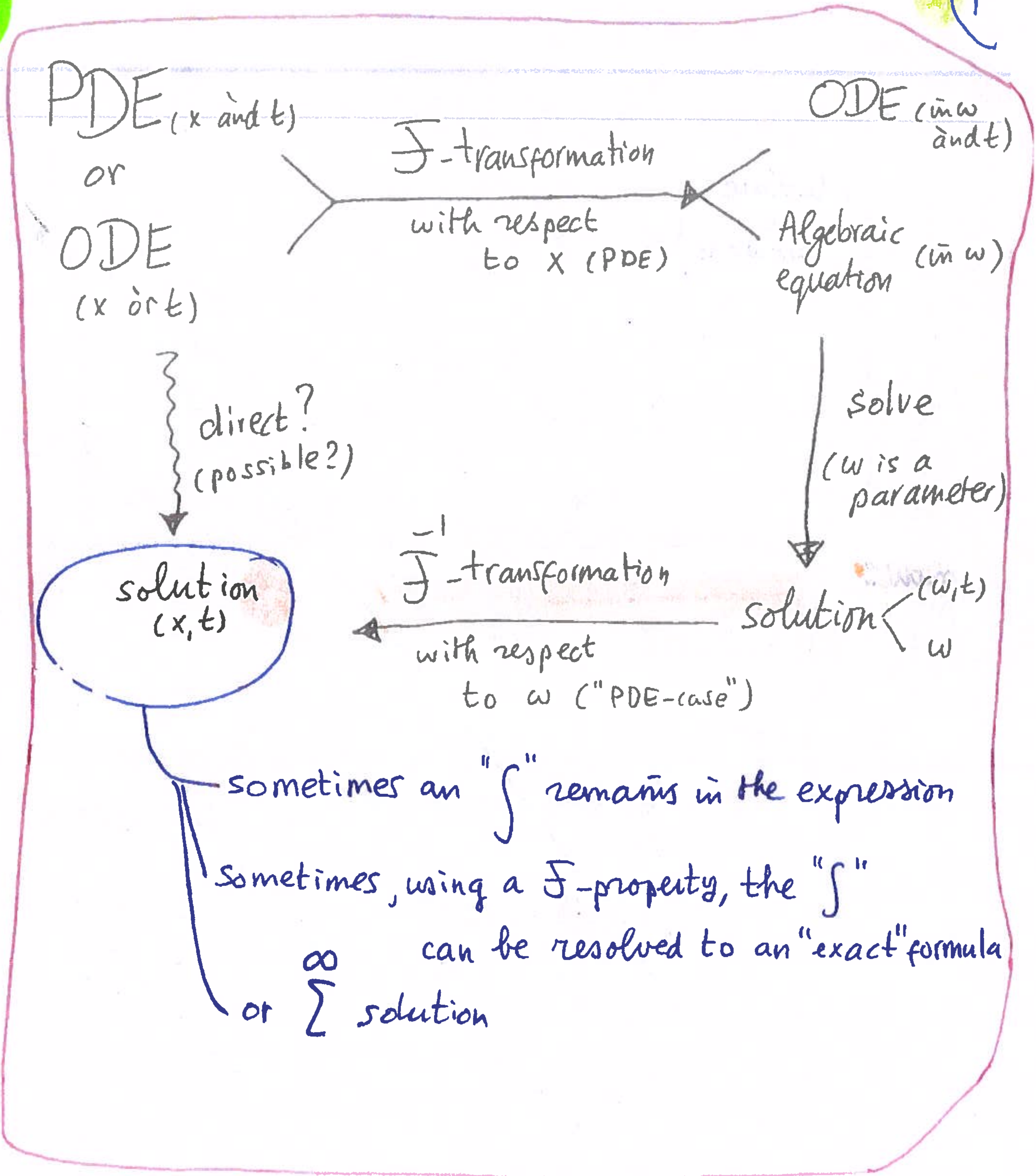

PDE (x and t)
or
ODE (x or t)
ODE (in ω and t)
ℱ-transformation
with respect to x (PDE)
Algebraic equation (in ω)
direct? (possible?)
solve
(ω is a parameter)
solution (x,t)
ℱ⁻¹-transformation
with respect to ω ("PDE-case")
solution (ω,t) / ω
sometimes an "∫" remains in the expression
sometimes, using a ℱ-property, the "∫" can be resolved to an "exact" formula
or ∑^∞ solution

Applications of $\mathcal{F}$-transformation:

* signals: sound, images, measurements
* solving differential equations
* ...

now:

Laplace transformation: $\mathcal{L}$

applications:

* engineering: electric circuits, digital signals, nuclear physics
* solving differential equations
* ...

Intermezzo

The Euler-Gamma function

("Gamma-function")

for $x > 0$ define: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt$

this integral is finite for all $x > 0$

Remember the "factorial":

$1! = 1$

$2! = 2 \cdot 1 = 2 = 2 \cdot 1!$

$3! = 3 \cdot 2 \cdot 1 = 6 = 3 \cdot 2!$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 = 4 \cdot 3!$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 = 5 \cdot 4!$

$\vdots$

etcetera

"product"

$n! = \prod_{k=1}^{n} k$

$= 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) n$

$\Rightarrow$ property: $(n+1)! = (n+1) \cdot n!$

for integer values of n

"define": $0! = 1$

Note that:

$$\Gamma(x+1) = \int_0^\infty t^{(x+1)-1} e^{-t}\, dt$$

$$= \int_0^\infty t^x e^{-t}\, dt$$

integration by parts

$$= \underbrace{\left[-t^x e^{-t}\right]_{t=0}^{t=\infty}}_{=0} - \int_0^\infty -x t^{x-1} e^{-t}\, dt$$

$+$

$$= \int_0^\infty x t^{x-1} e^{-t}\, dt$$

$$= x \underbrace{\int_0^\infty t^{x-1} e^{-t}\, dt}_{=\Gamma(x)} = x \cdot \Gamma(x)$$

Calculate:

$$\Gamma(1) = \int_0^\infty \underbrace{t^0}_{=1} e^{-t}\, dt = -e^{-t}\Big|_{t=0}^{t=\infty} = \underset{(--)}{0+1} = 1 = 0!$$

$$\Gamma(2) = 1 \cdot \Gamma(1) = 1 = 1!$$

$$\Gamma(3) = 2 \cdot \Gamma(2) = 2 \cdot 1 = 2!$$

$$\Gamma(4) = 3 \cdot \Gamma(3) = 3 \cdot 2 = 3!$$

$\vdots$

etcetera $\Longrightarrow$ $\boxed{\Gamma(n+1) = n!}$

$n = 0, 1, 2, 3, \ldots$

the Gamma-function is a <u>generalized</u> factorial !!!

What is $\Gamma(\frac{1}{2})$?

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^\infty t^{-\frac{1}{2}} e^{-t}\, dt = 2 \cdot \int_0^\infty \frac{1}{u} e^{-u^2} \cdot u\, du$$

change of variables
$t = u^2$
$dt = 2u\,du$
$t^{-1/2} = (u^2)^{-1/2} = \frac{1}{u}$

$$= 2 \int_0^\infty e^{-u^2}\, du$$

$\leftarrow$ see before

$$= 2 \cdot \frac{1}{2} \sqrt{\pi}$$

$$= \sqrt{\pi} = \cdots$$

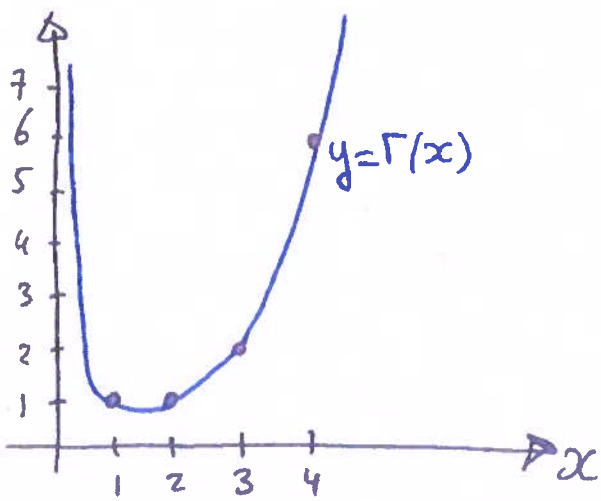
"$\left(\frac{1}{2}\right)!$"

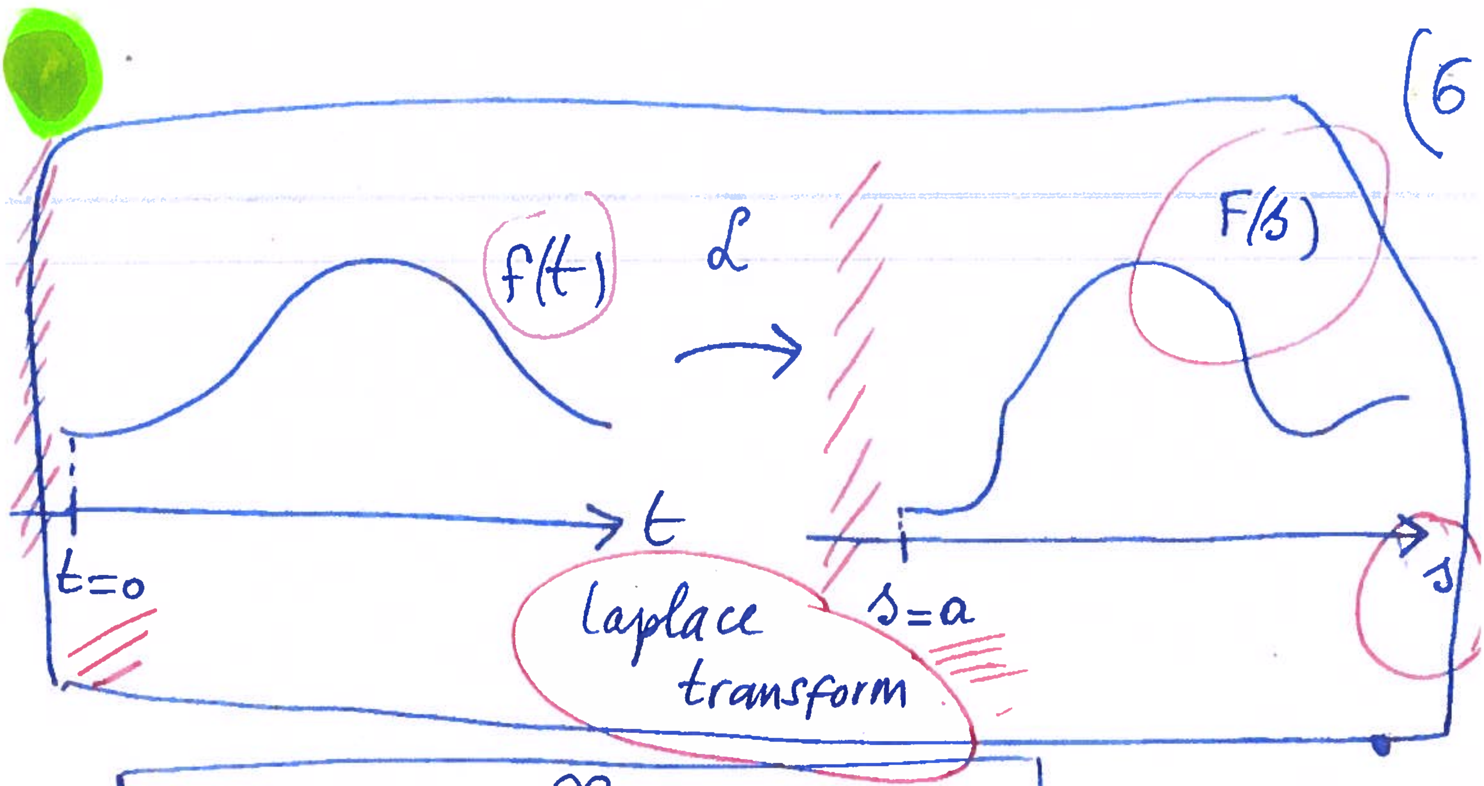
$$\Gamma\left(\tfrac{3}{2}\right) = \frac{1}{2} \cdot \Gamma\left(\tfrac{1}{2}\right) = \frac{1}{2} \sqrt{\pi} = \cdots$$

"$\left(\frac{3}{2}\right)!$" $\sim$ $$\Gamma\left(\tfrac{5}{2}\right) = \frac{3}{2} \cdot \Gamma\left(\tfrac{3}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi} = \frac{3}{4} \sqrt{\pi} = \cdots$$

etcetera ...

Graph of $\Gamma(x)$ for $x > 0$:





$$\boxed{\underbrace{\mathcal{L}(f(t))}_{\text{a function of } s} \overset{def.}{=} \int_0^\infty \underbrace{e^{-st}}_{\text{the kernel of the transformation}} f(t)\,dt = F(s)}$$

$s \geq a > 0$

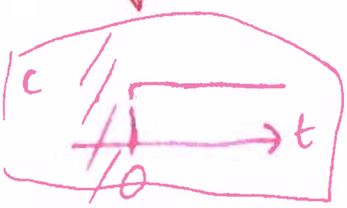
$\mathcal{L}(f(t))(s)$

better

(check the "ω" in the $\mathcal{F}$-transform)

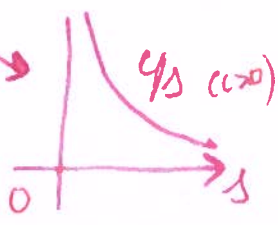
Example 1 $f(t) = c$ ← constant function

$$\Rightarrow \mathcal{L}(c) = \int_0^\infty e^{-st}\cdot c\,dt = c\int_0^\infty e^{-st}\,dt = c\,\frac{1}{-s}\,e^{-st}\Big|_{t=0}^{t=\infty}$$



$$= \frac{c}{-s}\,[0-1] = \frac{c}{s} \quad (s>0)$$

$s>0$

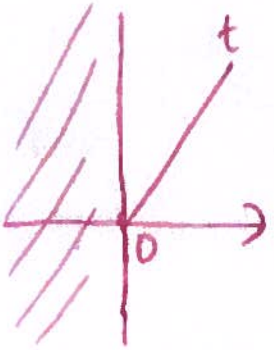


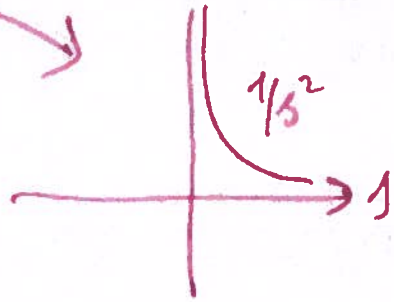
Example 2 $\quad f(t) = t$ linear function

$$\Rightarrow \mathcal{L}(t) = \int_0^\infty e^{-st}\, t\, dt = \left[-\frac{t}{s} e^{-st}\right]_{t=0}^{t=\infty} + (--)\int_0^\infty \frac{1}{s} e^{-st}\, dt$$

integration by parts

$$\overset{s>0}{=} \Big[\underbrace{\overset{e^{-\infty}=0}{0} + \overset{t=0}{0}}_{=0}\Big] + \frac{1}{s}\int_0^\infty e^{-st}\, dt$$





$$= -\frac{1}{s^2}\left[e^{-st}\right]_{t=0}^{t=\infty}$$

$$= -\frac{1}{s^2}\left[0 - 1\right]$$

$$\overset{---+}{=} \frac{1}{s^2} \quad (s>0)$$

Example 3 $\quad f(t) = e^{kt}$, k: a given positive constant

$$\Rightarrow \mathcal{L}(e^{kt}) = \int_0^\infty e^{-st} \cdot e^{kt}\, dt = \int_0^\infty e^{-(s-k)t}\, dt$$

now important

$$\overset{s>k}{=} -\frac{1}{s-k}\, e^{-(s-k)t}\Big|_{t=0}^{t=\infty}$$

$$= -\frac{1}{s-k}\Big[\overset{e^{-\infty}=0}{0} - \overset{e^{0}=1}{1}\Big] \overset{---+}{=} \frac{1}{s-k}$$

Example 4

$$f(t) = \sin(\alpha t), \quad \alpha \in \mathbb{R}$$
$$g(t) = \cos(\alpha t)$$

remember: $e^{i\alpha t} = \overbrace{\cos(\alpha t)}^{Re(\cdot)} + i \cdot \overbrace{\sin(\alpha t)}^{Im(\cdot)}$

$$f(t) = Im(e^{i\alpha t})$$
$$g(t) = Re(e^{i\alpha t}) \quad ; \quad \text{just calculated: in example 3} \quad \mathcal{L}(e^{kt}) = \frac{1}{s-k}$$

$$\Rightarrow \mathcal{L}(e^{i\alpha t}) = \frac{1}{s-i\alpha}, \quad s>0$$

take $k = i\alpha$

$$= \frac{s+i\alpha}{(s+i\alpha)(s-i\alpha)} = \frac{s+i\alpha}{s^2+\alpha^2}, \quad s>0$$

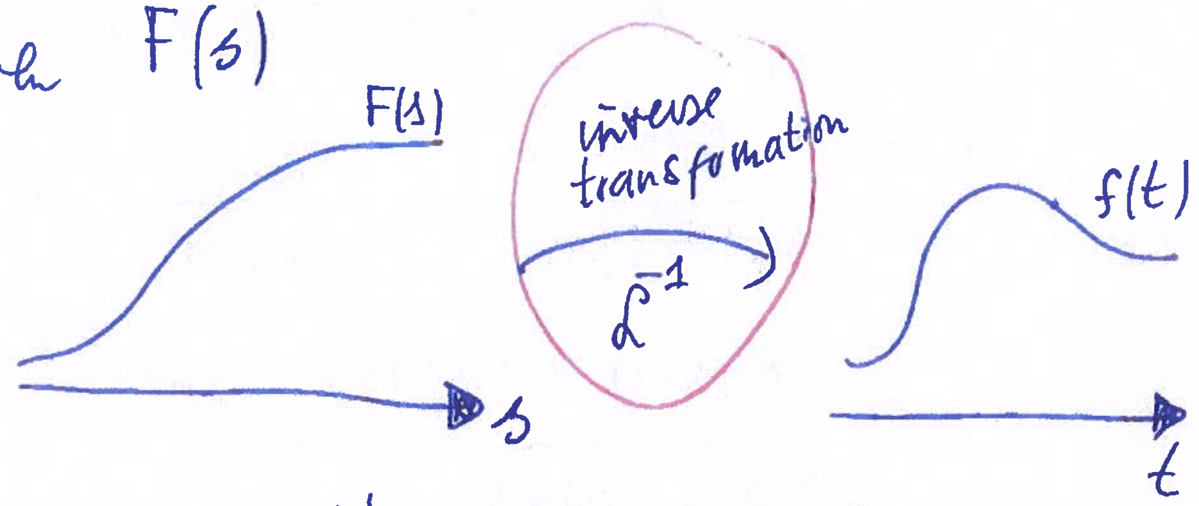
$$= \underbrace{\frac{s}{s^2+\alpha^2}}_{=Re(\mathcal{L}(e^{i\alpha t}))} + i \cdot \underbrace{\frac{\alpha}{s^2+\alpha^2}}_{=Im(\mathcal{L}(e^{i\alpha t}))}$$

$$\Rightarrow \begin{cases} \mathcal{L}(\sin(\alpha t)) = \dfrac{\alpha}{s^2+\alpha^2}, & s>0 \\ \mathcal{L}(\cos(\alpha t)) = \dfrac{s}{s^2+\alpha^2}, & s>0 \end{cases}$$

this calculation can also be done "directly" via integration by parts ----

Inverse Laplace transform

given $F(s)$



* Both $\mathcal{L}$ and $\mathcal{L}^{-1}$ are linear ← this can be checked very easily

example 1)

we have $\mathcal{L}(e^{kt}) = \dfrac{1}{s-k}$ (see before)

$\Rightarrow \quad \mathcal{L}^{-1}\left(\dfrac{1}{s-k}\right) = e^{kt}$ "create (→ table")

example 2) $\mathcal{L}^{-1}\left(\dfrac{1}{(s+3)(s-2)}\right) = ?$

* ↔ "breuksplitsen"

use partial fractions. ← $\boxed{\text{see later}}$

write $\dfrac{1}{(s+3)(s-2)} = \dfrac{-1/5}{s+3} + \dfrac{1/5}{s-2}$

useful, since we "understand" these functions

$\boxed{\text{how ?}}$ $$\frac{1}{(s+3)(s-2)} = \frac{A}{s+3} + \frac{B}{s-2}$$

and find appropriate A & B values

work out: $$\frac{A(s-2)}{(s-2)(s+3)} + \frac{B(s+3)}{(s+3)(s-2)}$$ (multiply)

$$= \frac{A(s-2) + B(s+3)}{(s-2)(s+3)}$$ (merge)

$$= \frac{(A+B)s + 3B - 2A}{(s-2)(s+3)}$$ (re-arrange)

* "must be" $$\frac{0.s + 1}{(s-2)(s+3)}$$ (set ...)

two equations with two unknowns

$$\Rightarrow \begin{cases} A+B=0 \xrightarrow{\text{solve}} B=-A \\ 3B-2A=1 \leftrightarrow -5A=1 \end{cases}$$

$$\boxed{\begin{aligned} A &= -1/5 \\ B &= 1/5 \end{aligned}}$$

We know (from earlier calculations or from supplied table/formula sheet)

that
$$\mathcal{L}(e^{-kt}) = \frac{1}{s+k} \Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s+k}\right) = e^{-kt}$$

use this information in our formula

$$\mathcal{L}^{-1}\left(\frac{1}{(s+3)(s-2)}\right) = \mathcal{L}^{-1}\left(\frac{-1/5}{s+3} + \frac{1/5}{s-2}\right)$$

linearity of $\mathcal{L}^{-1}$

$$= -\frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) + \frac{1}{5}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$

$$= -\frac{1}{5} e^{-3t} + \frac{1}{5} e^{+2t}$$

"table" "table"

---

example 3) $\mathcal{L}^{-1}\left(\dfrac{s+1}{s^2(s^2+9)}\right)$

"breuksplitsen"

$$\frac{s+1}{s^2(s^2+9)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C(s+1)}{s^2+9}$$

why this one? & &

how and why?

work out (multiply) and solve for A, B, C

three equations with three unknowns

$$\Rightarrow A = 1/9,\ B = 1/9,\ C = 1/9$$

$$\Rightarrow \mathcal{L}^{-1}(\ldots) = \frac{1}{9} + \frac{1}{9}t - \frac{1}{9}\cos(3t) - \frac{1}{27}\sin(3t)$$

Properties of Laplace transform

Linearity

$$\mathcal{L}(a\,f(t)+b\,g(t)) = a.\,\mathcal{L}(f(t))+b.\,\mathcal{L}(g(t))$$

example 1

$$\mathcal{L}(3t+2e^{3t})$$
$$= 3.\,\mathcal{L}(t)+2.\,\mathcal{L}(e^{3t})$$
$$= 3.\,\underbrace{\frac{1}{s^2}}_{(s>0)}+2.\,\underbrace{\frac{1}{s-3}}_{(s>3)}$$

$\underline{\underline{s>3}}$

example 2

$$\mathcal{L}(5-3t+4\sin(2t)-6e^{4t})$$
$$= \mathcal{L}(5)-3\,\mathcal{L}(t)+4.\,\mathcal{L}(\sin(2t))-6\,\mathcal{L}(e^{4t})$$
$$= \underbrace{\frac{5}{s}}_{s>0}-3.\,\underbrace{\frac{1}{s^2}}_{s>0}+4.\,\underbrace{\frac{2}{s^2+4}}_{s>0}-6.\,\underbrace{\frac{1}{s-4}}_{s>4}$$

see before

$s>4$

!!

Shift theorem

$$\mathcal{L}(e^{at} f(t)) = F(s-a) \quad \text{with} \quad \boxed{F(s) = \mathcal{L}(f(t))}$$

$s > \sigma + a$ $\qquad$ $s > \sigma$

example 1 $\quad \mathcal{L}(t e^{-2t})$

we know : $\mathcal{L}(t) \overset{\text{TABLE}}{=} F(s) = \dfrac{1}{s^2} \quad , s > 0$

$$\Rightarrow \mathcal{L}(t e^{-2t}) = F(s+2) = \frac{1}{(s+2)^2} \quad , s > -2$$

$(a = -2)$

example 2 $\quad \mathcal{L}(e^{-3t} \sin(2t))$

we know : $\mathcal{L}(\sin(2t)) \overset{\text{TABLE}}{=} F(s) = \dfrac{2}{s^2+4} \quad , s > 0$

$$\Rightarrow \mathcal{L}(e^{-3t} \sin(2t)) = F(s+3) = \frac{2}{(s+3)^2+4}$$

$(a = -3)$

$$= \frac{2}{s^2+6s+13} \qquad s > -3$$

derivative of a transform

If $F(s) = \mathcal{L}(f(t))$, $s > \sigma$

then $\mathcal{L}(t^n f(t)) = (-1)^n \dfrac{d^n F(s)}{ds^n}$, $s > \sigma$

example 1

we know $\mathcal{L}(\sin(3t)) = F(s) = \dfrac{3}{s^2+9}$, $s > 0$

$\Rightarrow \mathcal{L}(t \cdot \sin(3t)) = (-1)^1 \dfrac{d^1 F(s)}{ds^1}$,

(n=1)

$= -\dfrac{dF(s)}{ds} = \dfrac{6s}{(s^2+9)^2}$, $s > 0$

check!

example 2

we know $\mathcal{L}(e^t) = F(s) = \dfrac{1}{s-1}$, $s > 1$!

$\Rightarrow \mathcal{L}(t^2 e^t) = (-1)^2 \dfrac{d^2 F(s)}{ds^2} = \dfrac{d^2}{ds^2}\left(\dfrac{1}{s-1}\right)$

(n=2)

$= -\dfrac{d}{ds}\left(\dfrac{1}{(s-1)^2}\right)$

$= \dfrac{2}{(s-1)^3}$, $s > 1$

proof for $n=1$ (similar for $n = 2, 3, \ldots$): $\dfrac{d}{ds}\left[\mathcal{L}f(t))(s)\right] = \dfrac{d}{ds}\int_0^\infty f(t)e^{-st}dt = \int_0^\infty f(t)\dfrac{d}{ds}(e^{-st})dt$

$= -\int_0^\infty t f(t) e^{-st} dt = -\mathcal{L}(t f(t))(s)$

partieel integreren

Transforms of derivatives

$$\mathcal{L}(f'(t)) \overset{\text{def}}{=} \int_0^\infty f'(t)\, e^{-st}\, dt = \underbrace{f(t)\, e^{-st}\Big|_{t=0}^{t=\infty}}_{=[0-f(0)]} + s \cdot \int_0^\infty f(t)\, e^{-st}\, dt$$

integration by parts

$$= -f(0) + s \cdot \mathcal{L}(f(t))$$

$$= \boxed{-f(0) + s \cdot F(s)}$$

integr. by parts

$$\mathcal{L}(f''(t)) \overset{\text{def}}{=} \int_0^\infty f''(t)\, e^{-st}\, dt = \underbrace{e^{-st} f'(t)\Big|_{t=0}^{t=\infty}}_{=-f'(0)} + s \int_0^\infty f'(t)\, e^{-st}\, dt$$

$$= -f'(0) + s \cdot \mathcal{L}(f'(t))$$

$$= -f'(0) + s \cdot \left(-f(0) + s F(s)\right)$$

$$= -f'(0) - s f(0) + s^2 F(s)$$

above

gaan we gebruiken om DV en op te lossen !

etcetera

$$\boxed{\mathcal{L}(f^{(n)}(t)) = s^n F(s) - \sum_{i=1}^{n} s^{i-1} f^{(i-1)}(0)}$$

$n = 1, 2, 3, \ldots$

Inverse transform

$$\begin{tikzcd} f(t) \arrow[rr, bend left, "\mathcal{L}"] & & F(s) \arrow[ll, bend left, "\mathcal{L}^{-1}"] \end{tikzcd}$$

example 1 if $\mathcal{L}(e^{at}) = \dfrac{1}{s-a}$ → table

then $\mathcal{L}^{-1}\left(\dfrac{1}{s-a}\right) = e^{at}$

example 2 if $\mathcal{L}(\sin(wt)) = \dfrac{w}{s^2+w^2}$ → table

then $\mathcal{L}^{-1}\left(\dfrac{w}{s^2+w^2}\right) = \sin(wt)$

etcetera ....

$\mathcal{L}$ is linear

$\mathcal{L}^{-1}$ is linear as well: $\mathcal{L}^{-1}(\alpha F(s) + \beta G(s))$

$$= \alpha \mathcal{L}^{-1}(F(s)) + \beta \mathcal{L}^{-1}(G(s))$$

("breuksplitsen")

Explanation ⟷ partial fractions

Rule 1 the numerator (teller) must be of at least one degree less than the denominator (noemer) (otherwise this does not work)

Rule 2 for each linear factor $ax+b$ in the denominator (noemer), there is a partial fraction of the form $\frac{A}{ax+b}$ (A constant)

rule 1 ok

degree 1, degree 2

example: $\frac{3x}{(2x+1)(x+4)} = \frac{A}{2x+1} + \frac{B}{x+4} \rightsquigarrow \begin{cases} A = -\frac{3}{7} \\ B = \frac{12}{7} \end{cases}$

Rule 3 if a linear factor is repeated n times in the denominator, there will be n corresponding partial fractions with degree 1 to n

four

example: $\boxed{n=4}$ $\frac{5x^3+7x-9}{(x+1)^4} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{(x+1)^4}$

degree 1, degree 2, degree 3, degree 4

rule 1 ok ← degree 3, degree 4

$\boxed{n=2}$ $\frac{x+5}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2} \rightsquigarrow \begin{cases} A = 1 \\ B = 2 \end{cases}$

degree 1, degree 2

$\boxed{n=3}$ $\frac{2x^2-3}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1} \rightsquigarrow \begin{cases} A = -\frac{1}{8} \\ B = 9/4 \\ C = -1/2 \\ D = 1/8 \end{cases}$

degree 1, degree 2, degree 3, degree 1

$\boxed{n=1}$

Rule 4 corresponding to any quadratic factor ax^2+bx+c in the denominator, there will be a partial fraction of the form $\dfrac{Ax+B}{ax^2+bx+c}$

example: $\dfrac{x^3-2}{x^4-1} = \dfrac{x^3-2}{(x^2+1)(x^2-1)} = \dfrac{x^3-2}{(x^2+1)(x+1)(x-1)}$ simplify as far as possible

degree 3, degree 4, make it quadratic

$$= \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x-1}$$

$(A=\frac{1}{2},\ B=1,\ C=\frac{3}{4},\ D=-\frac{1}{4})$

Rule 4a $\boxed{\text{repeated}}$ quadratic factors in the denominator are dealt with in a similar way to repeated linear factors. (see Rule 3 for linear factors)

example: $\dfrac{x^2+1}{(x^2+x+1)^2} = \dfrac{Ax+B}{x^2+x+1} + \dfrac{Cx+D}{(x^2+x+1)^2}$

degree 2, degree 4 $(n=2)$, degree 2 $(n=1)$, degree 4 $(n=1)$

"Exercise": $\dfrac{1}{x^2(x-2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-2}$ (from the rules) how to get A, B, C

multiply both sides by $x^2(x-2)$:

$$1 = Ax(x-2) + B(x-2) + Cx^2$$

"take" $x=0 \Rightarrow B = -\frac{1}{2}$

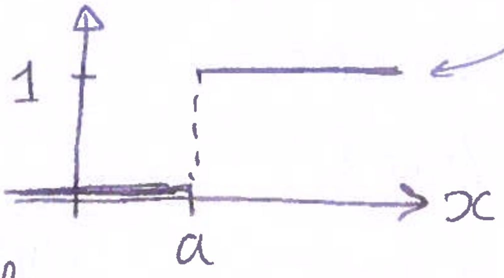
"take" $x=2 \Rightarrow C = 1/4$

$1 = Ax^2 - 2Ax + Bx - 2B + Cx^2$

comparing coefficients for $x^2 \Rightarrow A + C = 0 \Rightarrow A = -\frac{1}{4}$

extra

Heaviside function (unit step function)



$$\mathcal{H}(x-a) = \begin{cases} 0, & x < a \\ 1, & x \geq a \end{cases}$$

check notation

example:

$$\underbrace{\mathcal{H}(x-a) \cdot f(x-a)} = \begin{cases} 0, & x < a \quad (\text{because } \mathcal{H}=0) \\ f(x-a), & x \geq a \quad (\text{because } \mathcal{H}=1) \end{cases}$$

f : "signal", then : "same signal" with a "delay" of x

property

$$\mathcal{L}(\mathcal{H}(x-a) f(x-a))(s) = e^{-as} \cdot \underbrace{F(s)}_{= \mathcal{L}(f(x))(s)}$$

Convolution: $f * g(x) = \int_0^{x} f(x-\bar{x})\, g(\bar{x})\, d\bar{x}$!!! $(x \geq 0)$

(remember) but now slightly different

!!! instead of $\int_{-\infty}^{\infty}$ as in Fourier-transform

Property: $\boxed{\mathcal{L}(f * g) = \mathcal{L}(f) \cdot \mathcal{L}(g)}$

A

Voorbeeld 1

$$\boxed{\begin{cases} y' + 2y = 3 \\ y(0) = 5 \end{cases}}$$

Laplace $\Rightarrow$ transformatie

$$sY(s) - y(0) + 2Y(s) = \frac{3}{s}$$

invullen & herschrijven

$$\begin{cases} (s+2)\,Y(s) = \dfrac{3}{s} + 5 \\[2ex] Y(s) = \dfrac{3}{s(s+2)} + \dfrac{5}{s+2} \end{cases}$$

$$= 3\left[\frac{A}{s} + \frac{B}{s+2}\right] + 5 \cdot \frac{1}{s+2}$$

$$= 3\left[\frac{A(s+2) + Bs}{s(s+2)}\right] + 5 \cdot \frac{1}{s+2}$$

$$\Rightarrow (A+B)s + 2A \overset{\text{moet}}{=} 1$$

$$A + B = 0$$

$$A = 1/2, \quad B = -\frac{1}{2}$$

$$= 3 \cdot \frac{1}{2}\,\frac{1}{s} - 3\,\frac{1}{2} \cdot \frac{1}{s+2} + 5 \cdot \frac{1}{s+2}$$

$$= \frac{3}{2}\,\frac{1}{s} + \frac{7}{2}\,\frac{1}{s+2}$$

$$\Rightarrow y(t) = \frac{3}{2} + \frac{7}{2}e^{-2t}$$

check: $y' = -7e^{-2t}$

$$y' + 2y = \cancel{-7e^{-2t}} + 3 + \cancel{7e^{-2t}} = 3$$

$$y(0) = \frac{3}{2} + \frac{7}{2} = \frac{10}{2} = 5$$

voorbeeld 2

(B)

$$\boxed{\begin{cases} y' + 3y = e^{2t} \\ y(0) = 1 \end{cases}}$$

$$\mathcal{L}(y' + 3y) = s\,Y(s) - y(0) + 3\,Y(s) = (s+3)\,Y(s) - 1$$

$$\mathcal{L}(e^{2t}) = \frac{1}{s-2}$$

$$\Rightarrow (s+3)\,Y(s) - 1 = \frac{1}{s-2}$$

$$\Rightarrow \qquad Y(s) = \frac{1}{s+3} + \underbrace{\frac{1}{(s-2)(s+3)}}_{\text{schrijf als } \left\{\frac{A}{s-2} + \frac{B}{s+3}\right\}}$$

kruiselings vermenigvuldigen

$$= \frac{A(s+3) + B(s-2)}{(s-2)(s+3)}$$

moet gelden

$$\Rightarrow \quad 1 = A(s+3) + B(s-2)$$

dwz $\quad 1 = (A+B)s + 3A - 2B$

$$\Rightarrow \begin{cases} A + B = 0 \quad , B = -A \\ 3A - 2B = 1 \end{cases} \longrightarrow 3A + 2A = 1$$

$$5A = 1$$

$$A = 1/5$$

en $B = -1/5$

$$Y(s) = \frac{1}{s+3} + \frac{1}{5}\cdot\frac{1}{s-2} - \frac{1}{5}\cdot\frac{1}{s+3}$$

$$= \frac{4}{5}\cdot\frac{1}{s+3} + \frac{1}{5}\cdot\frac{1}{s-2}$$

neem inverse Laplace trafo (check tabel)

$$\Rightarrow \quad y(t) = \frac{4}{5}\cdot e^{-3t} + \frac{1}{5}\,e^{2t}$$

check: $y' = \frac{-12}{5}e^{-3t} + \frac{2}{5}e^{2t}$

$3y = \frac{12}{5}e^{-3t} + \frac{3}{5}e^{2t}$

$y' + 3y = \frac{5}{5}e^{2t} = e^{2t}$

$y(0) = \frac{4}{5} + \frac{1}{5} = 1$

voorbeeld 3

$$\begin{cases} y'' + 4y = 0 \\ y(0) = 1,\ y'(0) = 3 \end{cases}$$

C

Laplace transformatie $\Rightarrow$

$$s^2 Y(s) - s \underbrace{y(0)}_{=1} - \underbrace{y'(0)}_{=3} + 4Y(s) = 0$$

$$\Rightarrow Y(s) = \frac{s}{s^2+4} + \frac{3}{s^2+4} = \frac{3}{2} \cdot \frac{2}{s^2+2^2}$$

check tabel ("neem inverse Laplace transformatie")

$$\Rightarrow y(t) = \cos(2t) + \frac{3}{2} \sin(2t)$$

check (invullen) $\Rightarrow$ ∫

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extra om te oefenen

$$\begin{cases} y'' + 2y' + 5y = 0 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$$

(zie webpagina)

en

$$\begin{cases} y'' - 10y' + 9y = 5t \\ y(0) = -1,\ y'(0) = 2 \end{cases}$$

(" " ")

1^e vragenuur!

D

algemeen

$$\begin{cases} y'' + c_1 y' + c_2 y = c_3 \\ y(0) = A,\ y'(0) = B \end{cases}$$

constante (speciaal geval)

$\left.\begin{matrix} c_1 \\ c_2 \\ c_3 \\ A \\ B \end{matrix}\right\}$ gegeven

$$\Downarrow$$

$$\left[s^2 Y(s) - s\,\underbrace{y(0)}_{=A} - \underbrace{y'(0)}_{=B}\right] + c_1\left[s Y(s) - \underbrace{y(0)}_{=A}\right] + c_2 \cdot \left[Y(s)\right] = \frac{c_3}{s}$$

$$\Downarrow$$

$$(s^2 + c_1 s + c_2)\, Y(s) = \frac{c_3}{s} + sA + B + c_1 A$$

$$Y(s) = \frac{c_3}{s(s^2 + c_1 s + c_2)} + \frac{sA}{s^2 + c_1 s + c_2} + \frac{B + c_1 A}{s^2 + c_1 s + c_2}$$

"breuksplitsen"

$Y(s) = \text{-- -- -- -- --}$

$\Downarrow$ inverse toepassen Laplace (tabel gebruiken)

$y(t) = \text{--}$