$$\boxed{\text{Vraag 1}}$$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+10}$

comparison test : $\displaystyle\frac{1}{n^2+10} < \frac{1}{n^2}$ en $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2} < \infty$

(b) $\displaystyle\sum_{n=1}^{\infty} n^{-1/6}$

integral test : $\displaystyle\int_1^{\infty} x^{-1/6}\,dx = \frac{6}{5} x^{5/6} \Big|_{x=1}^{x=\infty} = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2n-1}{2^n}$

ratiotest : $\displaystyle\left(\frac{2(n+1)-1}{2^{n+1}}\right) \Big/ \left(\frac{2n-1}{2^n}\right) = \frac{2n+1}{2^{n+1}} \cdot \frac{2^n}{2n-1}$

$$= \frac{2+\frac{1}{n}}{2-\frac{1}{n}} \cdot \frac{1}{2} \underset{n\to\infty}{\to} \frac{1}{2} < 1$$

(d) $\displaystyle\sum_{n=0}^{\infty} n^{\frac{4}{3}}$

$\displaystyle\lim_{n\to\infty} n^{\frac{4}{3}} \neq 0$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$: dalende rij "$\frac{1}{n^3}$"
en "alternerend" $(-1)^n$

$\boxed{\text{Vraag 2}}$ $\begin{cases} y' = 4x - 2xy \\ y(0) = 5 \end{cases}$

Scheiding van variabelen

$$y' = 2x\,(2-y)$$

$$\Leftrightarrow \frac{dy}{dx} = 2x\,(2-y)$$

$$\Leftrightarrow \int \frac{dy}{2-y} = \int 2x\,dx$$

$$\Leftrightarrow -\ln(2-y) = x^2 + c$$

$$\Leftrightarrow 2-y = e^{-x^2}.\tilde{c}$$

$$\Leftrightarrow -y = \tilde{c}\,e^{-x^2} - 2$$

$$\Leftrightarrow y = 2 - \tilde{c}\,e^{-x^2}$$

$$y(0) = 2 - \tilde{c}.e^{0} = 2 - \tilde{c} = 5 \Rightarrow \tilde{c} = -3$$

$$\Downarrow$$

$$\boxed{y(x) = 2 + 3e^{-x^2}}$$

$\boxed{\text{Vraag 3}}$

$$E(T) = \cos(T^2) - 1 + \frac{1}{2}T^4$$
$$\rho(T) = T^2(T - \sin(T))^2$$

(a) $\cos(T) = 1 - \frac{1}{2}T^2 + \frac{1}{24}T^4 + \cdots$

$\sin(T) = T - \frac{1}{6}T^3 + \frac{1}{120}T^5 - \frac{1}{720}T^7 + \cdots$

$\Rightarrow \cos(T^2) = 1 - \frac{1}{2}T^4 + \frac{1}{24}T^8 + \cdots$

$\cos(T^2) - 1 + \frac{1}{2}T^4 = \frac{1}{24}T^8 + \cdots = E(T)$ t/m T^8

en $(T - \sin(T))^2 = \left(\frac{1}{6}T^3 - \frac{1}{120}T^7 + \frac{1}{720}T^9 \cdots\right)^2$

$= \frac{1}{36}T^6 + \cdots$

$T^2(T - \sin(T))^2 = \frac{1}{36}T^8 + \cdots = \rho(T)$ t/m T^8

(b) $$\lim_{T \to 0} \frac{E(T)}{\rho(T)} = \lim_{T \to 0} \frac{\frac{1}{24}T^8 + \cdots}{\frac{1}{36}T^8 + \cdots} = \frac{\frac{1}{24}}{\frac{1}{36}} = \frac{36}{24} = \frac{3}{2}$$

Vraag 4

$$\begin{cases} a' = -a + 2b & , a(0) = 1 \\ b' = a - 2b & , b(0) = 5 \end{cases}$$

---

matrix $\begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}$

<u>eigenwaarden:</u> $\begin{vmatrix} -1-\lambda & 2 \\ 1 & -2-\lambda \end{vmatrix} = 0$

$\Leftrightarrow 2 + \lambda + 2\lambda + \lambda^2 - 2 = 0$

$\Leftrightarrow \lambda^2 + 3\lambda = 0$

$\Leftrightarrow \lambda(\lambda + 3) = 0 \Leftrightarrow \lambda = 0$ of $\lambda = -3$

<u>eigenvectoren:</u>

bij $\lambda = 0$: $-x_1 + 2x_2 = 0 \Rightarrow x_1 = 2x_2$

$\Rightarrow \alpha . \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\alpha \neq 0$

bij $\lambda = -3$: $2x_1 + 2x_2 = 0 \Rightarrow x_1 = -x_2$

$\Rightarrow \beta . \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\beta \neq 0$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix}(t) = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{0.t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t}$$

$$\Rightarrow \begin{cases} a(t) = 2c_1 + c_2 e^{-3t} \\ b(t) = c_1 - c_2 e^{-3t} \end{cases}$$

$$\begin{cases} a(0) = 1 \\ b(0) = 5 \end{cases} \Rightarrow \begin{cases} 1 = 2c_1 + c_2 \\ 5 = c_1 - c_2 \end{cases} \xRightarrow{\text{oplossen}} \begin{cases} c_1 = 2 \\ c_2 = -3 \end{cases}$$

$$\Longrightarrow \begin{cases} a(t) = 4 - 3e^{-3t} \\ b(t) = 2 + 3e^{-3t} \end{cases}$$

$$\boxed{\text{Vraag } 5}$$

$$\mathcal{E} = 3x^4 - 4x^3 - 12x^2 + 12y^2 + 2023$$

(a)
$$\left.\begin{aligned} \mathcal{E}_x &= 12x^3 - 12x^2 - 24x \\ &= 12x\,(x^2 - x - 2) \end{aligned}\right\} \Rightarrow \operatorname{grad}(\mathcal{E}) = \begin{pmatrix} 12x(x^2-x-2) \\ 24y \end{pmatrix}$$
$$\mathcal{E}_y = 24y$$

(b)
$$\operatorname{grad}(\mathcal{E})\Big|_{(1,1)} = \begin{pmatrix} 12 \times (-2) \\ 24 \end{pmatrix} = \begin{pmatrix} -24 \\ 24 \end{pmatrix}$$

$$D_{\vec{b}}\,\mathcal{E}\Big|_{(1,1)} = \frac{\vec{b}}{|\vec{b}|} \bullet \operatorname{grad}(\mathcal{E})\Big|_{(1,1)} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \bullet \begin{pmatrix} -24 \\ 24 \end{pmatrix} = -\frac{72}{5} + \frac{96}{5}$$
$$\vec{b} = (3,4)^T \Rightarrow |\vec{b}| = 5 \qquad = \frac{24}{5}$$

(c)
$$\mathcal{E}_{xx} = 36x^2 - 24x - 24$$
$$\mathcal{E}_{yy} = 24 \qquad \Rightarrow \mathcal{H} = \begin{pmatrix} 36x^2 - 24x - 24 & 0 \\ 0 & 24 \end{pmatrix}$$
$$\mathcal{E}_{xy} = \mathcal{E}_{yx} = 0$$

en
$$\operatorname{div}(\operatorname{grad}(\mathcal{E})) = \mathcal{E}_{xx} + \mathcal{E}_{yy}$$
$$= 36x^2 - 24x \cancel{-24} \cancel{+24}$$
$$= 12x(3x-2)$$

(d)
$$\mathcal{E}_y = 0 \Rightarrow 24y = 0 \Rightarrow y = 0$$
$$\mathcal{E}_x = 0 \Rightarrow x = 0 \text{ of } x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = \Big\langle\begin{smallmatrix} 2 \\ -1 \end{smallmatrix} \Rightarrow \begin{matrix} (0,0) \\ (2,0) \\ (-1,0) \end{matrix}$$

(e)

	$\det(\mathcal{H})$	$\Delta\mathcal{E}$	
(0,0)	$-24^2<0$	0	ZADEL
(2,0)	>0	>0	MIN
(−1,0)	>0	>0	MIN

$\boxed{\text{Vraag 6}}$

$$\begin{cases} x' = y - 2xy \\ y' = 2xy - x \end{cases}$$

(a) kritieke punten: $\begin{cases} y(1-2x) = 0 \\ x(2y-1) = 0 \end{cases}$

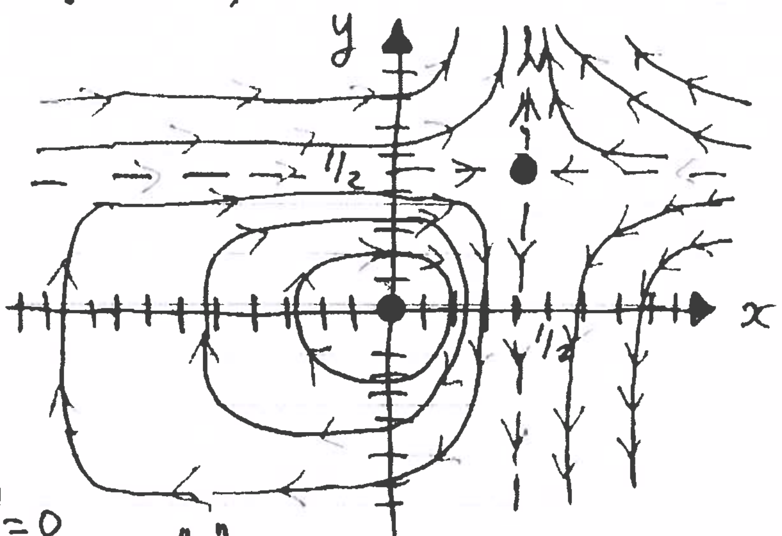
$\Rightarrow (0,0)$ en $(\frac{1}{2}, \frac{1}{2})$

(b) Jacobiaan: $\begin{pmatrix} -2y & 1-2x \\ 2y-1 & 2x \end{pmatrix}$

in $(0,0)$: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ centrumpunt $\begin{cases} \lambda_1 = i \\ \lambda_2 = -i \end{cases}$

in $(\frac{1}{2}, \frac{1}{2})$: $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$ zadelpunt $\begin{cases} \lambda_1 = -1 \\ \lambda_2 = 1 \end{cases}$

(c) $\dfrac{dy}{dx} = \dfrac{x(2y-1)}{y(1-2x)} = \begin{cases} \to \infty \text{ als } y=0 \text{ of } x = \frac{1}{2} \\ \to 0 \text{ als } x=0 \text{ of } y = \frac{1}{2} \end{cases}$



op $\begin{cases} x = -1 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x' = 0 \\ y' = 1 > 0 \end{cases} \Rightarrow$ "$\uparrow$"

$\boxed{\text{Vraag 7}}$

$$\begin{cases} y' = x^2 y \\ y(0) = 2023 \end{cases}$$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \Rightarrow y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$-x^2 y = -\sum_{n=0}^{\infty} a_n x^{n+2} = -\sum_{n=2}^{\infty} a_{n-2} x^n$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$\Rightarrow a_1 + 2a_2 x + \sum_{n=2}^{\infty} \left[(n+1)a_{n+1} - a_{n-2}\right] x^n = 0$$

$$\Rightarrow \boxed{a_1 = a_2 = 0} \text{ en } a_{n+1} = \frac{1}{n+1} a_{n-2} \quad n = 2, 3, 4, \ldots$$

$$\boxed{a_0 = y(0) = 2023}$$

$a_4 = 0, a_7 = 0$, etc ...

$a_5 = 0, a_8 = 0$, etc ...

$$a_3 = \frac{2023}{3} \quad a_6 = \frac{2023}{18}, \quad a_9 = \frac{2023}{162}$$

$$\Rightarrow \boxed{a_0 = 2023, a_1 = 0, a_2 = 0, a_3 = \frac{2023}{3}, a_4 = 0, a_5 = 0, a_6 = \frac{2023}{18}, a_7 = 0, a_8 = 0, a_9 = \frac{2023}{162}}$$

$$y(x) = 2023 + \frac{2023}{3} x^3 + \frac{2023}{18} x^6 + \frac{2023}{162} x^9 + \cdots \quad \left(\equiv 2023\, e^{\frac{1}{3}x^3}\right)$$

check Taylorreeks!!

Vraag 8

$$\begin{cases} u_t = u u_{xx} \\ u(0,t) = u(1,t) = \dfrac{1}{1-t} \quad (0<t<1) \end{cases}$$

aannname: $u(x,t) = \dfrac{X(x)}{1-t}$

$$\Rightarrow \quad u_t = X \cdot \frac{1}{(1-t)^2} = \frac{X}{1-t} \cdot \frac{X''}{1-t} = u u_{xx} \quad (t \neq 1)$$

$X \neq 0$

($\Rightarrow$ niet-triviale oplossingen)

$$\Rightarrow X'' = 1 \Rightarrow X' = x + c$$

$$X(x) = \frac{1}{2}x^2 + cx + d$$

$$u(0,t) = \left.\frac{\frac{1}{2}x^2 + cx + d}{1-t}\right|_{x=0} = \frac{d}{1-t} = \frac{1}{1-t} \Rightarrow d = 1$$

$$\Rightarrow u(x,t) = \frac{\frac{1}{2}x^2 + cx + 1}{1-t}$$

$$u(1,t) = \frac{\frac{1}{2} + c + 1}{1-t} = \frac{1}{1-t} \Rightarrow c = -\frac{1}{2}$$

$$\Rightarrow X(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

$$\Longrightarrow u(x,t) = \frac{\frac{1}{2}x^2 - \frac{1}{2}x + 1}{1-t}$$

$\boxed{\text{Vraag 9}}$ $\begin{cases} 2y'' - y' - y = 4xe^{2x} \\ y(0) = -\frac{28}{25}, \; y'(0) = \frac{39}{25} \end{cases}$

algemene <u>homogene</u> oplossing:

$$2y_h'' - y_h' - y_h = 0 \Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$y_h = e^{\lambda x} \qquad \Rightarrow \lambda_1 = 1, \lambda_2 = -\frac{1}{2}$$

$$\Rightarrow y_h = c_1 e^{x} + c_2 e^{-\frac{1}{2}x}$$

een <u>particuliere</u> oplossing:

$$\text{stel } y_p = (Ax+B)e^{2x} \Rightarrow y_p' = Ae^{2x} + 2(Ax+B)e^{2x}$$

$$= (A+2B+2Ax)e^{2x}$$

$$\Rightarrow y_p'' = 2Ae^{2x} + 2(A+2B+2Ax)e^{2x}$$

$$= e^{2x}(\cancel{2A+2A+4B+4Ax}) = e^{2x}(4A+4B+4Ax)$$

invullen in DV: $2e^{2x}(4A+4B+4Ax)$

$$- e^{2x}(A+2B+2Ax)$$

$$- e^{2x}(Ax+B) = 4xe^{2x}$$

delen door $e^{2x} \neq 0$: $8A + 8B + 8Ax - A - 2B - 2Ax - Ax - B = 4x$

$$\Leftrightarrow 7A + 5B + 5Ax = 4x$$

$$\Rightarrow \begin{cases} 5A = 4 & \Rightarrow A = 4/5 \\ 7A + 5B = 0 & \Rightarrow B = -\frac{28}{25} \end{cases} \Rightarrow y_p = \left(\frac{4}{5}x - \frac{28}{25}\right)e^{2x}$$

de <u>volledige</u> oplossing: (gebruik de twee beginvoorwaarde om c_1 en c_2 te bepalen)

$$y = c_1 e^{x} + c_2 e^{-\frac{1}{2}x} + \left(\frac{4}{5}x - \frac{28}{25}\right)e^{2x}$$

$$y(0) = c_1 + c_2 - \frac{28}{25} = -\frac{28}{25} \Rightarrow c_1 + c_2 = 0$$

$$y' = c_1 e^{x} - \frac{1}{2}c_2 e^{-\frac{1}{2}x} + \frac{4}{5}e^{2x} + 2\left(\frac{4}{5}x - \frac{28}{25}\right)e^{2x}$$

$$y'(0) = c_1 - \frac{1}{2}c_2 + \frac{4}{5} - \frac{56}{25} = \frac{39}{25} \Rightarrow c_1 - \frac{1}{2}c_2 = \frac{75}{25} = 3$$

$$\Rightarrow c_1 = 2, c_2 = -2 \quad \Rightarrow \boxed{y = 2e^{x} - 2e^{-\frac{1}{2}x} + \left(\frac{4}{5}x - \frac{28}{25}\right)e^{2x}}$$