# Vraag 1

(a) $\sum_{n=0}^{\infty} \frac{3n-1}{3^n}$, (b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, (c) $\sum_{n=0}^{\infty} e^{-n}$, (d) $\sum_{n=0}^{\infty} \sqrt{n^3}$,

(e) $\sum_{n=0}^{\infty} \left(-\frac{2}{\pi}\right)^n$

(a) convergent ("ratiotest")

$$\frac{\frac{3(n+1)-1}{3^{n+1}}}{\frac{3n-1}{3^n}} = \frac{3(n+1)-1}{3^{n+1}} * \frac{3^n}{3n-1} = \frac{3n+2}{3n-1} * \frac{1}{3}$$

$$= \underbrace{\frac{3+2/n}{3-1/n}}_{\substack{\to 1 \\ n\to\infty}} * \frac{1}{3} \to \frac{1}{3} \text{ voor } n \to \infty$$

(b) $\sqrt{n} < n$ ("comparison test")

$\Rightarrow \frac{1}{\sqrt{n}} > \frac{1}{n}$, $\sum_{n=1}^{\infty} \frac{1}{n}$ divergent $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ divergent ook

(c) $\int_1^{\infty} e^{-x}\,dx = -e^{-x}\Big|_{x=1}^{x=\infty} = 0 + e^{-1} = \frac{1}{e} < \infty$

("integraltest")

(d) $\lim_{n\to\infty} \sqrt{n^3} \neq 0 \Rightarrow$ divergentie

(e) $2 < \pi \Rightarrow \frac{2}{\pi}, \left(\frac{2}{\pi}\right)^2, \left(\frac{2}{\pi}\right)^3 \ldots$ dalende rij (positieve getallen)

vanwege "$-$" alternerend

$\Rightarrow$ convergentie

$\boxed{\text{Vraag 2}}$

$$E(T) = \cos(T) - \sin(T) - 1$$
$$\rho(T) = e^T - 1$$

(a) $$\cos(T) = 1 - \frac{T^2}{2!} + \frac{T^4}{4!} - \frac{T^6}{6!} \ldots$$
$$\sin(T) = T - \frac{T^3}{3!} + \frac{T^5}{5!} - \frac{T^7}{7!} \ldots$$

$$E(T) = -\frac{T^2}{2} + \frac{T^4}{24} + \ldots \quad - T + \frac{T^3}{6} - \frac{T^5}{120} + \ldots$$
$$= \underbrace{-T - \frac{T^2}{2} + \frac{T^3}{6} + \frac{T^4}{24} - \frac{T^5}{120}}_{} + \ldots$$

(b) $$e^T - 1 = 1 + T + \frac{T^2}{2!} + \frac{T^3}{3!} + \ldots \quad -1$$
$$= T + \frac{T^2}{2!} + \frac{T^3}{3!} \ldots$$

$$\lim_{T\to 0} \frac{E(T)}{\rho(T)} = \frac{-T - \frac{T^2}{2} + \frac{T^3}{6} \ldots}{T + \frac{T^2}{2} + \frac{T^3}{6} \ldots} = \lim_{T \to 0} \frac{-1 - \frac{T}{2} + \frac{T^2}{6} \ldots}{1 + \frac{T}{2} + \frac{T^2}{6} \ldots}$$

$$= -1$$

$\boxed{\text{Vraag 3}}$

$$\begin{cases} x e^{x^2} + y y' = 0 & , y > 0 \\ y(0) = 1 \end{cases}$$

$$\Rightarrow \quad y \frac{dy}{dx} = -x e^{x^2}$$

$$\Rightarrow \quad \int y \, dy = - \int x e^{x^2} dx$$ !

$$\Rightarrow \quad \frac{1}{2} y^2 = -\frac{1}{2} e^{x^2} + \tilde{c}$$

$$\Rightarrow \quad y = \pm \sqrt{-e^{x^2} + c}$$

$$y > 0 \quad \Rightarrow y = \sqrt{-e^{x^2} + c}$$

$$y(0) = \sqrt{-e^{0} + c} = \sqrt{-1 + c} = 1 \quad \Rightarrow \underline{c = 2}$$

$$\Rightarrow y = \sqrt{2 - e^{x^2}}$$

## Vraag 4

$$\begin{cases} y'' - y' - 6y = 5 - 6x \\ y(0) = 1, \ y'(0) = 0 \end{cases}$$

homogeen: $y_h'' - y_h' - 6y_h = 0$

$y_h = e^{\lambda x} \underset{\text{invullen}}{\Rightarrow} \lambda^2 - \lambda - 6 = 0$

$\Rightarrow (\lambda - 3)(\lambda + 2) = 0$

$\Rightarrow \lambda = 3, \ \lambda = -2$

$\Rightarrow y_h = c_1 e^{3x} + c_2 e^{-2x}$

particulier: $y_p = A + Bx \Rightarrow y_p' = B$ en $y_p'' = 0$

invullen

$\Rightarrow 0 - B - 6A - 6Bx = 5 - 6x$

$\Rightarrow \begin{cases} -B - 6A = 5 \\ -6B = -6 \end{cases} \Rightarrow \begin{matrix} B = 1 \\ A = -1 \end{matrix}$

$\Rightarrow y_p = -1 + x$

totaal: $y = c_1 e^{3x} + c_2 e^{-2x} - 1 + x$

IC's: $y(0) = c_1 + c_2 - 1 = 1 \Rightarrow c_1 + c_2 = 2$

$y' = 3c_1 e^{3x} - 2c_2 e^{-2x} + 1 \Rightarrow y'(0) = 3c_1 - 2c_2 + 1 = 0 \Rightarrow 3c_1 - 2c_2 = -1$

$\Rightarrow c_1 = \frac{3}{5}, \ c_2 = \frac{7}{5}$

$\Rightarrow y = \frac{3}{5} e^{3x} + \frac{7}{5} e^{-2x} - 1 + x$

# $\boxed{\text{Vraag 5}}$

$$\begin{cases} x' = -x + \gamma y, & \gamma \geq 0 \\ y' = y + \alpha x + \beta x^2, & \alpha \geq 0, \beta \in \mathbb{R} \end{cases}$$

(a) $\mathcal{J} = \begin{pmatrix} -1 & \gamma \\ \alpha + 2\beta x & 1 \end{pmatrix}$

(b) ① $\alpha = \gamma = 1, \beta = -1$

$$\begin{cases} x' = -x + y \\ y' = y + x - x^2 \end{cases} \rightarrow \begin{matrix} y = x \\ 2x - x^2 = 0 \end{matrix} \rightarrow (0,0),\ (2,2)$$

② $\alpha = \beta = 1, \gamma = 0$

$$\begin{cases} x' = -x \\ y' = y + x + x^2 \end{cases} \rightarrow \begin{matrix} x = 0 \\ y = 0 \end{matrix} \rightarrow (0,0)$$

(c) ① $\mathcal{J} = \begin{pmatrix} -1 & 1 \\ 1-2x & 1 \end{pmatrix}$

$(0,0)$ $\mathcal{J} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \det = -2 < 0$

<u>zadelpunt</u>

$(2,2)$ $\mathcal{J} = \begin{pmatrix} -1 & 1 \\ -3 & 1 \end{pmatrix} \rightarrow Sp = 0$, $\det = 2 > 0$, $Disc = -8 < 0$

<u>Centrumpunt</u>

② $\mathcal{J} = \begin{pmatrix} -1 & 0 \\ 1+2x & 1 \end{pmatrix} \xrightarrow{(0,0)} \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \rightarrow \det < 0 \Rightarrow$ zadelpunt

(d) $\boxed{A}$ $\begin{cases} x' = -x \\ y' = y \end{cases}$

$\alpha = \beta = \gamma = 0$

$\begin{matrix} x = 0 \\ y = 0 \end{matrix}$ $(0,0)$ $\mathcal{J} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow$ zadelpunt (lineair)

x en y-as zijn speciale oplossingen (aantr. en afstot.)

$\boxed{B}$ $\begin{cases} x' = -x \\ y' = y + x^2 \end{cases}$

$\alpha = 0$
$\beta = 1$
$\gamma = 0$

$(0,0)$ $\mathcal{J} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow$ zadelpunt

MAAR VERVORMD !!!

check enkele punten $(x,y)$ en bereken $dy/dx$ etcetera

$\alpha=\beta=0$

$$\begin{cases} x' = -x \\ y' = y \end{cases}$$

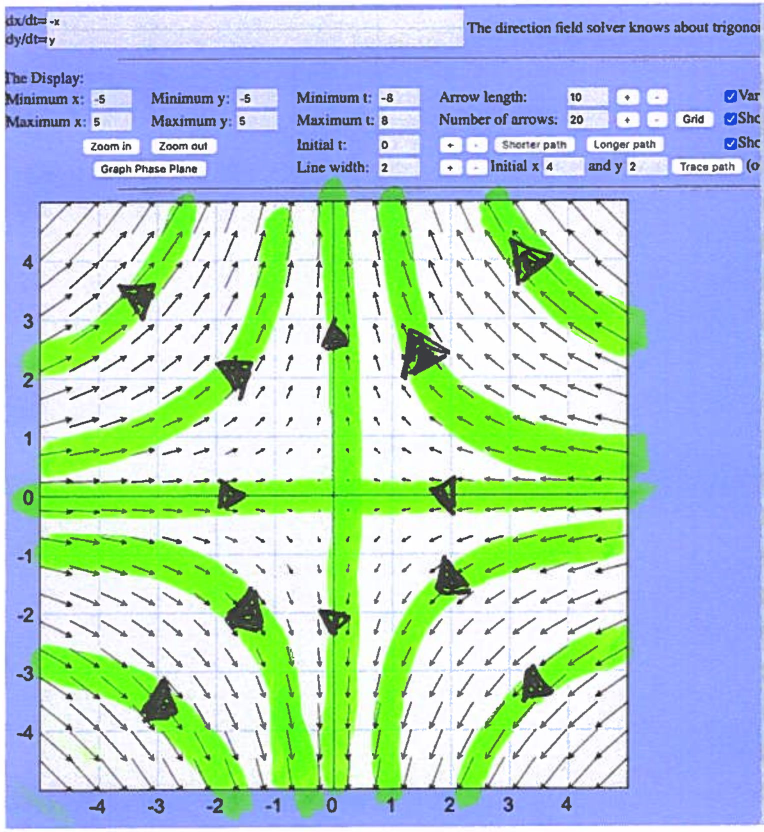
$\cancel{\alpha=1}, \beta=0$

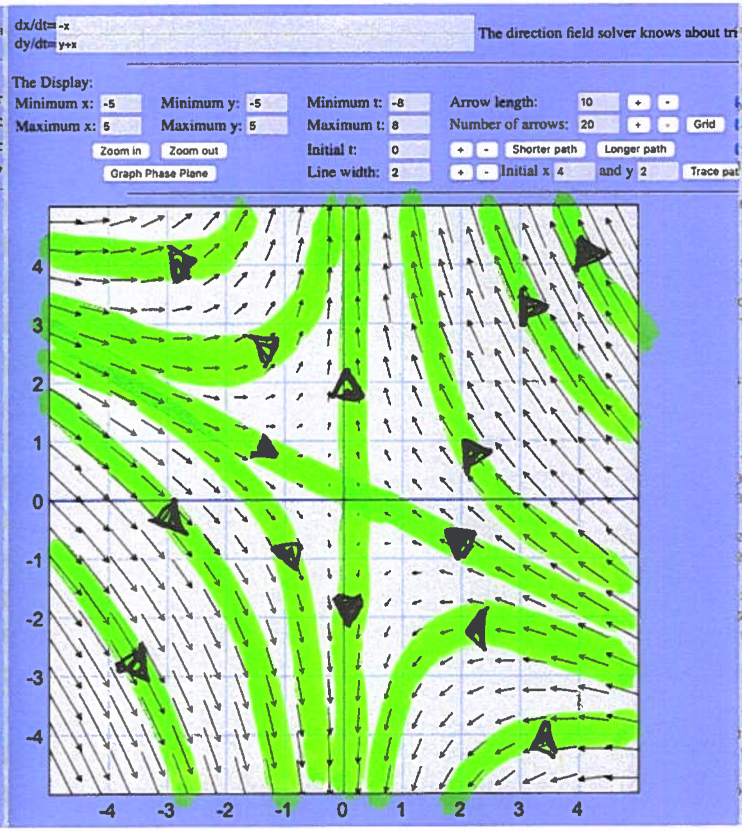
$$\left(\begin{cases} x' = -x \\ y' = y + x \end{cases}\right)$$

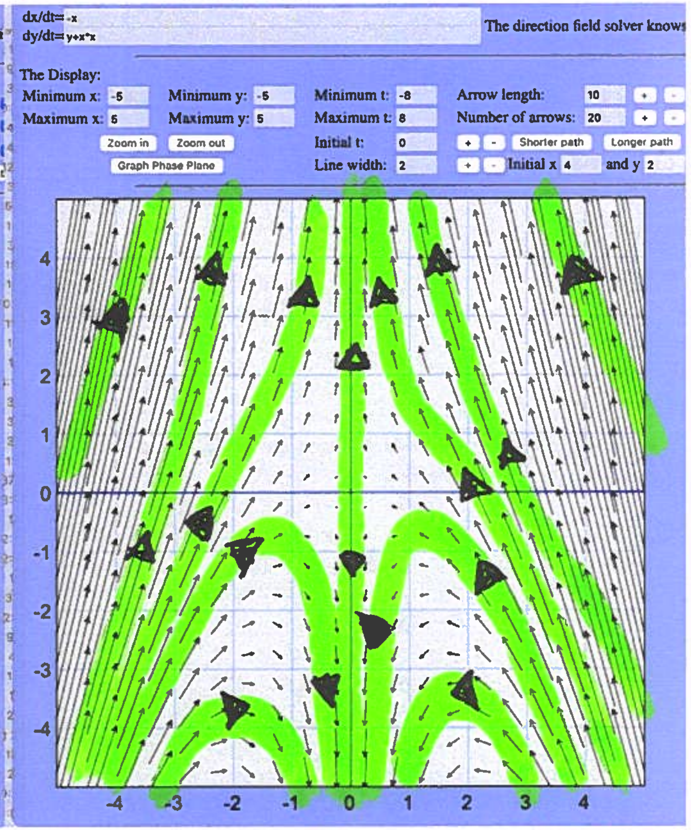
$\alpha=0, \beta=1$

$$\begin{cases} x' = -x \\ y' = y + x^2 \end{cases}$$

(a)




lineair

vervormd !!
(niet-lineair)

$\boxed{\text{Vraag 6}}$

$f(x) = x^2 + x - 2 = 0$ & $\begin{cases} x_{i+1} = \varphi(x_i) \quad i = 0, 1, 2, \ldots \\ x_0 = \ldots \end{cases}$

(a) $f(x) = 0 \iff x = \underbrace{-2}_{=a}$ of $x = \underbrace{+1}_{=b}$

1. $x = \frac{2}{x} - 1 \iff x^2 = 2 - x \iff x^2 + x - 2 = 0$ ✓

2. $x = \frac{x^2+2}{2x+1} \iff 2x^2 + x = x^2 + 2 \iff x^2 + x - 2 = 0$ ✓

3. $x = \sqrt{2-x} \iff x^2 = 2 - x \iff x^2 + x - 2 = 0$ ✓

4. $x = x^2 + 2x - 2 \iff x^2 + x - 2 = 0$ ✓

(b) $\varphi_1'(x) = -\frac{2}{x^2} \Rightarrow \varphi_1'(1) = -2$, $|\varphi_1'(1)| > 1$ $\underline{\text{div.}}$

$\varphi_2'(x) = \frac{2(x^2+2) - 2x(2x+1)}{(2x+1)^2} = \frac{-2x^2 - 2x + 4}{(2x+1)^2}$, $\varphi_2'(1) = 0$ $\underline{\underline{\text{CONV}}}$

$\varphi_3'(x) = -(2-x)^{-\frac{1}{2}} \Rightarrow \varphi_3'(1) = -1$, $|\varphi_3'(1)| = 1 \Rightarrow$ onduidelijk (ws. div.)

$\varphi_4'(x) = 2x + 2 \Rightarrow \varphi_4'(1) = 4$, $|\varphi_4'(1)| > 1$ $\underline{\underline{\text{div}}}$

(c) NR: $\varphi(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2+x-2}{2x+1} = \frac{x(2x+1) - (x^2+x-2)}{2x+1}$

$= \frac{x^2+2}{2x+1} \longrightarrow \underline{\underline{\varphi_3(x)}}$

voordeel NR: $\varphi'(b) = 0$

$\Rightarrow$ kwadratische convergentie

$\boxed{\text{Vraag 7}}$ $\mathcal{E} = x^3 - 6x^2 - 8y^2 + 2024$

(a) $\nabla \mathcal{E} = (\partial\mathcal{E}/\partial x, \partial\mathcal{E}/\partial y)^T = (3x^2 - 12x, -16y)^T$

$D_{\vec{b}}\,\mathcal{E}|_P = \nabla\mathcal{E}|_P \cdot \vec{b} = \frac{1}{\sqrt{2}}\begin{pmatrix}-9\\16\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}(9-16) = \frac{-7}{\sqrt{2}}$

$\vec{b} = \frac{(-1,-1)^T}{\|(-1,1)\|} = \frac{(-1,-1)^T}{\sqrt{2}} = (-\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})^T$

$P = (1,-1)$

(b) $\mathcal{H} = \begin{pmatrix}\mathcal{E}_{xx} & \mathcal{E}_{xy}\\ \mathcal{E}_{yx} & \mathcal{E}_{yy}\end{pmatrix} = \begin{pmatrix}6x-12 & 0\\ 0 & -16\end{pmatrix}$, $\nabla\cdot(\nabla\mathcal{E}) = \Delta\mathcal{E} = \mathcal{E}_{xx} + \mathcal{E}_{yy}$

$= 6x - 12 - 16 = 6x - 28$

(c) $\mathcal{E}_x = 0$ èn $\mathcal{E}_y = 0 \Rightarrow \begin{cases}3x^2 - 12x = 0\\ -16y = 0\end{cases} \Rightarrow y = 0$ èn $x = 0$ of $x = 4$

$\Rightarrow (0,0)$ en $(4,0)$

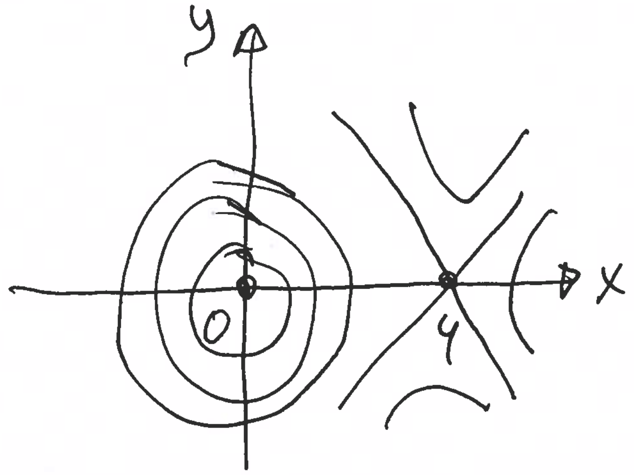
(d)

| | $\det(\mathcal{H})$ | $\Delta\mathcal{E}$ | karakter |
|---|---|---|---|
| $(0,0)$ | $>0$ | $<0$ | MAX |
| $(4,0)$ | $<0$ | $<0$ | ZADEL |

$\det(\mathcal{H}) = (6x-12) * (-16)$

$= 16(12 - 6x)$

(e)




MAX in $(0,0) \Rightarrow \mathcal{E} = 0 - 0 - 0 + 2024 = 2024$

Vraag 8

$$\begin{cases} y'' + 4y = \sin(2t) \\ y(0) = 0 \ , \ y'(0) = 0 \end{cases}$$

(a) $\mathcal{L}(\sin(2t)) = \dfrac{2}{s^2+4}$

$$\Rightarrow \quad s^2 \mathcal{L}(y) - s\,\underbrace{y(0)}_{=0} - \underbrace{y'(0)}_{=0} + 4\,\mathcal{L}(y) = \frac{2}{s^2+4}$$

$$\Rightarrow \quad s^2 \mathcal{L}(y) + 4\,\mathcal{L}(y) = \frac{2}{s^2+4}$$

$$\Rightarrow \quad \mathcal{L}(y) = \frac{2}{(s^2+4)^2}$$

(b) inverse van $\mathcal{L}$ toepassen (via tabel)

tabel nr 11

$$\Rightarrow \quad y(t) = \frac{1}{8}\sin(2t) - \frac{t}{4}\cos(2t)$$

$$\sin(2t) - 2t\cos(2t) = \frac{2 \cdot 2^3}{(s^2+2^2)^2}$$

"a = 2"

met extra factor $\frac{1}{8}$ om te "compenseren"