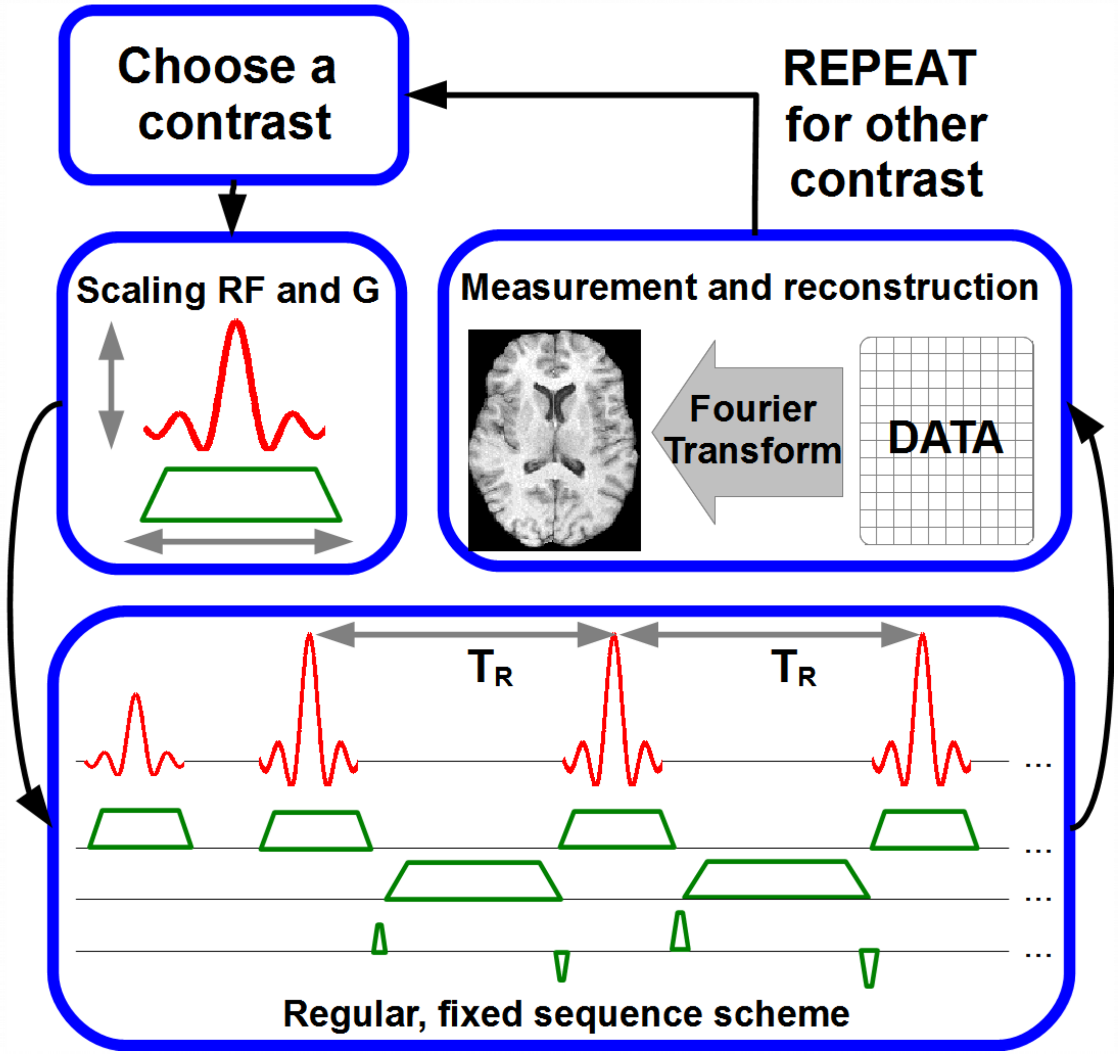
Which future for MRI?

- Typical MRI exam about 30 to 45 minutes
- This has not changed since introduction
- Aging population means increase of MRI exams

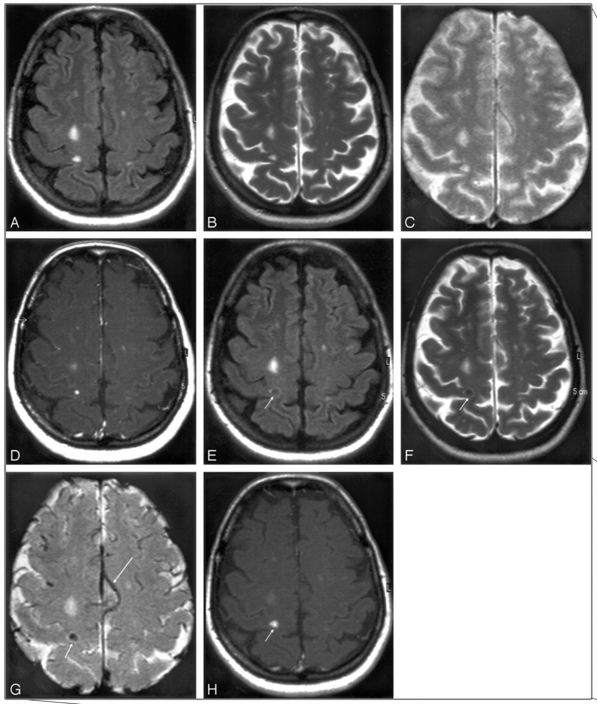
B_0

- Health care costs already under pressure
- Could MRI exam be max 10 minutes long?

Standard MRI exam: still (too) long…



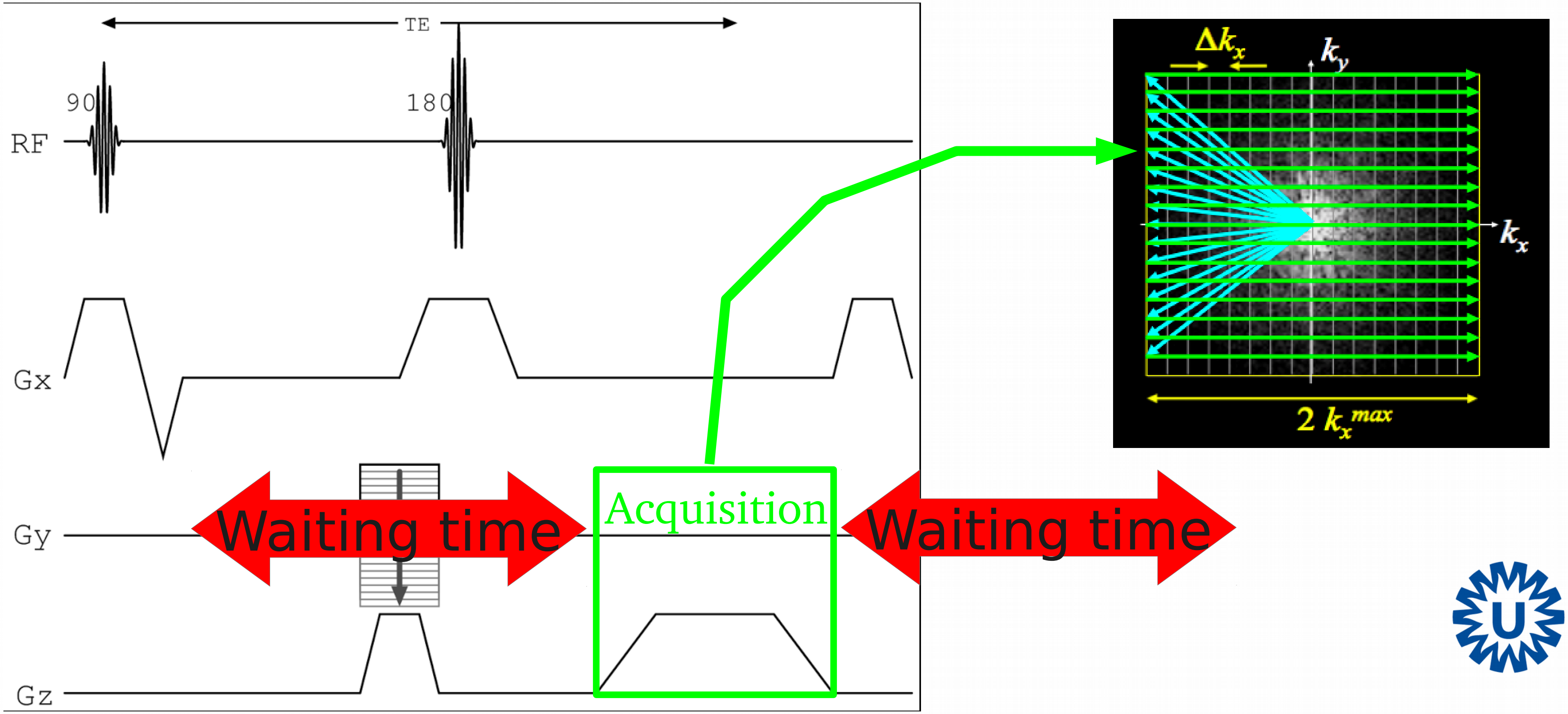
Standard MRI exam: still (too) long…



Time-(in)efficiency of MRI

$$\sigma(\vec{k}(t)) \propto \int_{\mathbb{V}} M_{\perp}(\vec{r}) e^{-2\pi i \vec{k}(t)\cdot\vec{r}} d\vec{r}$$

Problem: $M_{\perp}$ has to be brought to same value before each acquisition interval



Transient states

$$\sigma(t) \propto \int_{\mathbb{V}} M_{\perp}(\vec{r}, t) e^{-2\pi i \vec{k}(t) \cdot \vec{r}} d\vec{r}$$

No longer Fourier transform relationship between signal and magnetization

- flexibility in acquisition
- Reconstruction gets more complicated

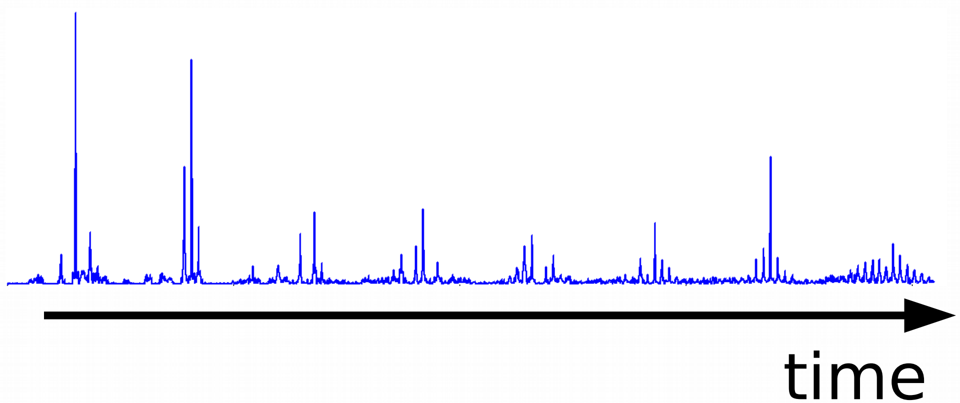
Time-domain inversion (MR-STAT)

Parameters

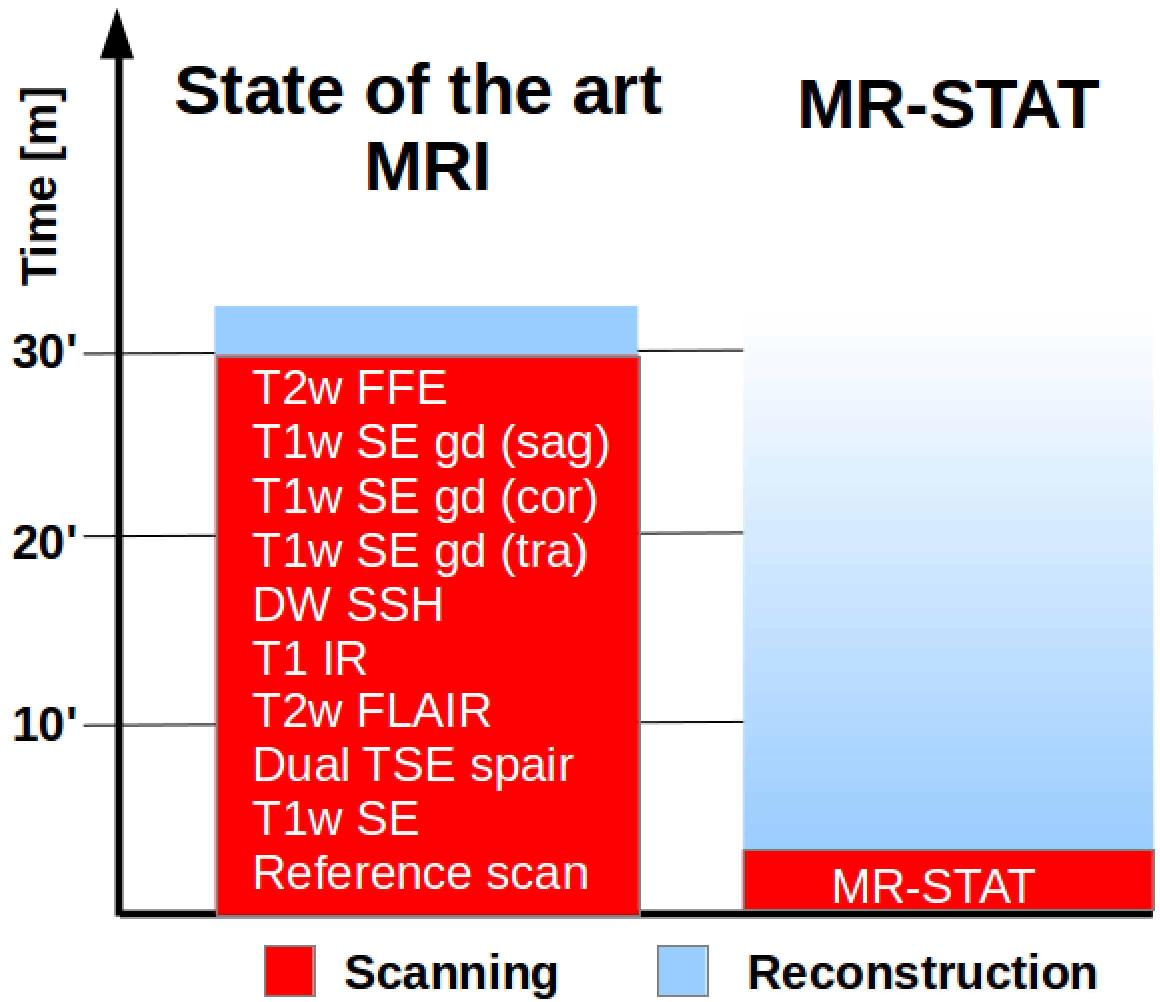
$$T_1, T_2, M_0, \ldots$$

System equations (Bloch and Faraday)

Inversion problem (nonlinear)



Save time during scanning!



Time-domain reconstruction

physical parameters: $\alpha \equiv \mathcal{B} M_0$ and $\vec{\beta} \equiv (T_1, T_2, B_1^+, \Delta B_0)$

data   Signal model

$$
\begin{aligned}
(\alpha^*, \vec{\beta}^*) \quad &= \arg\min_{\alpha,\vec{\beta}} \quad \int_{t\in\tau} \left| d(t) - s(\alpha, \vec{\beta}, t) \right|^2 dt, && \text{(Data consistency)} \\
\text{such that} \quad & s(\alpha, \vec{\beta}, t) = \int_V \alpha\, M_\perp(\vec{\beta}, t) d\vec{r}, \quad t \in \tau && \text{(Faraday's law)} \\
& \frac{d}{dt}\vec{M} = \Pi \vec{M} + \vec{c} && \text{(Bloch equation)} \\
& \vec{M}(\vec{\beta}, 0) = \vec{e}_3 && \text{(Initial condition)} \\
& \vec{\beta} \in \mathbb{B} && \text{(Physical bounds)}
\end{aligned} \tag{1}
$$

Time-domain reconstruction

- number of data points (time samples): $\sim 10^5, 10^6$
- number of unknowns (spatial domain): $N_{\text{voxels}} \times N_{\text{physical params}} \sim 10^5, 10^6$
- non-linear
- convexity
- ill-conditioning and convergence speed

A lot of interesting mathematical/computational problems!

Time-domain reconstruction

Parallel computing

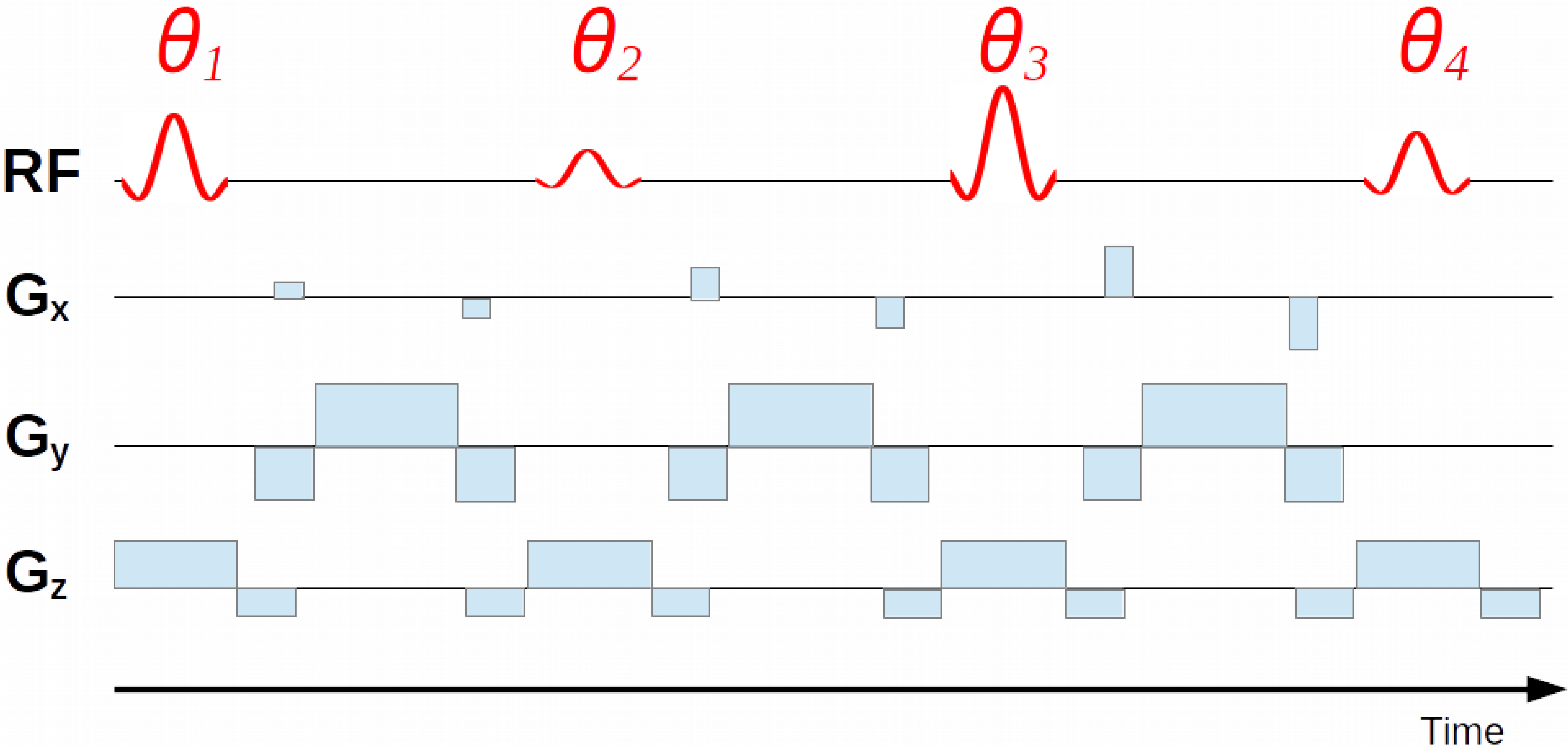
- number of data points (time samples): $\sim 10^5, 10^6$
- number of unknowns (spatial domain): $N_{\text{voxels}} \times N_{\text{physical params}} \sim 10^5, 10^6$
- non-linear Iterative methods with efficient derivative approximations
- convexity variable substitution ($T_2 \mapsto \log T_2, B_1 \mapsto \log B_1$)
- Convergence VARiable PROjection (exploit linear and nonlinear dependence)

A lot of interesting mathematical/computational problems!

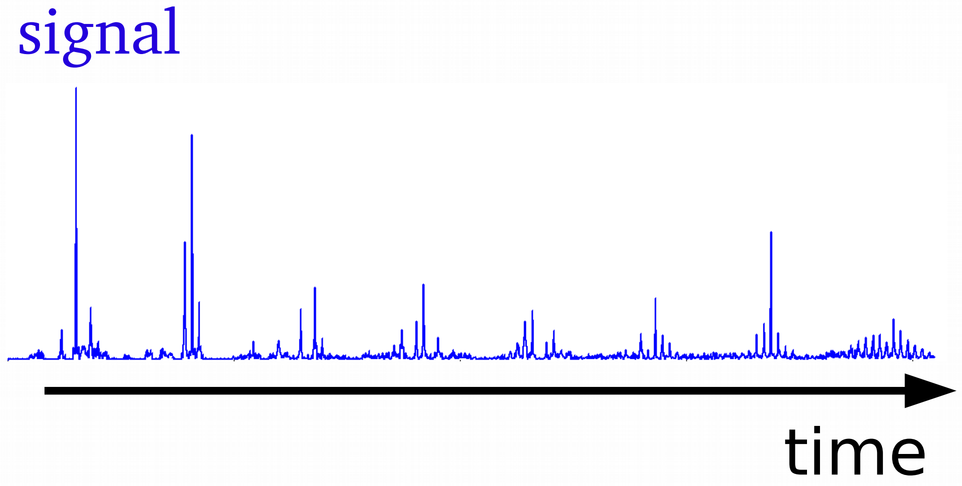
MR-STAT, example

- Numerical head model (realistic T_1,T_2,PD,B_1,B_0)
- 2D grid: N x N = 200 x 200
- Simulate the signal
- Add noise (realistic level)
- FFT in one dimension
- Resulting N independent subproblems to cluster of CPUs

MR-STAT sequence (no waiting times)

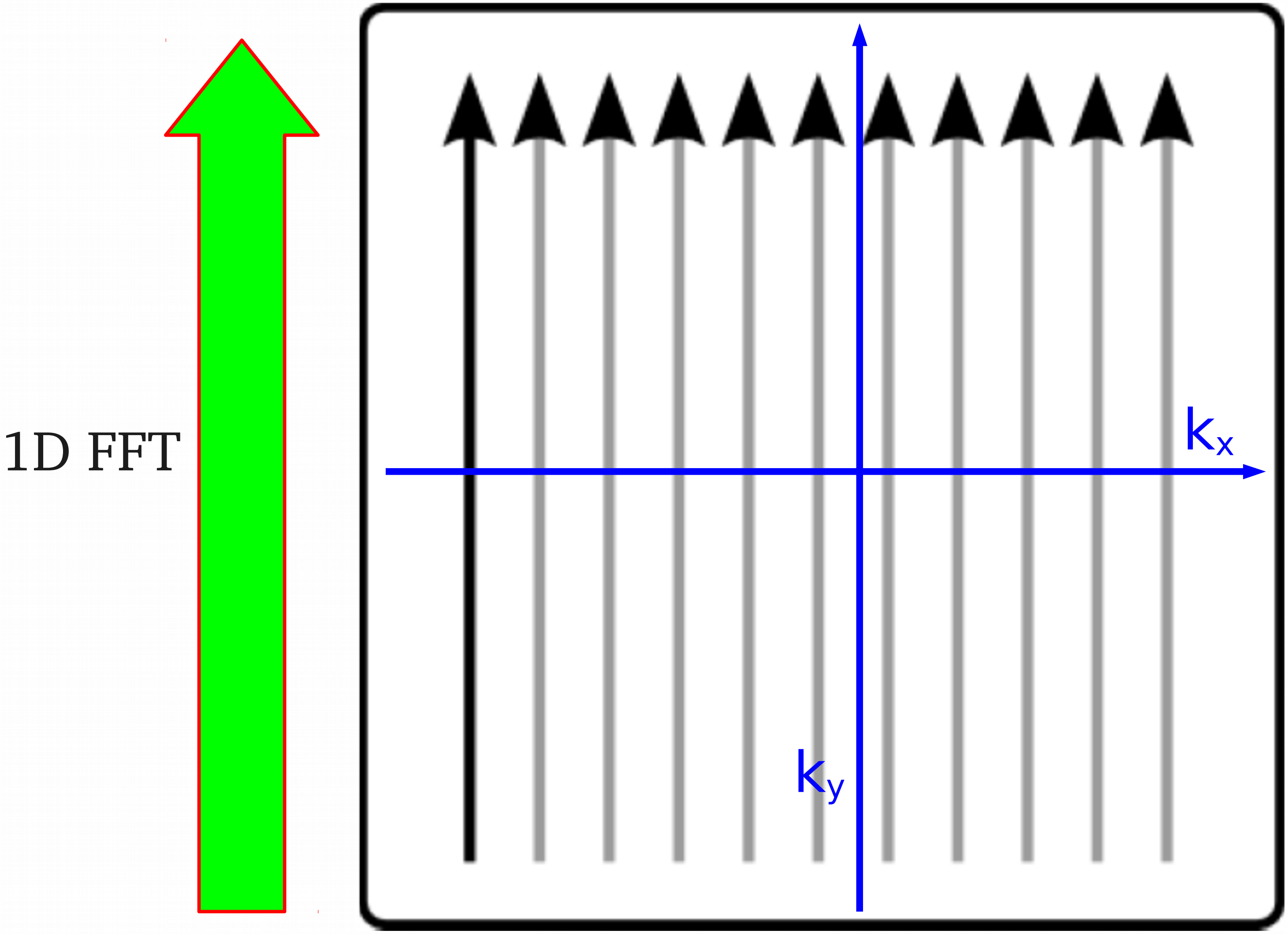


MR-STAT sequence (no waiting times)



Just 8 seconds for 1 slab !

K-space coverage and decoupling



Reconstruction

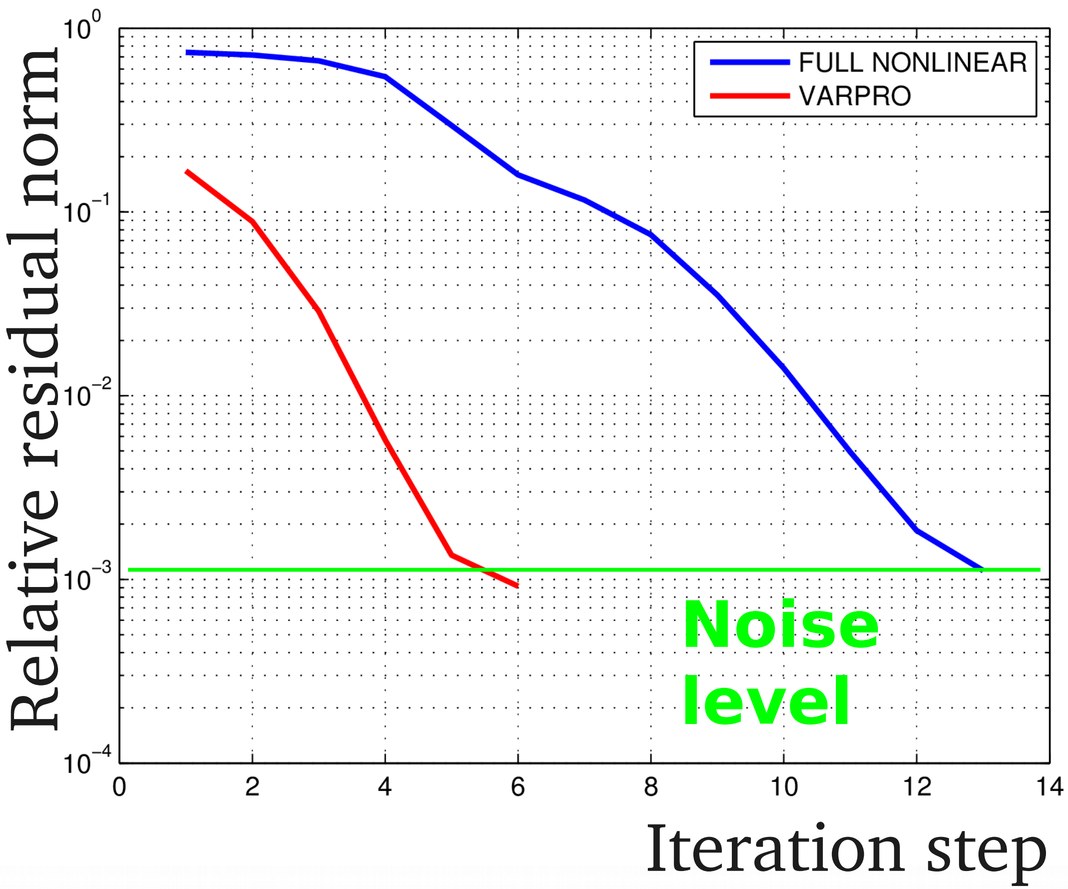
physical parameters: $\alpha \equiv \mathcal{B} M_0$ and $\vec{\beta} \equiv (T_1, T_2, B_1^+, \Delta B_0)$

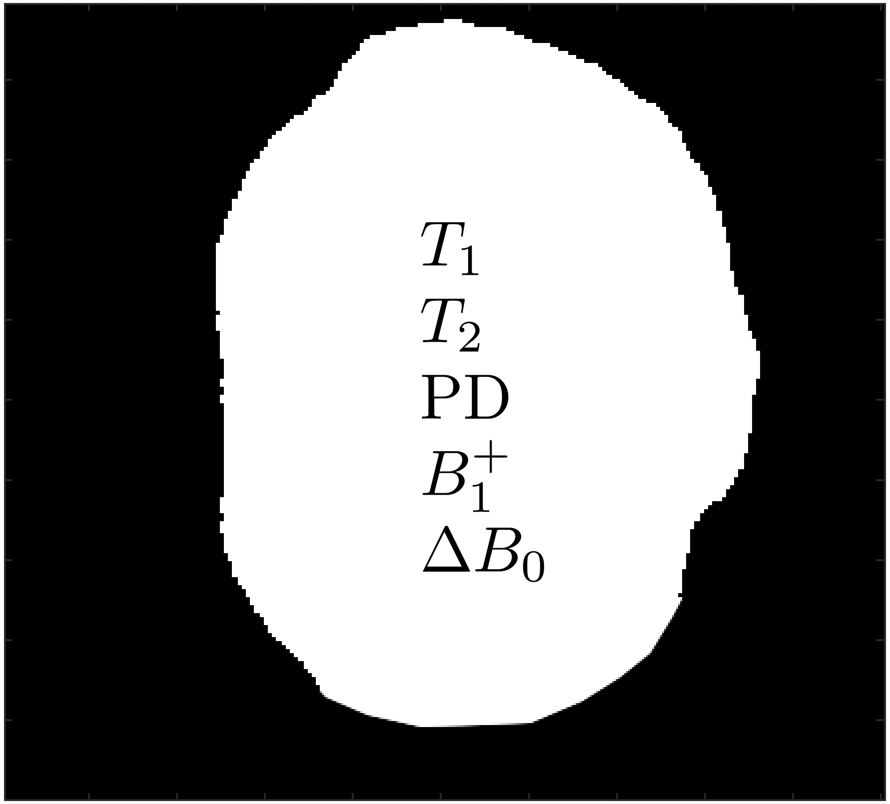
data   Signal model

$$
\begin{aligned}
(\alpha^*, \vec{\beta}^*) \quad &= \arg\min_{\alpha,\vec{\beta}} \quad \int_{t\in\tau} \left| d(t) - s(\alpha, \vec{\beta}, t) \right|^2 dt, && \text{(Data consistency)} \\
&\text{such that} \quad s(\alpha, \vec{\beta}, t) = \int_V \alpha\, M_\perp(\vec{\beta}, t) d\vec{r}, \quad t \in \tau && \text{(Faraday's law)} \\
&\frac{d}{dt}\vec{M} = \Pi \vec{M} + \vec{c} && \text{(Bloch equation)} \\
&\vec{M}(\vec{\beta}, 0) = \vec{e}_3 && \text{(Initial condition)} \\
&\vec{\beta} \in \mathbb{B} && \text{(Physical bounds)}
\end{aligned}
\tag{1}
$$

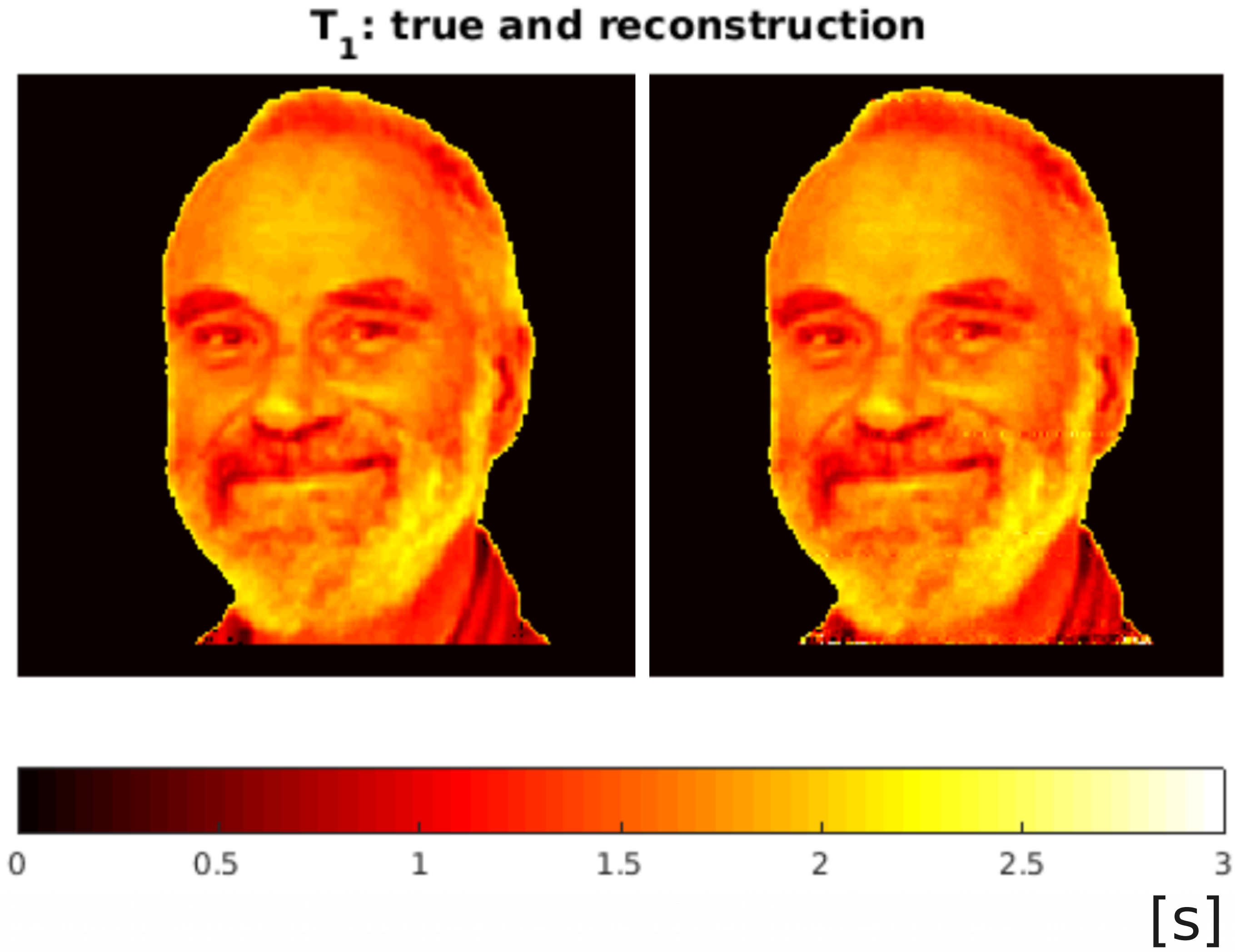
Computing

Wait about 1.5 hour (max: 10 iterations)

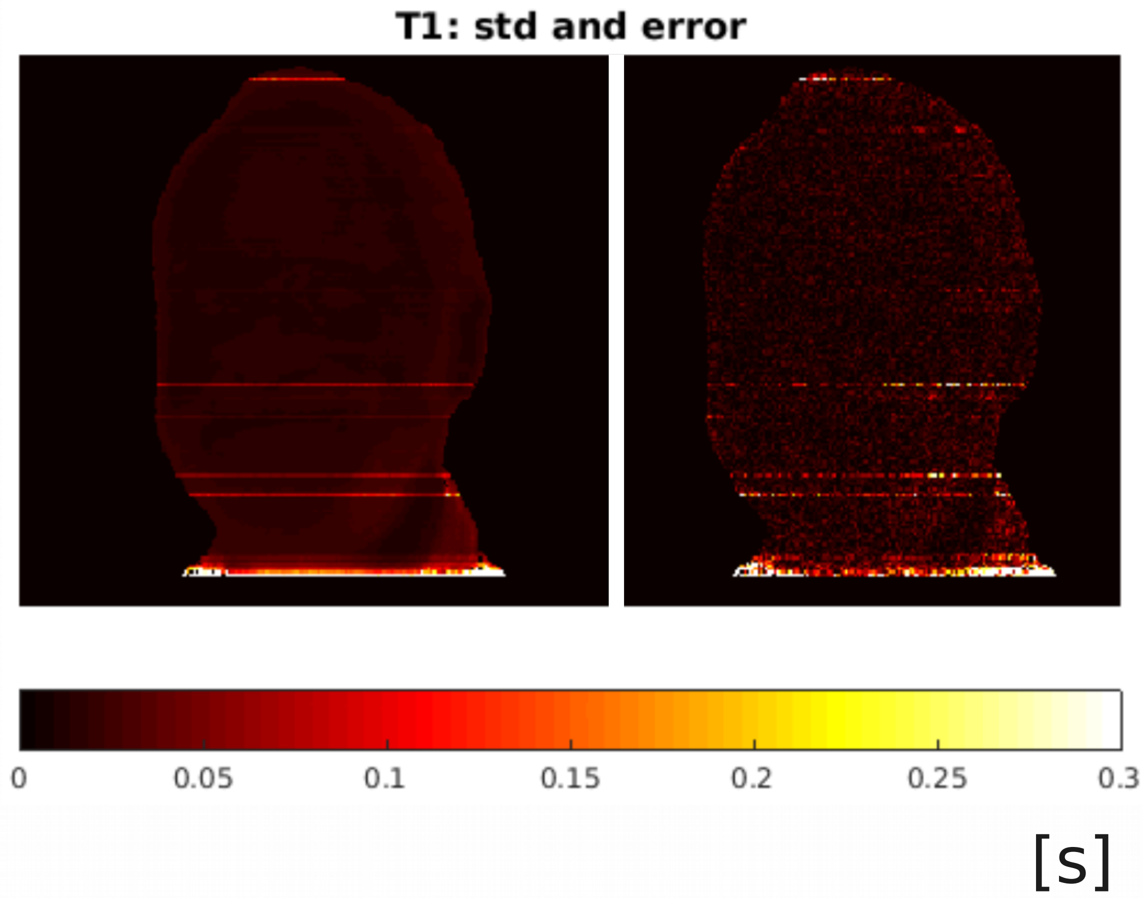



T_1
T_2
PD
B_1^+
ΔB_0

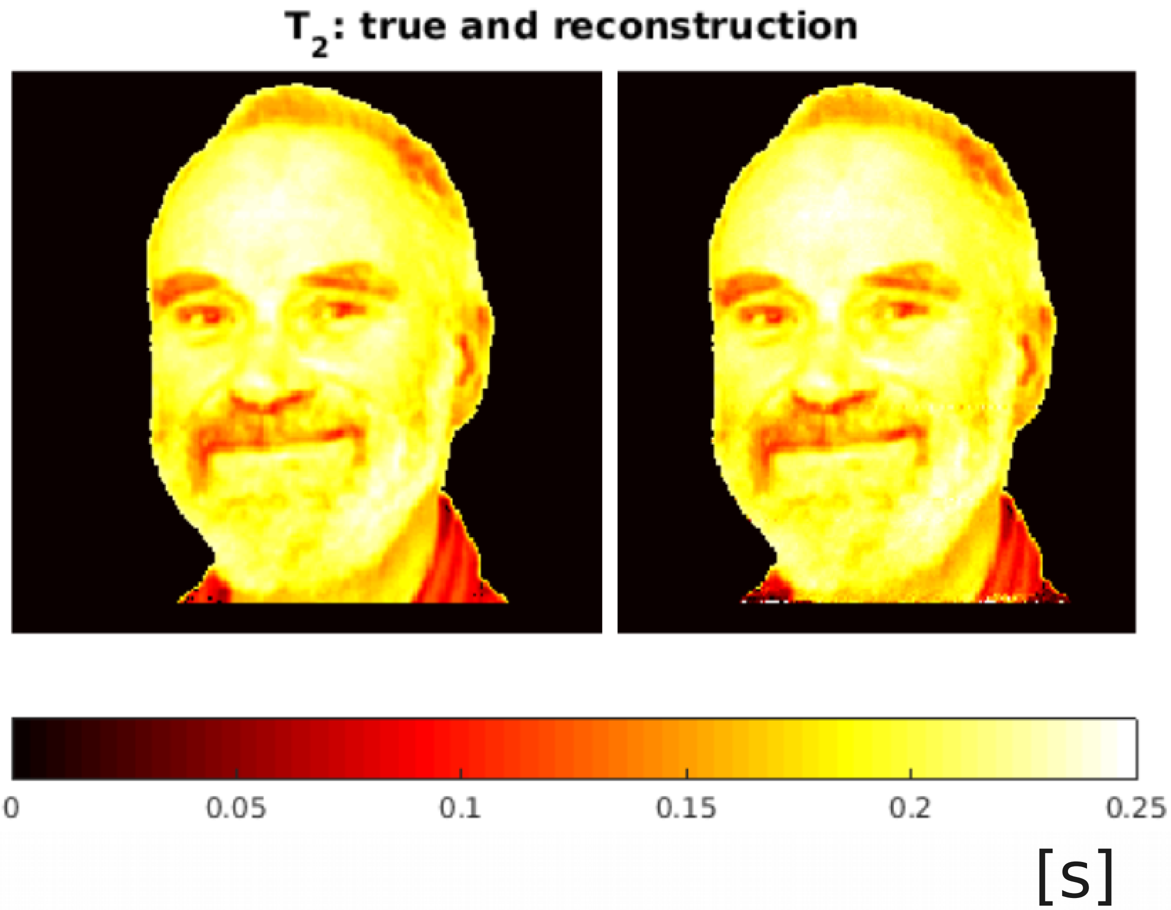
T_1



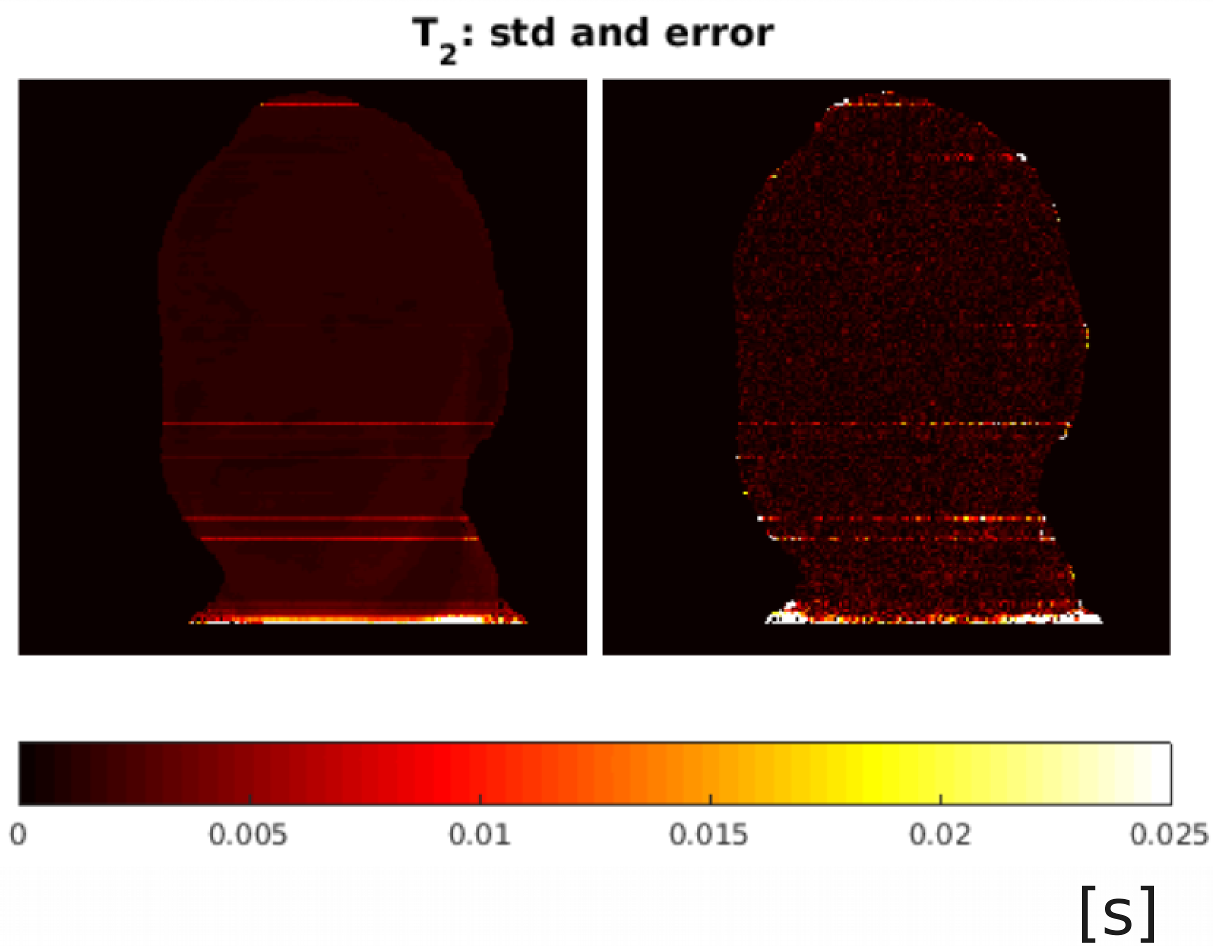
T1 error



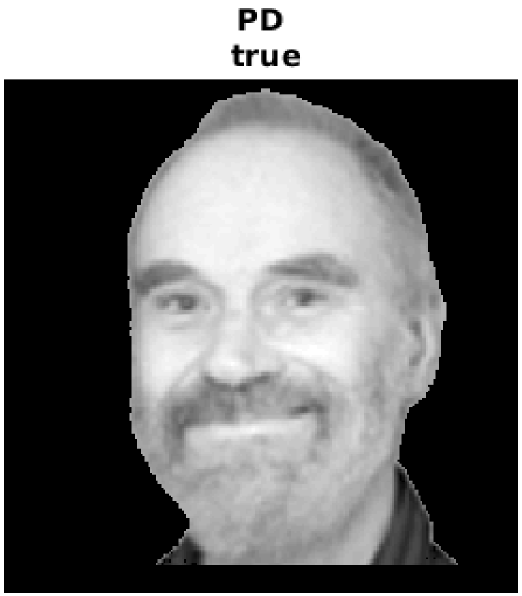
T_2

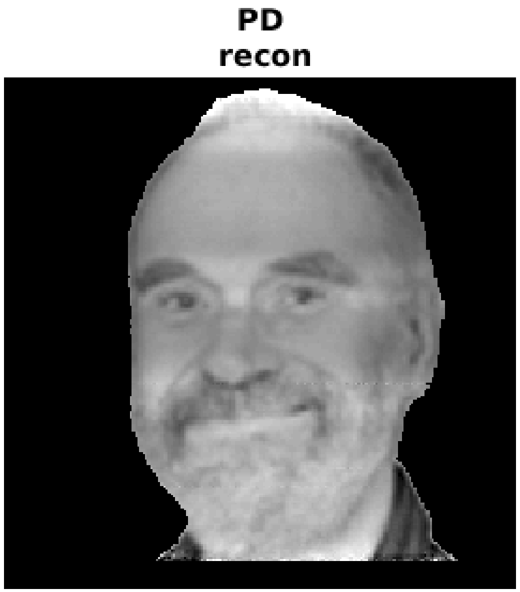


T_2 error

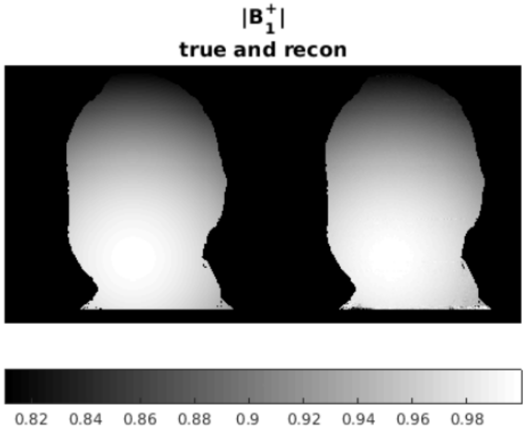


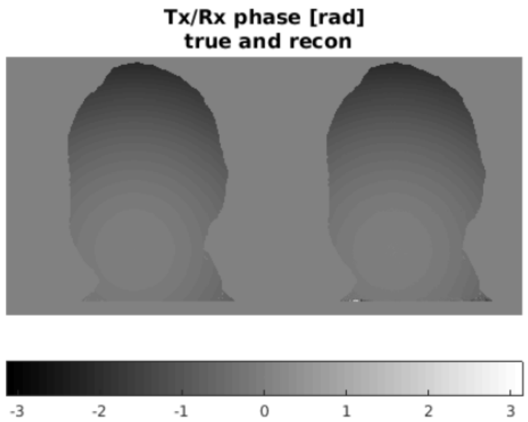
Proton density (linear dependency)

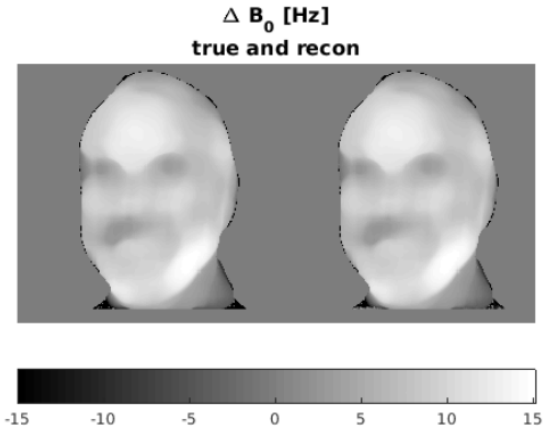




Other parameters







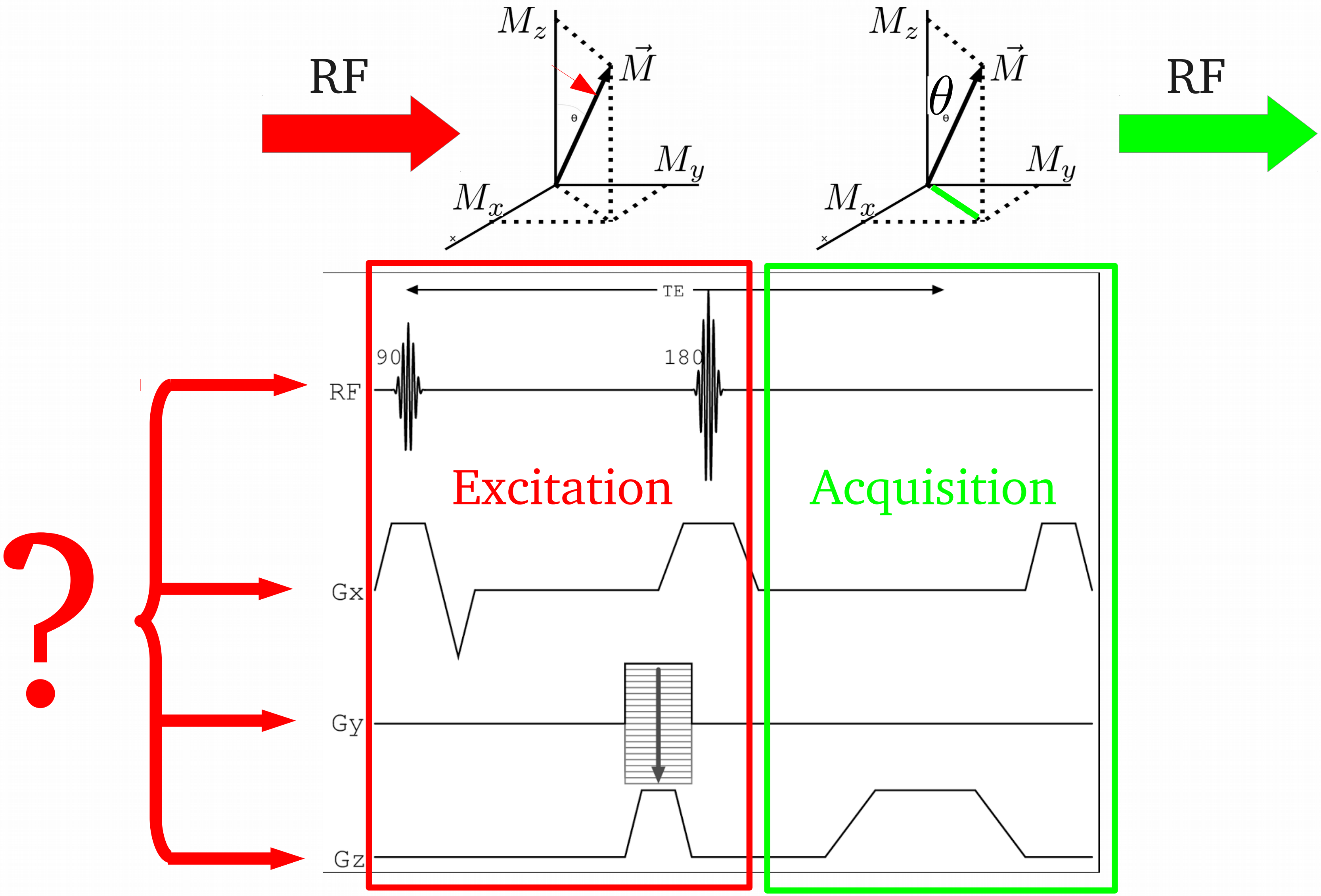
Future plans for MR-STAT

- Two PhD students (MSc in Mathematics) start working soon

  → Anna Kruseman

  → Oscar van der Heide
- + 1 post-doc (not appointed yet)

What I did **not** talk about:



Sequence design: (large-scale) control problems

Conclusion

Numerical mathematics/scientific computing is really hot in the field of MRI and will be fundamental to tackle future challenges!

Acknowledgment

- Personal thanks to Gerard for the following classes:

  Fourier & Wavelets Theory, Numerical Linear Algebra, Introductory Numerical Analysis, Lab Scientific Computing

- Supervision of my master thesis

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- Research collaboration & Co-supervision of student

Happy Birthday Gerard !

And thank you all for attention !