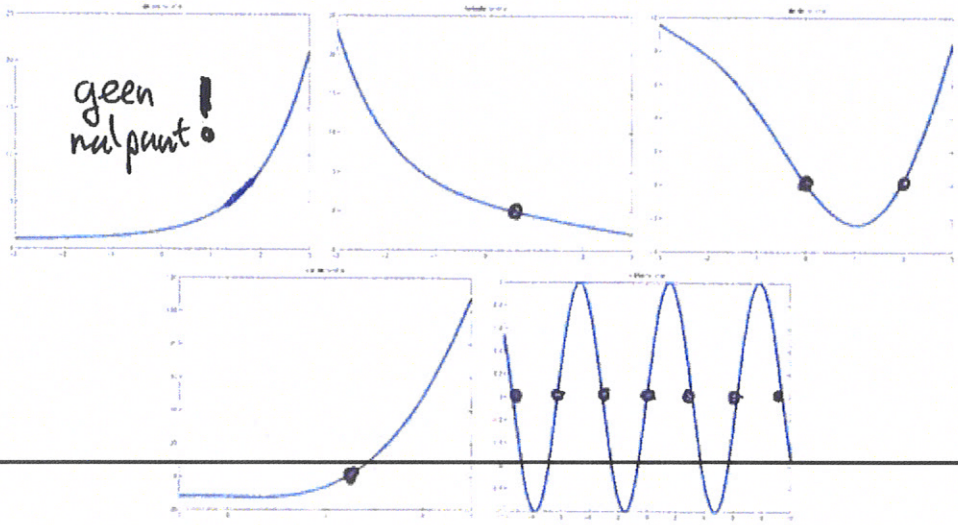
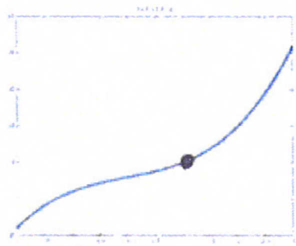
Opgave N1

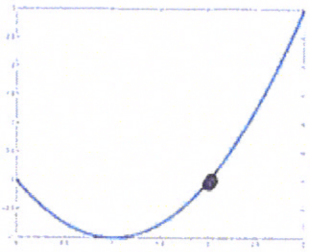


Opgave N2; $f(x) = x^3 + 3x - 4 = 0$; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$



i	x_i	$\lvert x_i - \alpha \rvert$	$\lvert f(x_i) \rvert$
0	2.00000000000000	1.00000000000000	10
1	1.33333333333333	0.33333333333333	2.37037037037034
2	1.04888888888889	0.04888888888889	0.30062055418382
3	1.00117515546039	0.00117515546039	0.00705507735629
4	1.00000069022454	0.00000069022454	4.141348669328693e-06
5	1.00000000000024	0.00000000000024	1.440625396753603e-12

Opgave N3; $f(x) = x^2 - 2x$; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

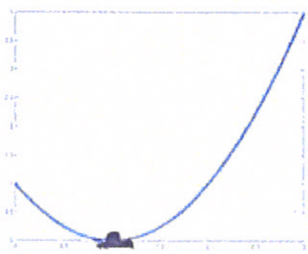


! $\Rightarrow$ 2e orde convergentie

$C = 0$

i	x_i	$\lvert x_i - \alpha \rvert$	$\lvert f(x_i) \rvert$	$\frac{x_{i+1}-\alpha}{x_i-\alpha}$	$\frac{x_{i+1}-\alpha}{(x_i-\alpha)^2}$
0	3.00000000000000	1.00000000000000	3.00000000000000	-	-
1	2.25000000000000	0.25000000000000	0.56250000000000	0.2500	0.2500
2	2.02500000000000	0.02500000000000	0.05062500000000	0.1000	0.4000
3	2.00030487804878	0.00030487804878	0.00060984904819	0.0121	0.4878
4	2.00000004646115	0.00000004646115	0.00000009292230	0.0001	0.4998

Opgave N3; $f(x) = x^2 - 2x + 1$; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$



! $\Rightarrow$ 1e orde convergentie

$C = 1$

i	x_i	$\lvert x_i - \alpha \rvert$	$\lvert f(x_i) \rvert$	$\frac{x_{i+1}-\alpha}{x_i-\alpha}$	$\frac{x_{i+1}-\alpha}{(x_i-\alpha)^2}$
0	2.00000000000000	1.00000000000000	1.00000000000000	-	-
1	1.50000000000000	0.50000000000000	0.25000000000000	0.5000	0.5000
2	1.25000000000000	0.25000000000000	0.06250000000000	0.5000	1.000
3	1.12500000000000	0.12500000000000	0.01562500000000	0.5000	2.000
4	1.06250000000000	0.06250000000000	0.00390625000000	0.5000	4.000
5	1.03125000000000	0.03125000000000	0.00097656250000	0.5000	8.000
6	1.01562500000000	0.01562500000000	0.00024414062500	0.5000	16.000

Opgave N4

Taylorontwikkeling:

$$x_{i+1} - \alpha = (x_i - \alpha)g'(\alpha) + \frac{1}{2}(x_i - \alpha)^2 g''(\alpha) + \ldots \approx (x_i - \alpha)g'(\alpha)$$

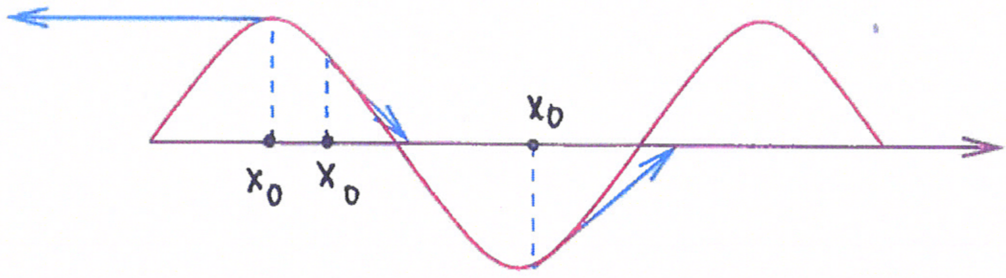
Voor NR , als $f'(\alpha) \neq 0 \Rightarrow g'(\alpha) = 0$: de fout $\approx \frac{1}{2}\frac{x_{i+1}-\alpha}{(x_i-\alpha)^2} g''(\alpha)$ kwadratische afname

Als $f'(\alpha) = 0$, dan $g'(\alpha) = "\frac{0}{0}"$; is dus, voor dit voorbeeld:

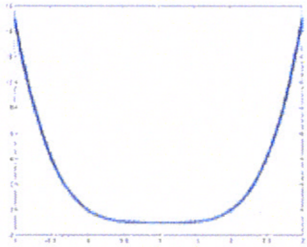
$$g'(1) = \lim_{x \to 1} \frac{f(x)f''(x)}{(f'(x))^2} = f''(1) \lim_{x \to 1} \frac{f(x)}{(f'(x))^2} =$$

$$2 \lim_{x \to 1} \frac{f'(x)}{2f'(x)f''(x)} = 2 \lim_{x \to 1} \frac{1}{2f''(x)} = 2\frac{1}{4} = \frac{1}{2} \neq 0 \Rightarrow \text{lineaire afname van de fout}$$

Opgave N5; $f(x) = \cos(x)$; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$



Opgave N6; $f(x) = (x-1)^4 - 1$; $x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$



i	x_i	$\lvert x_i - \beta \rvert$	$\lvert f'(x_i) \rvert$
0	1.50000000000000	0.50000000000000	0.50000000000000
1	1.33333333333333	0.33333333333333	0.14814814814815
2	1.22222222222222	0.22222222222222	0.04389574759945
3	1.14814814814815	0.14814814814815	0.01300614743687
...	...	...	
11	1.00867076495792	0.00867076495792	0.00000260754753
12	1.00578050997194	0.00578050997194	0.00000077260667
13	1.00385367331463	0.00385367331463	0.00000022892050
14	1.00256911554309	0.00256911554309	0.00000006782830

Opgave N7

Beginconcentratie $f(0) = 12$

We willen weten voor welke t geldt dat

$10\,e^{-3t} + 2\,e^{-5t} = 6$

m.a.w.: we willen de nulpunten uitrekenen van de functie $g(t) = 10\,e^{-3t} + 2\,e^{-5t} - 6$

Pas NR toe op $g(t)$

Antwoord: $t \approx 0.211$

Find a solution to 8 significant figures of the following equations by the Newton–Raphson method starting in every case with $x_0 = 1$ (see Exercises 9 to 11):

12. $x^2 - \ln x = 2$

Write $f(x) = x^2 - \ln x - 2$. Then $f'(x) = 2x - 1/x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{\,2} - \ln x_n - 2}{2x_n - 1/x_n}$$

The calculation is summarized in Table 6:

Table 6

n	x_n	$f(x_n)/f'(x_n)$	x_{n+1}
0	1	−1	2
1	2	0.37338652	1.62661348
2	1.62661348	0.06040327	1.56621021
3	1.56621021	0.00174647	1.56446373
4	1.56446373	0.00000148	**1.56446226**
5	1.56446226	0.00000000	**1.56446226**

Therefore $x = 1.5644623$

13. $e^{-x} = \tan x$

Write $f(x) = e^{-x} - \tan x$. Then $f'(x) = -e^{-x} - \sec^2 x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{e^{-x_n} - \tan x_n}{e^{-x_n} - \sec^2 x_n}$$

The calculation is summarized in Table 7:

Table 7

n	x_n	$f(x_n)/f'(x_n)$	x_{n+1}
0	1	0.313578539	0.686421461
1	0.686421461	0.145291364	0.541130097
2	0.541130097	0.009714010	0.531416087
3	0.531416087	0.000025230	**0.531390857**
4	0.531390857	0.000000000	**0.531390857**

Therefore $x = 0.53139086$

14. $x^3 - 3x^2 + 6x = 5$

Write $f(x) = x^3 - 3x^2 + 6x - 5$. Then $f'(x) = 3x^2 - 6x + 6$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{\ 3} - 3x_n^{\ 2} + 6x_n - 5}{3x_n^{\ 2} - 6x_n + 6}$$

The calculation is summarized in Table 8:

Table 8

n	x_n	$f(x_n)/f'(x_n)$	x_{n+1}
0	1	−0.333333333	1.333333333
1	1.333333333	0.011111111	1.322222222
2	1.322222222	0.000036867	**1.322185355**
3	1.322185355	0.000000000	**1.322185355**

Therefore $x = 1.3221854$

15. Given one root, x_1, of a polynomial of degree n, a second root can be obtained by first dividing the polynomial by the factor $(x - x_1)$ to give a polynomial of degree $n-1$. Show that the computed root of the cubic in Exercises14 is the only real one.

The computed root of the cubic in Exercise 14 is $x = 1.322$ to 4 significant figures.
The cubic can then be written as

$$f(x) = x^3 - 3x^2 + 6x - 5 = (x - 1.322)(ax^2 + bx + c)$$

and the quadratic can be obtained by algebraic division (Section 2.6):

$$\begin{array}{r|l}
 & x^2 - 1.678x + 3.782 \\
\hline
x - 1.322 & x^3 - 3x^2 \quad + 6x - 5 \\
 & \underline{x^3 - 1.322x^2} \\
 & -1.678x^2 + 6x \quad -5 \\
 & \underline{-1.678x^2 + 2.218x} \\
 & 3.782x - 5 \\
 & \underline{3.782x - 5} \\
 & 0
\end{array}$$

The roots of the quadratic are

$$x^2 - 1.678x + 3.782 = 0 \text{ when } x = \frac{1}{2}\left[1.678 \pm \sqrt{1.678^2 - 4 \times 3.782}\right] = \frac{1}{2}\left[1.678 \pm \sqrt{-12.31}\right]$$

The discriminant is negative, so that the roots of the quadratic are complex, and the cubic has only the one real root.