Trigonometric Identities

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad \cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad \sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos\left(a - \frac{\pi}{2}\right) = \sin a \qquad \sin\left(a + \frac{\pi}{2}\right) = \cos a$$

$$\cos^2 a = \frac{1+\cos 2a}{2} \qquad \sin^2 a = \frac{1-\cos 2a}{2}$$

$$\cos 2a = \cos^2 a - \sin^2 a \qquad \sin 2a = 2 \sin a \cos a$$

$$\cos a + \sin a = \sqrt{2}\cos(a - \frac{\pi}{4}) \qquad \cos a + \sin a = \sqrt{2}\sin(a + \frac{\pi}{4})$$

$$\alpha \cos a + \beta \sin a = \sqrt{\alpha^2+\beta^2}\cos(a-b), \text{ where } \cos b = \frac{\alpha}{\sqrt{\alpha^2+\beta^2}} \text{ and } \sin b = \frac{\beta}{\sqrt{\alpha^2+\beta^2}}$$

Complex Numbers

$$z = x + iy \qquad \overline{z} = x - iy \qquad \mathrm{Re}\,(z) = x, \quad \mathrm{Im}\,(z) = y$$

$$z + \overline{z} = 2\,\mathrm{Re}\,(z) = 2x \qquad z - \overline{z} = 2i\,\mathrm{Im}\,(z) = 2iy \qquad |z| = \sqrt{z\cdot\overline{z}} = \sqrt{x^2+y^2}$$

$$|z|^2 = z\cdot\overline{z} = x^2 + y^2 \qquad z^2 = x^2 - y^2 + 2ixy \qquad \frac{1}{z} = \frac{\overline{z}}{z\cdot\overline{z}} = \frac{x-iy}{x^2+y^2} \; (z \neq 0)$$

Triangle Inequality: If z and w are any complex numbers, then

$$|z \pm w| \le |z| + |w| \quad \text{and} \quad \big|\,|z| - |w|\,\big| \le |z \pm w|$$

Euler's Identity and Related Identities: If x is any real number, then

$$e^{ix} = \cos x + i \sin x \qquad e^{-ix} = \cos x - i \sin x \qquad e^{-ix} = \overline{(e^{ix})} \qquad |e^{ix}| = 1$$

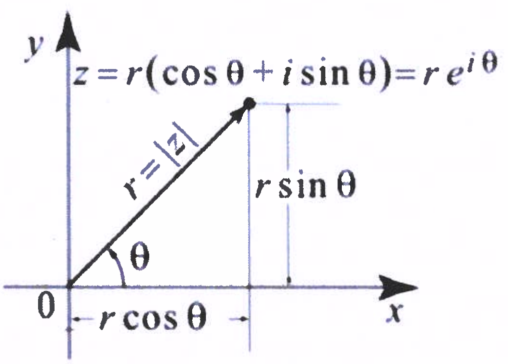
$$\cos x = \frac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \frac{e^{ix}-e^{-ix}}{2i} \qquad (\cos nx + i \sin nx)^n = e^{inx} = \cos nx + i \sin nx$$

Polar Representation of Complex Numbers

$$z = x + iy = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$r = |z| = \sqrt{x^2+y^2}$$

$$x = r\cos\theta, \quad y = r\sin\theta$$



Voorbeelden van Taylorreeksen <u>ROND 0</u>

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \qquad \text{alle } x$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \qquad \text{alle } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad \text{alle } x$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n \qquad |x| < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n \qquad |x| < 1$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \qquad |x| < 1$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} x^n \qquad |x| < 1$$

2

In 1D kennen we de Taylorontwikkeling van de functie f rond het punt a:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots$$

Vergelijk dit met de *2D-Taylorontwikkeling* van de functie $f(x,y)$ rond het punt (a,b):

$$f(x,y) = f(a,b) + (x-a)\tfrac{\partial f}{\partial x}(a,b) + (y-b)\tfrac{\partial f}{\partial y}(a,b) +$$
$$\tfrac{1}{2!}\{(x-a)^2\tfrac{\partial^2 f}{\partial x^2}(a,b) + (y-b)^2\tfrac{\partial^2 f}{\partial y^2}(a,b) + 2(x-a)(y-b)\tfrac{\partial^2 f}{\partial x \partial y}\} +$$
$$\tfrac{1}{3!}\{\ldots.\} + \ldots$$

Verder herinneren we ons de *gradient* van een functie $f(x,y)$:

$$\text{grad } f = \nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}.$$

De Jacobiaan van een *vector*functie $(f_1(x,y), f_2(x,y))$ wordt gegeven door de 2×2-matrix:

$$\mathcal{J} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}.$$

De *Hessiaan* van een functie $f(x,y)$ is gedefinieerd als de $2 \times 2-$matrix:

$$\mathcal{H} = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}.$$

De matrix $\mathcal{H}$ is symmetrisch, want

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

Een verband tussen $\mathcal{H}$, $\mathcal{J}$ en ∇f wordt uitgewerkt in opgave N8 en luidt:

$$\mathcal{H} = \mathcal{J}(\text{grad} f).$$

De functie $z = f(x, y)$ stelt een 2D-oppervlak in 3D voor (denk aan een 'vervormd velletje papier').
Voor constante waarden van z kunnen we ook een contourplot maken (zie figuur 2) voor een verband tussen een 3D-plot en een contourplot van hetzelfde oppervlak.
De punten (x, y) waarvoor $\nabla f = \vec{0}$ heten 'stationaire punten' van f en die punten komen in aanmerking voor een minimum of maximum. Voor een minimum in 2D moet gelden:

$$\nabla f = \vec{0} \quad \text{en} \quad \begin{cases} \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2 = \det(\mathcal{H}) > 0 \\ \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f > 0 \end{cases}$$

terwijl voor een maximum geldt:

$$\nabla f = \vec{0} \quad \text{en} \quad \begin{cases} \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2 = \det(\mathcal{H}) > 0 \\ \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f < 0 \end{cases}$$

Als

$$\nabla f = \vec{0} \quad \text{en} \quad \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2 = \det(\mathcal{H}) < 0$$

hebben we te maken met een zogenaamd zadelpunt. Voor alle overige situaties is het niet direkt duidelijk hoe de functie zich gedraagt rondom het 'stationaire punt'. Een nadere analyse is dan nodig om het gedrag van f te begrijpen rond dit soort speciale punten.

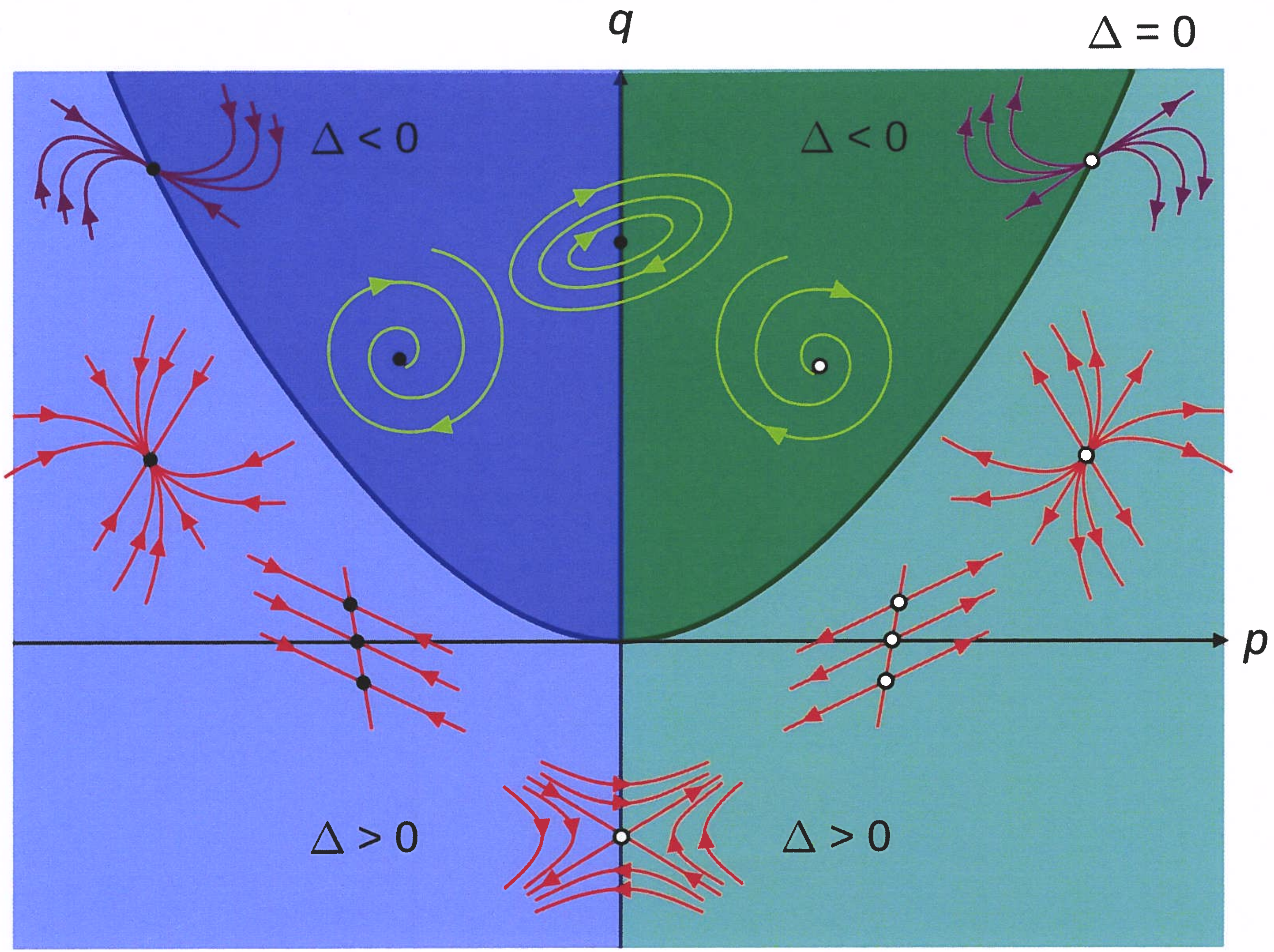
Separation of Variables

If f and g are continuous functions, then the differential equation

$$\frac{dy}{dx} = f(x)g(y)$$

has a general solution of

$$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx + C.$$



$$\frac{dx}{dt} = Ax + By$$

$$\frac{dy}{dt} = Cx + Dy$$

$p = A + D$ = spoor$\left[\begin{pmatrix} A & B \\ C & D \end{pmatrix}\right]$

$q = AD - BC$ = det$\left[\begin{pmatrix} A & B \\ C & D \end{pmatrix}\right]$

$\Delta = p^2 - 4q$

Fourier reeks

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(n\frac{\pi x}{L}\right) \right]$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right) dx, n = 0, 1, 2, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right) dx, n = 1, 2, \ldots$$

Fourier transformatie

Suppose that f is piecewise smooth on every finite interval and that $\int_{-\infty}^{\infty} |f(x)|\,dx < \infty$. Then f has the following **Fourier integral representation**

$$f(x) = \int_0^\infty [A(\omega)\cos\omega x + B(\omega)\sin\omega x]\,d\omega \quad (-\infty < x < \infty),$$

where, for all $\omega \geq 0$,

$$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt; \qquad B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt.$$

The integral in (2) converges to $f(x)$ if f is continuous at x and to $\frac{f(x+)+f(x-)}{2}$ otherwise.

FOURIER TRANSFORM

$$\widehat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx \qquad (-\infty < \omega < \infty)$$

and

INVERSE FOURIER TRANSFORM

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{i\omega x}\widehat{f}(\omega)\,d\omega \qquad (-\infty < x < \infty).$$

The Fourier transform is a linear operation; that is, for any integrable functions f and g and any real numbers a and b,

$$\mathcal{F}(af + bg) = a\mathcal{F}(f) + b\mathcal{F}(g).$$

(i) Suppose $f(x)$ is piecewise smooth, $f(x)$ and $f'(x)$ are integrable, and $f(x) \to 0$ as $|x| \to \infty$, then

$$\mathcal{F}(f') = i\omega\,\mathcal{F}(f).$$

(ii) If in addition $f''(x)$ is integrable, and $f'(x)$ is piecewise smooth and $\to 0$ as $|x| \to \infty$, then

$$\mathcal{F}(f'') = i\omega\mathcal{F}(f') = -\omega^2\mathcal{F}(f).$$

(iii) In general, if f and $f^{(k)}(x)$ $(k = 1, 2, \ldots, n-1)$ are piecewise smooth and tend to 0 as $|x| \to \infty$, and f and its derivatives of order up to n are integrable, then

$$\mathcal{F}(f^{(n)}) = (i\omega)^n\mathcal{F}(f).$$

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12 APPENDIX A: integratieformules

1. Common Integrals

Indefinite Integral

Method of substitution

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integrals of Rational and Irrational Functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int c\,dx = cx + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$\int \sqrt{x} dx = \frac{2x\sqrt{x}}{3} + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

Integrals of Trigonometric Functions

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\tan x + \sec x| + C$$

$$\int \sin^2 x\,dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}(x + \sin x \cos x) + C$$

$$\int \tan^2 x\,dx = \tan x - x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

Integrals of Exponential and Logarithmic Functions

$$\int \ln x\,dx = x \ln x - x + C$$

$$\int x^n \ln x\,dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$

2. Integrals of Rational Functions

Integrals involving ax + b

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} \qquad (\text{for } n \neq -1)$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$$

$$\int x(ax+b)^n\,dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} \qquad (\text{for } n \neq -1, n \neq -2)$$

$$\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$

$$\int \frac{x}{(ax+b)^n}\,dx = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} \qquad (\text{for } n \neq -1, n \neq -2)$$

$$\int \frac{x^2}{ax+b}\,dx = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\ln|ax+b|\right)$$

$$\int \frac{x^2}{(ax+b)^2}\,dx = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$

$$\int \frac{x^2}{(ax+b)^3}\,dx = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right)$$

$$\int \frac{x^2}{(ax+b)^n}\,dx = \frac{1}{a^3}\left(-\frac{(ax+b)^{3-n}}{n-3} + \frac{2b(a+b)^{2-n}}{n-2} - \frac{b^2(ax+b)^{1-n}}{n-1}\right) \qquad (\text{for } n \neq 1,2,3)$$

$$\int \frac{1}{x(ax+b)}\,dx = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{1}{x^2(ax+b)}\,dx = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{1}{x^2(ax+b)^2}\,dx = -a\left(\frac{1}{b^2(a+xb)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right)$$

Integrals involving $ax^2 + bx + c$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\,arctg\,\frac{x}{a}$$

$$\int \frac{1}{x^2-a^2}\,dx = \begin{cases} \frac{1}{2a}\ln\frac{a-x}{a+x} & for\,|x|<|a| \\ \frac{1}{2a}\ln\frac{x-a}{x+a} & for\,|x|>|a| \end{cases}$$

$$\int \frac{1}{ax^2+bx+c}dx = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\arctan\dfrac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } 4ac-b^2>0 \\ \dfrac{2}{\sqrt{b^2-4ac}}\ln\left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| & \text{for } 4ac-b^2<0 \\ -\dfrac{2}{2ax+b} & \text{for } 4ac-b^2=0 \end{cases}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

$$\int \frac{mx+n}{ax^2+bx+c}dx = \begin{cases} \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| + \dfrac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\dfrac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } 4ac-b^2>0 \\ \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| + \dfrac{2an-bm}{a\sqrt{b^2-4ac}}\operatorname{arctanh}\dfrac{2ax+b}{\sqrt{b^2-4ac}} & \text{for } 4ac-b^2<0 \\ \dfrac{m}{2a}\ln\left|ax^2+bx+c\right| - \dfrac{2an-bm}{a(2ax+b)} & \text{for } 4ac-b^2=0 \end{cases}$$

$$\int \frac{1}{\left(ax^2+bx+c\right)^n}dx = \frac{2ax+b}{(n-1)\left(4ac-b^2\right)\left(ax^2+bx+c\right)^{n-1}} + \frac{(2n-3)2a}{(n-1)\left(4ac-b^2\right)}\int\frac{1}{\left(ax^2+bx+c\right)^{n-1}}dx$$

$$\int \frac{1}{x\left(ax^2+bx+c\right)}dx = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int\frac{1}{ax^2+bx+c}dx$$

3. Integrals of Exponential Functions

$$\int xe^{cx}dx = \frac{e^{cx}}{c^2}(cx-1)$$

$$\int x^2e^{cx}dx = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)$$

$$\int x^n e^{cx}dx = \frac{1}{c}x^n e^{cx} - \frac{n}{c}\int x^{n-1}e^{cx}dx$$

$$\int \frac{e^{cx}}{x}dx = \ln|x| + \sum_{i=1}^{\infty}\frac{(cx)^i}{i\cdot i!}$$

$$\int e^{cx}\ln x\,dx = \frac{1}{c}e^{cx}\ln|x| + E_i(cx)$$

$$\int e^{cx}\sin bx\,dx = \frac{e^{cx}}{c^2+b^2}(c\sin bx - b\cos bx)$$

$$\int e^{cx}\cos bx\,dx = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx)$$

$$\int e^{cx}\sin^n x\,dx = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos bx) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}dx$$

5. Integrals of Trig. Functions

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$\int \sin^3 x dx = \frac{1}{3}\cos^3 x - \cos x$$

$$\int \cos^3 x dx = \sin x - \frac{1}{3}\sin^3 x$$

$$\int \frac{dx}{\sin x} xdx = \ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\cos x} xdx = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\sin^2 x} xdx = -\cot x$$

$$\int \frac{dx}{\cos^2 x} xdx = \tan x$$

$$\int \frac{dx}{\sin^3 x} = -\frac{\cos x}{2\sin^2 x} + \frac{1}{2}\ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2\cos^2 x} + \frac{1}{2}\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \sin x \cos x dx = -\frac{1}{4}\cos 2x$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x$$

$$\int \sin x \cos^2 x dx = -\frac{1}{3}\cos^3 x$$

$$\int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32}\sin 4x$$

$$\int \tan x dx = -\ln|\cos x|$$

$$\int \frac{\sin x}{\cos^2 x} dx = \frac{1}{\cos x}$$

$$\int \frac{\sin^2 x}{\cos x} dx = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| - \sin x$$

$$\int \tan^2 x dx = \tan x - x$$

$$\int \cot x dx = \ln|\sin x|$$

$$\int \frac{\cos x}{\sin^2 x} dx = -\frac{1}{\sin x}$$

$$\int \frac{\cos^2 x}{\sin x} dx = \ln\left|\tan\frac{x}{2}\right| + \cos x$$

$$\int \cot^2 x dx = -\cot x - x$$

$$\int \frac{dx}{\sin x \cos x} = \ln|\tan x|$$

$$\int \frac{dx}{\sin^2 x \cos x} = -\frac{1}{\sin x} + \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\sin x \cos^2 x} = \frac{1}{\cos x} + \ln\left|\tan\frac{x}{2}\right|$$

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \tan x - \cot x$$

$$\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \qquad m^2 \neq n^2$$

$$\int \sin x \cos^n x dx = -\frac{\cos^{n+1} x}{n+1}$$

$$\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1}$$

$$\int \arcsin x dx = x \arcsin x + \sqrt{1 - x^2}$$

$$\int \arccos x dx = x \arccos x - \sqrt{1 - x^2}$$

$$\int \arctan x dx = x \arctan x - \frac{1}{2}\ln\left(x^2 + 1\right)$$

$$\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \frac{1}{2}\ln\left(x^2 + 1\right)$$